FACULTY OF ENGINEERING AND COMPUTER SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING

ENGR-391 NUMERICAL METHODS FOR ENGINEERS

Student's Name: ______________________________

I.D.: ______________________________

Duration 3 hours

- Read carefully all questions
- Write all the steps you need to find the solution
- Please do not write in red (colour used for correction)
- Write all your answers in the foreseen places
- Everything not readable will NOT be corrected
- No questions will be answered during exam : answer to your best knowledge all questions

Points:

1	2	3	4	5	6	7
/5	/10	/5	/5	/10	/10	/5

PROBLEM 1 [5 marks]

We want to compute a positive root of

$$f(x) = x^3 + 4x^2 - 10$$

a) Give an interval of length 1 containing a positive root of f(x).

b) Find the root in your interval from a) to 3 correct significant digits using the bisection algorithm.

a_i	b_i	$\frac{a_i + b_i}{2}$	Signe of $f(a_i)$	Signe of $f(b_i)$

Root of f(x) with 3 correct significant digits:

PROBLEM 2 [10 marks]

Consider following equations

$$3x + 2y + z = 9$$
$$6x + 6y + 4z = 0$$
$$12x + 14y + 11z = -4$$

a) Write the system in Matrix form

b) Decompose the A matrix in LU

Hint: Check your answers at each step!

c) Solve the system using your LU decomposition

Show the different steps of your calculations.

PROBLEM 3 [5 marks]

a) Find the conditioning number of the matrix A in the following system, in function of the parameter $\delta > 0$:

$$\begin{bmatrix} 1 & 1 \\ 1 & 1+\delta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 2+\delta \end{bmatrix}$$

b) Determine the minimal value of the parameter δ such you can solve the system (1) with at least 12 significant digits on a computer using 16 significant digits.

PROBLEM 4 [5 marks]

Compute the following integral:

$$I = \int_{1}^{2} \ln(x)dx$$

using Romberg's method. Therefore complete following table

0.34657359027				
0.37601934919				
0.38369950940				
0.38564390995				

Give your final answer (the value of I) with as many significant digits as you can guarantee to be correct.

PROBLEM 5 [10 marks]

Consider the following initial value problem:

$$\frac{dy}{dx} = y - x^2 + 1$$
$$y(0)=0.5$$

Solve this initial value problem from x=0 to x=2 using h=0.2 using the following Adams-Bashforth 4-step method:

$$w_{i+1} = w_i + \frac{h}{4}[55f(x_i, w_i) - 59f(x_{i-1}, w_{i-1}) + 37f(x_{i-2}, w_{i-2}) - 9f(x_{i-3}, w_{i-3})]$$

Hint: To generate the needed values to start the method you can use the exact solution of the initial value problem which is $y(x) = (1+x)^2 - 0.5e^x$.

x_i	w_i	$f(x_i, w_i)$

PROBLEM 6 [10 marks]

Consider following data.

x_i	0	1	2	3	4	5
y_i	2.1	7.7	13.6	27.2	40.9	61.1

We want to fit following model to the data:

$$y = a_0 + a_1 x + a_2 x^2$$

Write down, in matrix form, the normal equations that have to be solved in order to find the constants a_0, a_1 and a_2 (i.e. give the 3x3 system that have to be solved (no need to solve the system)

PROBLEM 7 [5 marks]

The following table gives a few explicit and implicit multi-step formulas to solve initial value problems.

Order	M-step	Formula	Local Error
1	1	$w_{i+1} = w_i + h\, f(t_i, w_i)$	$(1/2)h^2 y''(\xi)$
2	2	$w_{i+1} = w_i + (h/2)[\, 3 f(t_i, w_i) - f(t_{i-1}, w_{i-1})\,]$	$(5/12)h^3 y'''(\xi)$
3	3	$w_{i+1} = w_i + (h/12)[\, 23 f(t_i, w_i) - 16 f(t_{i-1}, w_{i-1}) + 5 f(t_{i-2}, w_{i-2})\,]$	$(3/8)h^4 y^{iv}(\xi)$
4	4	$w_{i+1} = w_i + (h/24)[\, 55 f(t_i, w_i) - 59 f(t_{i-1}, w_{i-1}) + 37 f(t_{i-2}, w_{i-2}) - 9 f(t_{i-3}, w_{i-3})\,]$	$(251/720)h^5 y^{v}(\xi)$

Order	M-step	Formula	Local Error
2	1	$w_{i+1} = w_i + (h/2)[\, f(t_{i+1}, w_{i+1}) + f(t_i, w_i)\,]$	$-(1/12)\, h^3 y'''(\xi)$
3	2	$w_{i+1} = w_i + (h/12)[\, 5 f(t_{i+1}, w_{i+1}) + 8 f(t_i, w_i) - f(t_{i-1}, w_{i-1})\,]$	$-(1/24)h^4 y^{iv}(\xi)$
4	3	$w_{i+1} = w_i + (h/24)[\, 9 f(t_{i+1}, w_{i+1}) + 19 f(t_i, w_i) - 5 f(t_{i-1}, w_{i-1}) + f(t_{i-2}, w_{i-2})\,]$	$-(19/720)h^5 y^{v}(\xi)$
5	4	$w_{i+1} = w_i + (h/720)[\, 251 f(t_{i+1}, w_{i+1}) + 646 f(t_i, w_i) - 264 f(t_{i-1}, w_{i-1}) + 106 f(t_{i-2}, w_{i-2}) - 19 f(t_{i-3}, w_{i-3})\,]$	$-(3/160)h^6 y^{vi}(\xi)$

a) Based on these tables, give a predictor-corrector scheme in local truncation error 4

b) For the scheme you proposed in a), derive a formula allowing the estimation of the local truncation error Δ in each step.

Derivation:

$\Delta =$