MAT1322 - winter 2014

Assignment 8 (MAT 1322)

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Feedback: Details Report

MAT1322 - winter 2014, Assignment 8 (MAT 1322)
Nicolas Salcedo, 3/25/14 at 12:52 AM

Question 1: *Score 0/1*

A function $f(x)$ is represented by the following Taylor series expansion about the point $x = 1$

$$f(x) = 4 + 2(x-1) - 4(x-1)^2 - 3(x-1)^3 + \cdots$$

Incorrect

What is $f^{(3)}(1)$?

Your Answer: No answer
Correct Answer: -18
Comment:

The coefficient of $(x-1)^3$ is $\frac{f^{(3)}(1)}{3!}$.
Therefore, $f^{(3)}(1) = \text{-}3 \cdot (3!) = \text{-}18$

Question 2: *Score 0/2*

Your response	Correct response
The function	The function
$f(x) = (1+2x)\,e^{2x^2}$	$f(x) = (1+2x)\,e^{2x^2}$
is represented by the MacLaurin series expansion	is represented by the MacLaurin series expansion
$f(x) = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4 + \cdots$	$f(x) = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4 + \cdots$
What are its coefficients?	What are its coefficients?
$c_0 =$ No answer (0%)	$c_0 =$ 1
$c_1 =$ No answer (0%)	$c_1 =$ 2
$c_2 =$ No answer (0%)	$c_2 =$ -2
$c_3 =$ No answer (0%)	$c_3 =$ -4
$c_4 =$ No answer (0%)	$c_4 =$ 2

Original text

Contribute a better translation

Incorrect

Total grade: 0.0×1/5 + 0.0×1/5 + 0.0×1/5 + 0.0×1/5 + 0.0×1/5 = 0% + 0% + 0% + 0% + 0%

Comment:

We have

$$e^{2x^2} = \sum_{n=0}^{\infty} \frac{(\text{-}2x^2)^n}{n!} = 1 + (\text{-}2)x^2 + \frac{(\text{-}2)^2}{2}x^4 + \frac{(\text{-}2)^3}{6}x^6 + \cdots$$

Therefore,

$$(1+2x)e^{2x^2} = \sum_{n=0}^{\infty} \frac{(\text{-}2)^n}{n!}x^{2n} + 2x\sum_{n=0}^{\infty} \frac{(\text{-}2)^n}{n!}x^{2n}$$
$$= \sum_{n=0}^{\infty} \frac{(\text{-}2)^n}{n!}x^{2n} + \sum_{n=0}^{\infty} \frac{2(\text{-}2)^n}{n!}x^{2n+1}$$
$$= (1 + (\text{-}2)x^2 + (2)x^4 + \cdots) + (2x + (\text{-}4)x^3 + \cdots)$$
$$= 1 + 2x + (\text{-}2)x^2 + (\text{-}4)x^3 + (2)x^4 + \cdots$$

Question 3: *Score 0/1*

Your response	Correct response
The MacLaurin series expansions for the two functions f and g begin thus:	The MacLaurin series expansions for the two functions f and g begin thus:
$f(x) = 3 + 3x - 4x^2 + 3x^3 + \cdots$	$f(x) = 3 + 3x - 4x^2 + 3x^3 + \cdots$
$g(x) = 1 - 2x + 4x^2 - 4x^3 + \cdots$	$g(x) = 1 - 2x + 4x^2 - 4x^3 + \cdots$
Their quotient $\frac{f(x)}{g(x)}$ may also be represented by a Maclaurin series expansion.	Their quotient $\frac{f(x)}{g(x)}$ may also be represented by a Maclaurin series expansion.
$\frac{f(x)}{g(x)} = c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots$	$\frac{f(x)}{g(x)} = c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots$
What are its coefficients?	What are its coefficients?

Incorrect

$c_0 =$ No answer (0%)	$c_0 =$ 3
$c_1 =$ No answer (0%)	$c_1 =$ 9
$c_2 =$ No answer (0%)	$c_2 =$ 2
$c_3 =$ No answer (0%)	$c_3 =$ -17

Total grade: 0.0×1/4 + 0.0×1/4 + 0.0×1/4 + 0.0×1/4 = 0% + 0% + 0% + 0%

Comment:

First solution: By dividing $f(x)$ by $g(x)$ we find

$$\begin{aligned} f(x) &= 3 + 3x - 4x^2 + 3x^3 + \cdots \\ &= 3\,(1 - 2x + 4x^2 - 4x^3 + \cdots) \\ &\quad + 9x - 16x^2 + 15x^3 + \cdots \\ &= (3 + 9x)(1 - 2x + 4x^2 - 4x^3 + \cdots) \\ &\quad + (2)x^2 + (\text{-}21)x^3 + \cdots \\ &= (3 + 9x + (2)x^2)(1 - 2x + 4x^2 - 4x^3 + \cdots) \\ &\quad + (\text{-}17)x^3 + \cdots \\ &= (3 + 9x + (2)x^2 + (\text{-}17)x^3 + \cdots)(1 - 2x + 4x^2 - 4x^3 + \cdots). \end{aligned}$$

Therefore,

$$\frac{f(x)}{g(x)} = 3 + 9x + (2)x^2 + (\text{-}17)x^3 + \cdots$$

Second solution: Let us write

$$\frac{f(x)}{g(x)} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

since $f(x) = g(x)\dfrac{f(x)}{g(x)}$ the unknowns c_0 , c_1 , c_2 , c_3 satisfy

$$\begin{aligned} f(x) &= 3 + 3x - 4x^2 + 3x^3 + \cdots \\ &= (1 - 2x + 4x^2 - 4x^3 + \cdots)(c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots) \\ &= c_0 + (c_1 - 2c_0)x + (c_2 - 2c_1 + 4c_0)x^2 + (c_3 - 2c_2 + 4c_1 - 4c_0)x^3 + \cdots \end{aligned}$$

By comparing the coefficients, we deduce

$c_0 = 3,$
$c_1 - 2c_0 = 3$
$c_2 - 2c_1 + 4c_0 = -4$
$c_3 - 2c_2 + 4c_1 - 4c_0 = 3$

In resolving this system, we obtain successively $c_0 = 3$, $c_1 = 9$, $c_2 = 2$ and $c_3 = \text{-}17$

Question 4: *Score 0/1*

Compute

$$\lim_{x \to 0} \frac{e^{3x^2} - 1}{x^2}$$

Incorrect

with the aid of a MacLaurin series expansion.

Your Answer: No answer
Correct Answer: 3
Comment:

For all x real we have

$$e^{3x^2} - 1 = -1 + \sum_{n=0}^{\infty} \frac{(3x^2)^n}{n!} = 3x^2 + (9/2)x^4 + \cdots$$

We deduce

$$\lim_{x \to 0} \frac{e^{3x^2} - 1}{x^2} = \lim_{x \to 0} (3 + (9/2)x^2 + \cdots) = 3.$$

Question 5: *Score 0/2*

Your response	Correct response
Find the first coefficients of the MacLaurin series expansion $\dfrac{1}{(1-3x^2)^3} = c_0 + c_1 x^2 + c_2 x^4 + c_3 x^6 + \cdots$	Find the first coefficients of the MacLaurin series expansion $\dfrac{1}{(1-3x^2)^3} = c_0 + c_1 x^2 + c_2 x^4 + c_3 x^6 + \cdots$

Incorrect

$c_0 =$ No answer (0%)	$c_0 =$ 1
$c_1 =$ No answer (0%)	$c_1 =$ 9
$c_2 =$ No answer (0%)	$c_2 =$ 54
$c_3 =$ No answer (0%)	$c_3 =$ 270

Total grade: 0.0×1/4 + 0.0×1/4 + 0.0×1/4 + 0.0×1/4 = 0% + 0% + 0% + 0%

Comment:

The binomial formula gives us

$$\begin{aligned} \frac{1}{(1+y)^3} &= (1+y)^{-3} \\ &= \sum_{n=0}^{\infty} \binom{-3}{n} y^n \\ &= 1 + (-3)y + \frac{(-3)(-3-1)}{2} y^2 + \frac{(-3)(-3-1)(-3-2)}{3!} y^3 + \cdots \\ &= 1 - 3y + 6y^2 - 10y^3 + \cdots \end{aligned}$$

for all y such that $|y| < 1$

By setting $y = -3x^2$, we deduce that

$$\frac{1}{(1-3x^2)^3} = \sum_{n=0}^{\infty} \binom{-3}{n} (-3x^2)^n$$
$$= 1 - 3(-3x^2) + 6(-3x^2)^2 - 10(-3x^2)^3 + \cdots$$
$$= 1 + 9x^2 + 54x^4 + 270x^6 + \cdots$$

Question 6: *Score 0/2*

Your response	Correct response
Find the first coefficients of the MacLaurin series expansion $\int \sqrt{4+9x^2}dx = C + c_1 x + c_2 x^3 + c_3 x^5 + \cdots$ where C represents the constant of integration. $c_1 =$ No answer (0%) $c_2 =$ No answer (0%) $c_3 =$ No answer (0%)	Find the first coefficients of the MacLaurin series expansion $\int \sqrt{4+9x^2}dx = C + c_1 x + c_2 x^3 + c_3 x^5 + \cdots$ where C represents the constant of integration. $c_1 =$ 2 $c_2 =$ 3/4 $c_3 =$ -81/320

Incorrect

Total grade: 0.0×1/3 + 0.0×1/3 + 0.0×1/3 = 0% + 0% + 0%

Comment:

We have

$$\sqrt{4+9x^2} = 2\sqrt{1+\frac{9}{4}x^2}$$
$$= 2(1+\frac{9}{4}x^2)^{\frac{1}{2}}$$
$$= 2\sum_{n=0}^{\infty} \binom{1/2}{n} (\frac{9}{4}x^2)^n$$
$$= 2(1 + \frac{1}{2}(\frac{9}{4}x^2) + \frac{(\frac{1}{2})(\frac{1}{2}-1)}{2}(\frac{9}{4}x^2)^2 + \cdots)$$
$$= 2 + (9/4)x^2 - (81/64)x^4 + \cdots$$

if $\left|\frac{9}{4}x^2\right| < 1$, that is to say if $|x| < \frac{2}{3}$

We deduce that

$$\int \sqrt{4+9x^2}dx = C + \frac{9}{4}x + \frac{9/4}{3}x^3 - \frac{81/64}{5}x^5 + \cdots$$
$$= C + 2x + (3/4)x^3 - (81/320)x^5 + \cdots$$

if $|x| < \frac{2}{3}$

Question 7: *Score 0/1*

The level curves of the function

$f(x,y) = 16x^2 + 9y^2 - 2$ **Incorrect**

are :

Your Answer:
Correct Answer:
ellipses

Comment:

The equation of the level curve $f(x,y) = C$ is

$$16x^2 + 9y^2 = C + 2\,.$$

It consists of an ellipse passing through the points $(\pm\frac{\sqrt{C+2}}{4}, 0)$ and $(0, \pm\frac{\sqrt{C+2}}{3})$
