Principles of Metal Manufacturing Processes

Solutions Manual

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Chapter 2 - Solidification & Casting Processes

Problem 2.3

Macroporosity caused by improper feeding system/riser design; to eliminate locate risers so that last metal to freeze is in riser; use Chvorinov's rule to evaluate.

Microporosity caused by liquid metal trapped between dendrites, so that liquid is isolated from feeding system; to reduce microsporosity cast eutectic alloys or near eutectic alloys that have short mushy zones and/or use grain refiners during casting.

Gas Porosity caused by large reduction in solubility of hydrogen (primarily) from liquid to solid metal; to reduce, cast in vacuum or 'degas' before casting.

Aspiration caused by decreasing pressure in feeding system which causes formation and injestion of gas bubbles into molten metal; to reduce - keep all vertical sections of feeding system completely full of metal at all times and keep feeding system pressure >1 atm at all locations.

Problem 2.4

formation of dendrites explained in section 2.3.1;
Dendrite formation is usually not desirable because of the increased propensity for micro-porosity, and in addition, the dendritic structure does not have isotropic properties. In alloys, coring is also associated with a dendritic structure.

Problem 2.5

Solidification porosity is likely to occur in the regions of the casting that solidify last, often this will be at or near the centre of the cast section. Solidification porosity typically will have an irregular geometry. Gas porosity can be found throughout the as cast part and, due to internal gas pressure will most often have a spherical shape (solidification porosity will contain a vacuum).

Problem 2.7

(a) $T_M = 660°C$, $T_o = 20°C$, t = 300 seconds,

From Table 2.3 $k_m = 0.60$ W/m·°C, $\rho_m = 1.5$ g/cm^3, $C_m = 1.16$ J/g°C

From equation 2.3:

$$\alpha_{th} = \frac{k_m}{C_m \cdot \rho_m} = \frac{0.6\times10^{-2}}{1.16 \cdot 1.5} = 34.5\times10^{-4}\ cm^2/second$$

α_{th}, the thermal diffusivity, is calculated using the properties of the sand mould, since the temperature gradient is in the sand and the metal is assumed to be at a uniform temperature.

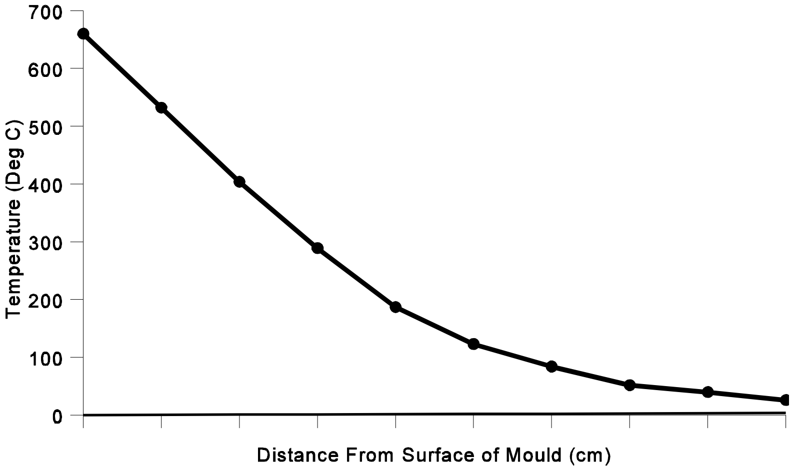
From equation 2.7, with t = 300 s, α_{th} = 34.5x10^{-4} cm^2/s:

$$T(x,t) = T_M + (T_o - T_M)\, erf\left(\frac{x}{2\sqrt{\alpha_{th} t}}\right) = 660°C + (20°C - 660°C)\, erf\left(\frac{x}{2.04}\right)$$

Solve for various *x* with the aid of Table 2.2:

x	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	4.0
T(*x*, 300)	660	532	404	289	187	123	84	52	40	26

Plot these values:



(b) Use equation 2.10, with k_{mould} = 0.6 W/m·°C, C_{mould} = 1.16 J/g°C, ρ_{mould} = 1.5 g/cm^3

$$\vec{J}_{m_{x=0}} = \sqrt{\frac{k_m \cdot C_m \cdot \rho_m}{\pi t}} \cdot (T_M - T_o) = \sqrt{\frac{0.6\times10^{-2}\cdot 1.16 \cdot 1.5}{\pi \cdot 300}} \cdot 640 = 2.1\ W\cdot cm^2$$

(c) Use equation 2.12, with H_f = 396 J/g and ρ=2.7 g/cm^3 from Table 2.3 and J = 2.1 W·cm^2

$$\vec{J} = \rho_c \cdot \Delta H_f \cdot \frac{dS}{dt} \quad \Rightarrow \quad \frac{dS}{dt} = \frac{2.1}{2.7\cdot 396} = 0.00195\ cm/s$$

(d) use equation 2.13: $S = \frac{2}{\sqrt{\pi}} \frac{T_M - T_o}{\rho_c \Delta H_f} \sqrt{k \rho_m C_m t} = 1.18\ cm$

Problem 2.8

(a) thickness = 100 mm, so distance from centre of casting to mould is 50 mm.

$$\frac{V}{A} = \frac{w \times l \times 50mm}{w \times l} = 0.05\ m$$

From Table 2.3: T_M = 1540°C, ΔH_f = 280 J/g, ρ_C = 7.9 g/cm^3,
k_m = 0.6 W/m·°C, ρ_m = 1.5 g/cm^3, C_m = 1.16 J/g·°C

Using equation 2.15:

$$t = \left[\frac{\pi}{4}\left(\frac{\rho_C \Delta H_f}{T_m - T_0}\right)^2 \frac{1}{k_m \rho_m C_P}\right]\left(\frac{V}{A}\right)^2$$

$$= \left[\frac{\pi}{4}\left(\frac{7.9\times10^6\ g/m^3 \times 280\ J/g}{1515^\circ C}\right)^2 \frac{1}{0.6\ J/s\cdot m\cdot{}^\circ C \times 1.5\times10^6 g/m^3 \times 1.16\ J/g\cdot{}^\circ C}\right]\times(0.05\ m)^2$$

$$= 4007\ s \approx 1.1\ hour$$

(b) From Table 2.3: C_{Pl} = 0.77 J/g·°C

$C_{Pl}\cdot\Delta T$ = 0.77 · 60 = 46.2 J/g

So total equivalent latent heat ΔH_f = 46.2 + 280 = 326.2 J/g

so time becomes: $t = 4007\,s\left(\frac{326.2}{280}\right)^2 = 5440\ s \approx 1.5\ hour$

Problem 2.9

V_c = 10 x 10 x 10 = 1000 cm^3

A_c = 6 x 10 x 10 - $\pi d^2/4$ = 600 - $\pi d^2/4$ d = diameter of riser

$$t_c = C\left(\frac{V_c}{A_c}\right)^2 \qquad t_r = 2C\left(\frac{V_r}{A_r}\right)^2$$

factor of 2 is to account the effect of the insulator placed around the riser.

require that:

$$\left(\frac{V_c}{A_c}\right)^2 < 2\left(\frac{V_r}{A_r}\right)^2$$

$$\frac{V_c}{A_c} < 1.41\frac{V_r}{A_r}$$

$$\frac{1000}{600 - \pi\frac{d^2}{4}} < 1.41\frac{d}{4}$$

$$1000 \le 212d - 0.276d^3$$

solve by trial and error to get that d≈ 4.9 cm.

Problem 2.10

use Chvorinov's rule, equation 2.17.

$A_c = \pi 0.05^2$ = 0.00785 m^2

circular casting: $V = \pi r^2 l = \pi 0.05^2\cdot 0.5$ = 0.0039 m^3

$A = 2\pi rl + 2\pi r^2 = 2\pi 0.05\cdot0.5 + 2\pi 0.05^2$ = 0.172 m^2

$$\left.\frac{V}{A}\right|_C^2 = \left(\frac{0.0039}{0.172}\right)^2 = 0.00051$$

From equations for elipse provided: $a = 2b \Rightarrow K = \pi 2b^2$

$$K = A_c = 0.00785\ m^2 \Rightarrow \pi 2b^2 = 0.00785$$

$$b = \sqrt{\frac{0.00785}{2\pi}} = 0.035\ m^2 \Rightarrow a = 0.07\ m$$

$$P = 2\pi\sqrt{\frac{0.07^2 + 0.035^2}{2}} = 0.347\ m$$

$A_e = 0.347 \cdot 0.5 + 2\pi \cdot 0.07 \cdot 0.035 = 0.189\ m^2$

$$\left.\frac{V}{A}\right|_e^2 = \left(\frac{0.0039}{0.189}\right)^2 = 0.000426$$

Ratio of times: $\frac{t_e}{t_c} = \frac{42.5}{51} = 0.83$

The ellipse shape solidifies in 83% of the time required for solidification of the circular shape.

Problem 2.11

advantages of eutectic alloys:	low melting temperature
	minimal microporosity
	short freezing range (short mushy zone)
disadvantages of eutectic alloys:	poor properties
	eutectic structure
	large risers required because of macroporosity

Problem 2.12

(a) *coring*: 2% Si alloy is more prone to coring, since it will have a much larger mushy zone, in which the composition of the solid & liquid phases have a considerably different composition causing segregation between dendritic and interdendritic regions.

(b) *feeding*:the 12%Si alloy is easier to feed since it is closer to the eutectic composition. Therefore, it has a narrower mushy zone, minimizing dendritic growth that can hinder molten metal feeding.

(c) *microporosity*: the 2%Si alloy is more prone to microporosity, since it has a larger mushy zone and consequently longer dendritic arms. This makes it more difficult for liquid to feed into the solidification shrinkage that forms between dendritic arms.

Problem 2.13

Cool metal to as low a temperature as possible (but sufficiently above melting temperature to ensure that no localized solidification occurs prior to casting or in the liquid metal feeding system), since this will keep the concentration of dissolved hydrogen in the molten metal as low as possible. While the metal is still molten, any undissolved hydrogen gas can easily form bubbles and escape from the surface of the molten metal.

Problem 2.14

$$[H] = K\sqrt{p_{H_2}} \qquad \text{so} \qquad \frac{[H]_{atm}}{[H]_{vacuum}} = \frac{K\sqrt{p_{H_{2atm}}}}{K\sqrt{p_{H_{2vacuum}}}}$$

$$p_{H_{2atm}} = 10^{-3}\ atm.$$

$$p_{H_{2vacuum}} = 10^{-9}\ atm.$$

$$\text{so} \qquad \frac{[H]_{atm}}{[H]_{vacuum}} = \sqrt{\frac{10^{-3}}{10^{-9}}} = 1000$$

Problem 2.15

(a) for each 100 g of Mg: 26 - 18 = 8 cm^3 of H_2 rejected from solution upon solidification. volume of 100 g of Mg is 100/1.74 = 57.5 cm^3

$$\text{percent porosity} = \frac{8}{57.5+8}\times 100 = 12.2\%$$

(b)

$$[H] = K\sqrt{p_{H_2}}$$

$$\frac{26\ cm^3}{100\ g} = K\sqrt{1\ atm}$$

$$K = \frac{26}{100\sqrt{1\ atm}} = 0.26$$

$$0.26\sqrt{p_{H_2}} = \frac{18\ cm^3}{100\ g} \quad \Rightarrow \quad p_{H_2} = 0.48\ atm.$$

Problem 2.16

see section 2.7.1

In *gray cast iron*, carbon exists as graphite flakes that have a high aspect ratio. This leads to brittleness due to stress concentrations at the sharp ends of the graphite flakes, but good castability, since the formation of graphite causes a volume expansion during solidification. In *white cast iron*, the carbon exists as iron carbide (Fe_3C) which is inherently brittle. The extended homogenization of *malleable cast iron* causes the formation of graphite nodules, rather than the iron carbide of white cast iron or the graphite flakes of gray cast iron. As a result malleable cast iron has improved strength and ductility. *Nodular cast iron* is a compositionally modified version of gray cast iron. The addition of Mg causes the formation of graphite nodules rather than graphite flakes, thereby improving ductility.

Chapter 3 - Stress and Strain During Deformation

Problem 3.1

Once the strain exceeds the point corresponding to the UTS, necking begins and the local cross sectional area decreases. Hence, the load carrying capacity of the material decreases. In engineering stress the decreasing cross sectional area is not taken into account (always use inital area) and therefore decreases. However, for the true stress calculation, the decreasing area is taken into account and even though the load carrying capacity is decreases the true stress increases because the material that is necking is still strain hardening.

Problem 3.2

e = 25%; $\frac{l_f - l_o}{l_o} = 0.25$ RA = 50%: $\frac{A_o - A_f}{A_o} = 0.5$

so: $l_f - l_o = 0.25 l_o$ and $\varepsilon = \ln\left(\frac{1}{1-RA}\right) = \ln\left(\frac{1}{1-0.5}\right) = 0.693$

$$\frac{l_f}{l_o} = 1.25$$

$$\varepsilon = \ln\frac{l}{l_o} = \ln 1.25 = 0.223$$

Yes necking occurs: because the local strain at the failure point (ε=0.693) is more than the uniform strain ε = 0.223.

Problem 3.3

d_o = 12.8 mm P_{max} = 53.4 kN area ratio A_m/A_o = 0.6

area at P_{max}: $A_m = 0.6 \times \pi \cdot 12.8^2/4 = 77.2\ mm^2 = 7.72 \times 10^{-5}\ m^2$

at P_{max}, true stress is: $\sigma_t = P_{max}/A_m = 53400\ N/7.72 \times 10^{-5}\ m^2 = 691$ MPa

true strain at P_{max}: $\varepsilon = \ln A_o/A_m = \ln 1/0.6 = 0.51$

at P_{max}: ε = n (equation 3.23)
so: n = 0.51

using Hollomon equation 3.13: $\sigma_t = K\varepsilon^n \Rightarrow K = \frac{\sigma_t}{\varepsilon^n} = \frac{691\ MPa}{0.51^{0.51}} = 974\ MPa$

Mean true flow stress, equation 3.19:

$$\sigma_{tm} = \frac{K\varepsilon^n}{n+1} = \frac{974 \cdot 0.51^{0.51}}{1.51} = 457 \ MPa$$

Problem 3.4

(a) $A_o = 5\ cm^2$ $A_{UTS} = 4\ cm^2$ UTS = 250 MPa

From equation 3.11: $\varepsilon_{UTS} = \ln\frac{A_o}{A_{UTS}} = \ln\frac{5}{4} = 0.223$

From equation 3.23: n = ε

From equation 3.28: $\sigma_{t_{UTS}} = UTS \exp(n) = 250 \exp(0.223) = 312.5 \ MPa$

From Hollomon equation 3.13: $\sigma_t = K\varepsilon^n \Rightarrow K = \frac{312.5}{0.223^{0.223}} = 437 \ MPa$

(b) $t_i = 2$ cm $t_f = 1.8$ cm

ε = ln 2/1.8 = 0.11

Volume, V = 20 cm x 5 cm x 2 cm = 200 cm^3 = $2x10^{-4}$ m^3

Ideal work of deformation, from equation 3.33:

$$W = \frac{KV\varepsilon^{n+1}}{n+1} = \frac{437\times10^6 \cdot 2\times10^{-4} \cdot 0.11^{1.223}}{1.223} = 4.8 \ kJ$$

(c) Calculation of part (b) *under*estimates the actual power, since it does not take into account friction or redundant work.

(d) Yield strength will be approximately the maximum stress reached during the prior deformation process:

$$\sigma_{\varepsilon=0.11} = 437 \cdot 0.11^{0.223} = 264 \ MPa$$

approximate yield stress after deformation is 264 MPa.

Problem 3.5

h_o = 1 cm h_f = 0.3 cm, $\dot{\varepsilon}$ = 1x10^{-2} s^{-1}

$$\varepsilon = \ln\frac{h_o}{h_f} = \ln\frac{1}{0.3} = 1.2$$ and for <u>constant</u> strain rate $$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{\Delta\varepsilon}{\Delta t}$$

so $$\Delta t = \frac{\Delta\varepsilon}{\dot{\varepsilon}} = \frac{1.2}{0.01} = 120 \ seconds$$

Problem 3.6

at UTS ε=n, therefore, $$\sigma_{t_{UTS}} = 50 \cdot 0.25^{0.25} = 35.4 \ ksi$$

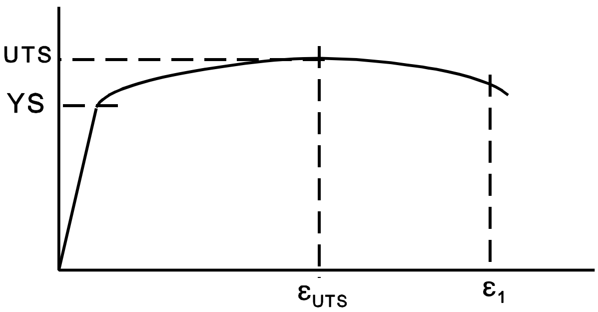
since: $$\sigma_t = \sigma_a \exp(n) \Rightarrow \sigma_a = \frac{\sigma_t}{\exp(n)} = \frac{35.4}{\exp(0.25)} = 27.5 \ ksi$$

calculate difference between calculated & measured UTS:

$$\Delta\% = \frac{28 - 27.5}{28} \times 100 = 1.8\%$$

so K & n determined from experimental plot were accurate to within 1.8%.

Problem 3.7



$\varepsilon = \ln\frac{l}{l_o}$ is only valid up to the ε_{UTS}, the region in which deformation is homogeneous. This is because ε is calculated based on the sample length and it is assumed that deformation is uniform along the whole length, i.e. no necking occurs (necking begins at ε_{UTS}). $\varepsilon = \ln\frac{A_o}{A}$ is valid up to about ε_1, or somewhat less than ultimate failure. At $\varepsilon > \varepsilon_1$, calculation of the true strain using the area ratio becomes inaccurate due to internal voids which reduce the cross sectional area & necking causes lateral stresses, so the loading in no longer uniaxial.

Problem 3.8

D_1 = 5 mm
D_2 = 4 mm σ = 490 MPa
D_3 = 3 mm σ = 603 MPa
D_4 = 2 mm σ = ?

$$\varepsilon_1 = \ln\frac{5^2}{4^2} = 0.45$$

$$\varepsilon_2 = \ln\frac{4^2}{3^2} = 0.57$$

$$\varepsilon_1 + \varepsilon_2 = 1.02$$

$$\varepsilon_3 = \ln\frac{3^2}{2^2} = 0.81$$

$$\sigma = K\varepsilon^n$$
$$\ln\sigma = \ln K + n\ln\varepsilon$$

at ε_1: $\ln 490 = \ln K + n\ln 0.45$

at ε_1+ε_2: $\ln 603 = \ln K + n\ln 1.02$

two equations and two unknowns, solve to get: *n* = 0.26 and *K* = 599.9 MPa

$$\sigma = 599.6\,\varepsilon^{0.26}$$
$$at\ \varepsilon_3:\quad \sigma = 599.9\,(0.81)^{0.26} = 567.9\ MPa$$

so yield strength at the 2 mm diameter is about 568 MPa.

Problem 3.9

d_o = 10 mm d_1 = 8 mm

if it assumed that volume remains constant, then from equation 3.10:

$$\frac{l_o}{l_1} = \frac{A_1}{A_o} = \frac{8^2}{10^2} = 0.64$$

using equation 3.4 calculate engineering strain: $$e = \frac{l_1}{l_o} - 1 = \frac{1}{0.64} - 1 = 0.56$$

True Strain: $$\varepsilon = \ln\frac{d_o^2}{d_1^2} = \ln\frac{10^2}{8^2} = 0.44$$

Assumption is that deformation is homogeneous, i.e. no necking occurs.

Problem 3.10

From example E3.3 it is known: $\varepsilon_1+\varepsilon_2+\varepsilon_3=0$.

$$\varepsilon_1 = \ln\frac{89}{76} = 0.158 \qquad \varepsilon_2 = \ln\frac{11.9}{12.7} = -0.065 \qquad \varepsilon_3 = \ln\frac{7.1}{7.6} = -0.068$$

$\varepsilon_1+\varepsilon_2+\varepsilon_3$ = 0.158-0.065-0.068 = 0.025
To be completely accurate the three strains should sum to zero not 0.02, but probably the measurements reasonably accurate.

Problem 3.11

d = 10 mm, P_{max} = 5 kN, RA = 20%

$$A_o = \pi \frac{10^2}{4} = 78.5\ mm^2$$

$$A_{P_{max}} = 78.5 \cdot 0.8 = 62.8\ mm^2$$

$$\sigma_{t_{P_{max}}} = \frac{5000\ N}{62.8\times10^{-6}\ m^2} = 79.6\ MPa$$

$$\varepsilon_{P_{max}} = \ln\left(\frac{1}{1-RA}\right) = \ln\left(\frac{1}{1-0.2}\right) = 0.223$$

since the P_{max} corresponds to the UTS then $n = \varepsilon_{Pmax} = 0.223$.

Calculate K: $$\sigma = K\varepsilon^n \quad \Rightarrow \quad K = \frac{\sigma_{t_{P_{max}}}}{\varepsilon_{P_{max}}{}^n} = \frac{79.6\ MPa}{0.223^{0.223}} = 111\ MPa$$

At $\varepsilon_2 = \frac{1}{2}n = 0.112$: $$\sigma_2 = 111\ (0.112)^{0.223} = 68.1\ MPa$$

$$\varepsilon_2 = \ln\left(\frac{1}{1-RA_2}\right)$$

$$\exp(\varepsilon_2) = \frac{1}{1-RA_2}$$

$$1.12 = \frac{1}{1-RA_2}$$

$$RA_2 = 0.107$$

$$A_2 = A_o(1-RA_2) = 78.5(1-0.107) = 70.1\ mm^2$$

$$P_2 = 70.1\ mm^2 \times 68.1\ MPa = 4774\ N$$

Problem 3.12

$\dot{\varepsilon} = 10^{-3}\ s^{-1}$, $$\varepsilon_{2/3} = \ln\frac{1}{2/3} = 0.41$$

since strain rate is constant, therefore: $$\Delta t_{2/3} = \frac{\varepsilon}{\dot{\varepsilon}} = \frac{0.41}{10^{-3}} = 410\ s$$

$$\varepsilon_{1/3} = \ln\frac{1}{1/3} = 1.1 \quad \Rightarrow \quad \Delta t_{1/3} = \frac{1.1}{10^{-3}} = 1100\ s$$

Problem 3.13

ratio of stresses is 3, and ratio of strain rates is 10, therefore:

$$\sigma = C\dot{\varepsilon}^m$$
$$\frac{3}{1} = \left(\frac{10}{1}\right)^m$$
$$m = 0.47$$

If the criterion of m>0.5 is used for superplasticity (page 3 - 19) then the material is not superplastic.

Problem 3.14

Hot Deformation equation is: $\sigma_t = 20\times10^6\,\dot{\varepsilon}^{0.5} = 20\times10^6\left(\frac{dh}{h\,dt}\right)^{0.5}$

$$F = \sigma A = 20\times10^6\left(\frac{dh}{hdt}\right)^{0.5}\frac{V}{h}$$
$$F(dt)^{0.5} = 20\times10^6 V\cdot\frac{dh^{0.5}}{h^{1.5}}$$
$$F^2\cdot dt = \left(20\times10^6\right)^2\cdot V^2\cdot\frac{dh}{h^3}$$
$$F^2\int_0^t dt = \left(20\times10^6\right)^2\cdot V^2\int_h^{h_o}\frac{dh}{h^3}$$
$$F^2\cdot t = \left(20\times10^6\right)^2\cdot V^2\left[\frac{1}{2}\left(\frac{1}{h_o^2}-\frac{1}{h^2}\right)\right]$$

t = 20 second, F = 5000 N, h_o = 0.1 m, V = 50x10^{-6} m^3

substituting these values into the last equation gives h = 3.3 cm.

Problem 3.15

(a) l_o = 30 cm, A = 1cm^2 = 0.0001 m^2, m = 10 kg

Force on rod: F = 10 kg x 9.81 m/s^2 = 98.1 N

True Stress in Rod, σ_t = F/A = 98.1 N / 0.0001 m^2 = 0.981 MPa

Use equation given in question to calculate strain rate:

$$\left(\frac{d\varepsilon}{dt}\right)^{½} = \frac{\sigma_t}{200} = \frac{0.981}{200}$$

$$\frac{d\varepsilon}{dt} = 2.4\times10^{-5}\ s^{-1}$$

after 1 hour or 3600 seconds:

$$\Delta\varepsilon = \dot{\varepsilon}\cdot t = 2.4\times10^{-5}\times 3600 = 0.086$$

Note: can only use $\Delta\varepsilon = \dot{\varepsilon}$t because of assumption of negligible change in area. This assumption implies a small strain and therefore it can be assumed that $\dot{\varepsilon}$ is at least nearly constant over the small ε applied.

$$0.086 = \ln\frac{l}{30}$$

$$l = 32.7\ cm$$

(b) deformation is superplastic since $m = 0.5$.

(c)
- i grain size < 10 μm
- ii temperature > 0.4 T_M (absolute temperature)
- iii well controlled slow strain rate ≈10^{-4} s^{-1}
- iv grain size stable at high temperature

Problem 3.16

(a) $l_o = 5$ cm, $D_o = 1.28$ cm, Y.S. = 345 MPa, UTS = 485, $e_f = 18\%$

Load at yielding:

$$Y.S. = \frac{load_{YS}}{A_o}$$

$$load_{YS} = 345\times10^6\ \frac{N}{m^2}\times\pi\cdot\frac{(1.28\times10^{-2}\ m)^2}{4} = 44400\ N$$

Load at UTS:

$$load_{UTS} = 485\times10^6\ \frac{N}{m^2}\times1.28\times10^{-4}m^2 = 62400\ N$$

(b) at UTS, $e_{UTS} = 15\%$

$$l_{UTS} = l_o(1+e_{UTS}) = 5\ cm(1.15) = 5.75\ cm$$

from volume constancy:

$$l_oA_o = l_{UTS}A_{UTS}$$

$$A_{UTS} = \frac{5\cdot1.28}{5.75} = 1.11\ cm^2$$

$$D_{UTS} = \sqrt{\frac{A_{UTS}\cdot4}{\pi}} = \sqrt{\frac{1.11\cdot4}{\pi}} = 1.19\ cm$$

(c)

$$\sigma_{t_{UTS}} = \frac{62400\ N}{0.000111\ m^2} = 562\ MPa$$

(d) at strains $\leq e_{UTS}$:

$$\varepsilon = \ln(e+1)$$
$$\varepsilon_{UTS} = \ln 1.15 = 0.14$$

at UTS: ε = *n*=0.14 and σ_t = 562 MPa, so $$K = \frac{\sigma_t}{\varepsilon^n} = \frac{562}{0.14^{0.14}} = 740 \ MPa$$

Chapter 4 - Bulk Deformation Processes

Problem 4.1

Advantages of hot forming - greater deformation strains possible
- lower forming energy

Disadvantages of hot forming - surface not as good as after cold forming
- lower strength than cold formed products

Problem 4.2

h_o = 4 cm, d_o = 2 cm, ε_f = 1.5, μ = 0.1, Y.S. = 800 MPa

$$\varepsilon_f = \ln\frac{h_o}{h_f} \quad \Rightarrow \quad \frac{h_o}{h_f} = e^{1.5}$$

$$h_f = \frac{4}{e^{1.5}} = 0.89$$

using volume constancy, calculate the mean final cross sectional area, A_m and diameter, d_m:

$$A_o h_o = A_m h_f$$

$$A_m = \frac{A_o h_o}{h_f}$$

$$d_m^2 = \frac{d_o^2 h_o}{h_f} = \frac{4 \cdot 4}{0.89} = 17.9 \ cm^2 \qquad \frac{d_m}{h_f} = \frac{4.24}{0.89} = 4.76$$

$$d_m = 4.24 \ cm$$

Use Figure 4.7 to estimate $Q_a \approx 1.2$ and use Hollomon equation given in question to calculate the true stress at the end of the forging stroke:

$$\sigma_{t_f} = 400 \times 1.5^{0.5} = 489 \ MPa$$

the actual workpiece-platens interface stress at the end of the forging stroke will be:

$$\sigma_a = Q_a \sigma_{t_f} = 1.2 \times 489 = 587 \ MPa$$

since the yield stress of the platens is 800 MPa (>587 MPa) the cylinder fractures prior to the platens yielding.

Problem 4.3

(a) d_o or D = 10 cm, l = 500 mm, d_f = 5 cm,
from Table 3.1 for AA-1100, K = 140 MPa, n = 0.25 and Y.S. = 35 MPa

extrusion strain: $$\varepsilon = \ln\frac{d_o^2}{d_f^2} = \ln\frac{10^2}{5^2} = 1.39$$

also from equation 4.22 extrusion ratio R_e = 4
mean true stress during extrusion (from equation 3.32):

$$\sigma_{tm} = \frac{K\varepsilon^n}{n+1} = \frac{140\cdot 1.39^{0.25}}{1.25} = 121.5\ MPa$$

For estimation of frictional component estimate that τ = ½Y.S. = 17.5 MPa (Tresca's criterion). Use this estimate because at frictional interface between billet and chamber wall, the billet is initially undeformed and once the load reaches the shear yield strength internal deformation of the billet will occur due to sticking friction conditions.

use equation 4.27 to calculate extrusion pressure:

$$p_p = \sigma_{tm}\left[0.8 + 1.2\ln R_e\right] + \frac{\tau 4l}{D} = 121.5\left[0.8 + 1.2\cdot 1.39\right] + \frac{17.5\times 4\times 0.5}{0.1} = 650\ MPa$$

extrusion force = 650 MPa x $\pi(0.1)^2/4 \approx$ 5.1 MN

(b) extrusion ram will not deform since it has a yield strength of 1000 MPa and the stress at the ram billet interface is 650 MPa.

Problem 4.4

(a) true strain for one pass: $$\varepsilon = \ln\left(\frac{1}{1-RA}\right) = \ln\left(\frac{1}{1-0.35}\right) = 0.43$$

since true strains are additive, therefore the strain for seven passes = 7 x 0.43 = 3.01

(b) $$\varepsilon = \ln\frac{l}{l_o} \quad \Rightarrow \quad l = e^{3.01} l_o = 20.3 l_o$$

(c) $$e = \frac{\Delta l}{l_o} = \frac{20.3 l_o - l_o}{l_o} = 19.3$$

Problem 4.5

(a) h_o = 3 mm, h_f = 2.2 mm, D = 0.25 m, w = 1.5 m, μ = 0.05, SAE 1045

from Table 3.1 for SAE 1045 steel K = 950 MPa and n = 0.12

$$\sigma_{tm} = \frac{950 \cdot \varepsilon^{0.12}}{1.12} = \frac{950 \cdot \left(\ln \frac{3}{2.2}\right)^{0.12}}{1.12} = 750 \;\; MPa$$

$$L = \sqrt{R(h_o - h_1)} = \sqrt{0.125(0.003 - 0.0022)} = 0.01 \;\; m$$

mean height during rolling $h = \frac{3 + 2.2}{2} = 2.6 \;\; mm$

$$\frac{h}{L} = \frac{0.0026}{0.10} = 0.26 < 1$$

therefore, process is plane strain deformation, use Q_p from figure 4.9.

$$\frac{L}{h} = 3.85, \quad \mu = 0.05 \quad \Rightarrow \quad Q_p \approx 1.0$$

$$F_S = L w Q_p \sigma_{tm} = 0.01 \times 1.5 \times 1.0 \times 750 = 11.3 \;\; MN$$

(b) $$Torque = F_S \cdot L = 11.3\, MN \times 0.01\, m = 113 \;\; kN \cdot m$$

$$\omega = \frac{4\; m/s}{0.125\; m} = 32\; rad/s$$

$$Power = \omega \cdot T = 32 \frac{rad}{s} \times 113 \times 10^3\; N \cdot m = 3.6\, MW \approx 4800 \;\; horsepower$$

(c) Use a 4-hi mill, because of the wide strip (1.5 m) and small diameter rolls. The large separating force would otherwise cause bending of work rolls which would produce sheet with severe camber. The stiff back up rolls of a four hi mill will reduce the magnitude of the camber by minimizing the work roll deflection.

Problem 4.6

(a) Power = 1000 hp = 746 kW, ω = 100 rpm = 10.5 rad/s, R = 0.3 m, Δh = 0.005 m
From Table 3.1 for AA-1100, K = 140 MPa and n = 0.25

$$Torque = \frac{Power}{\omega} = \frac{746{,}000\; W}{10.5\; rad/s} = 71\; kN \cdot m$$

T = F·L, and F = $Lw\sigma_{tm}$, so T = $L^2 w \sigma_{tm}$

calculate σ_{tm}: $$\sigma_{tm} = \frac{K\varepsilon^n}{n+1} = \frac{140\times10^6}{1.25}\left[\ln\frac{25}{20}\right]^{0.25} = 77\ MPa$$

calculate contact length: $$L = \sqrt{R\Delta h} = \sqrt{0.3\times0.005} = 0.038\ m$$

$$w = \frac{T}{L^2\sigma_{tm}} = \frac{71\ kN\cdot m}{0.038^2\times77\ MPa} = 0.64\ m$$

The maximum width that can be rolled at full power is 64 cm.

(b) deformation is homogeneous, since friction coefficient is very low. also, h/L = 5 /38 = 0.013 < 1.

Problem 4.7

Small diameter rolls reduce the contact area in the roll gap and thus, reduce the roll forces. However, as the width of the strip increases, small diameter rolls tend to deflect more and therefore, often need to be supported by larger back up rolls. Although not mentioned in the text, another disadvantage of smaller rolls is the 'refusal' of the strip by the rolls, i.e., relatively thick strip cannot be fed into small diameter rolls. It can be shown that for each roll radius there is a minimum strip exit thickness that is achievable. Thus, the smaller the roll, the thinner the strip that may be produced. For the rolling of light gauges (e.g. aluminium foil) this is a major factor determining the roll radius.

Problem 4.8

(a) $A_o = 20\ mm^2$, $A_f = 12\ mm^2$, P = 3.5 kN

at UTS $$\varepsilon = \ln\frac{20}{12} = 0.51$$

at UTS, ε = n = 0.51, and $$\sigma_{t_{UTS}} = \frac{3.5\ kN}{12\ mm^2} = 292\ MPa$$

calculate K using Hollomon equation: $$K = \frac{292}{0.51^{0.51}} = 411.6\ MPa$$

for rolling operation given: h_o = 1 mm and h_f = 0.8 mm, so $$\varepsilon_r = \ln\frac{1}{0.8} = 0.22$$

volume of sheet rolled = 10 cm x 2 cm x 1 cm = 2 cm^3

Ideal work of deformation $$W_i = \frac{VK\varepsilon^{n+1}}{n+1} = \frac{2\ cm^3\cdot411.6\ MPa\cdot0.22^{1.51}}{1.51} = 55\ N\cdot m$$

This calculation underestimates the actual deformation work since friction has not been taken into account.

(b) w = 0.5 m, v = 50 m/min, D = 500 mm, μ = 0.1

$$\sigma_{tm} = \frac{K\varepsilon_r^n}{n+1} = \frac{411 \cdot 0.22^{0.51}}{1.51} = 126\ MPa$$

$$h_{av} = \frac{1+0.8}{2} = 0.9\ mm$$

$$L = \sqrt{R \cdot \Delta h} = \sqrt{250 \cdot 0.2} = 7.1\ mm \quad \Rightarrow \quad \frac{h}{L} = \frac{0.9}{7.1} = 0.126 < 1$$

rolling is homogeneous and from Figure 4.9, $Q_P \approx 1.8$

$$F = L w Q_P \sigma_{tm} = 7.1\ mm \cdot 500\ mm \cdot 1.8 \cdot 126\ MPa = 0.8\ MN$$

$$T = F \cdot L = 0.8\ MN \cdot 0.0071\ m = 5.7\ kN \cdot m \qquad \omega = \frac{50 m/\min}{0.25\ m} = 3.33 rad/s$$

$$Power = 5.7\ kN \cdot m \times 3.3\ rad/s = 18.8\ kW \approx 25\ horsepower$$

(c) to reduce power required:
- slow down rolling speed,
- decrease size of work rolls,
- roll reduction in two passes rather than one pass.

Problem 4.9

P = 100 kW, ω = 60 rpm, h_o = 4 mm, F_r = 1000 kN, μ = 0.1

(a) Power = ωT (equation 4.40), ω = 60 rpm = 2π rad/s

$$Torque = \frac{100000\ N \cdot m/s}{2\pi/s} = 15915\ N \cdot m$$

Torque = $F_r L$ (equation 4.39)

$$L = \frac{Torque}{F_r} = \frac{15915\ N \cdot m}{1{,}000{,}000\ N} = 16\ mm$$

equation given: $(h_o - h_1)_{max} = \mu^2 R$ and $L = \sqrt{R(h_o - h_1)}$

combine to get: $$L = \sqrt{\frac{(h_o - h_1)}{\mu^2}(h_o - h_1)} = \frac{h_o - h_1}{\mu}$$

so $h_o - h_1 = \mu L = 0.1 \times 16\ mm = 1.6\ mm$,
therefore $h_1 = 4 - 1.6 = 2.4\ mm$

(b) $$\varepsilon = \ln\frac{4}{2.4} = 0.51 \quad \text{and} \quad \sigma_{tm} = \frac{K\varepsilon^n}{n+1} = \frac{200\,(0.51)^{0.18}}{1.18} = 150\ MPa$$

$$\frac{L}{h} = \frac{16\,mm}{\frac{4+2.4}{2}} = 5$$ so from Figure 4.9, $Q_P \approx 1.2$

$$Torque = \frac{100{,}000\ W}{2\pi} = 15923\ N{\cdot}m \quad \text{and} \quad F_r = \frac{15923\ N{\cdot}m}{0.016\ m} = 995{,}187\ N$$

Note that since rolling at full power and maximum speed, the rolling force is very close to the maximum allowable of 1,000 kN.

$F_r = L{\cdot}w{\cdot}Q_P{\cdot}\sigma_{tm}$

so: $$w = \frac{F_r}{L\,Q_P\,\sigma_{tm}} = \frac{995{,}187\ N}{0.016\ m \times 1.2 \times 150 \times 10^6\ Pa} = 0.34\ m$$

Chapter 5 - Sheet Forming Processes

Problem 5.1

High *n* is desirable, since this avoids localized deformation and necking begins when $\varepsilon = n$. So a higher *n* allows a larger strain to be applied before failure.

Problem 5.2

$\varepsilon_f = 0.3$, t = 0.5 mm

calculate reduction in area according to equation 3.12:

$$\varepsilon_f = \ln\left(\frac{1}{1 - RA}\right) \quad \Rightarrow \quad RA = 1 - \frac{1}{e^{\varepsilon_f}} = 0.26$$

since RA > 0.2 use equation 5.6 to estimate minimum radius:

$$R_m = t\frac{(1 - RA)^2}{2RA - RA^2} = 0.5\frac{(1 - 0.26)^2}{2 \cdot 0.26 - 0.26^2} = 0.60 \; mm$$

Problem 5.3

UTS = 200 MPa, l = 500 mm, t = 0.3 mm,

From Figure 5.8: for wiping die K = 0.33, w = 30.3 mm,
for v-die K = 1.33, w = 50 mm.

Using equation 5.8: $$F = \frac{K \cdot l \cdot t^2 \cdot UTS}{w}$$

so for v - die F ≈ 239 N and for the wiping die, F ≈ 98 N

Problem 5.4

The alloy that is best for sheet forming will have the highest *r* value. A larger *r* value indicates that the material is more resistant to thinning than deformation in the plane of the sheet. That is: planar strains are large compared to through thickness strains. For sheet metal forming it is desirable to have low through thickness strains to minimize local necking.

		$\varepsilon_w = \ln w/w_o$	$\varepsilon_t = \ln t/t_o$	$r = \varepsilon_w/\varepsilon_t$
Alloy A				
	0	-0.143	-0.083	1.72
	45	-0.174	-0.073	2.38
	90	-0.113	-0.139	0.81
Alloy B				
	0	-0.068	-0.198	0.34
	45	-0.083	-0.186	0.45
	90	-0.055	-0.248	0.22

$$r_m = \frac{r_0 + r_{90} + 2r_{45}}{4}$$

$$r_{m_{Alloy\ A}} = \frac{1.72 + 0.81 + 2 \cdot 2.38}{4} = 1.82$$

$$r_{m_{Alloy\ B}} = \frac{0.34 + 0.22 + 2 \cdot 0.45}{4} = 0.37$$

use Alloy A for sheet forming.

Problem 5.5

Stretch forming requires high *n*, so choose AA-1100. For stretch forming deformation is limited by the onset of necking, which occurs when ε = *n*, so maximize *n*.

Problem 5.6

r_i = 5 mm, r_0 = 7.5 mm, r_{90} = 3.8 mm.

strain in the rolling direction, $\varepsilon_0 = \ln\frac{7.5}{5} = 0.405$

strain in the transverse direction, $\varepsilon_{90} = \ln\frac{3.8}{5} = -0.27$

from example E3.3: $\varepsilon_0 + \varepsilon_{90} + \varepsilon_t = 0$

$$\varepsilon_t = -0.40 + 0.27 = -0.13$$

$$r_{90} = \frac{\varepsilon_{90}}{\varepsilon_t} = \frac{0.27}{0.13} = 2.1$$

Problem 5.7

(a) $w_o = 10$ mm, $w_1 = 9$ mm, $t_o = 1$ mm, $t_1 = 0.93$ mm.

$$\varepsilon_w = \ln\frac{w_1}{w_o} = \ln\frac{9}{10} = -0.105 \qquad \varepsilon_t = \ln\frac{t_1}{t_o} = \ln\frac{0.93}{1} = -0.073$$

(b) $$\varepsilon_l + \varepsilon_t + \varepsilon_w = 0$$
$$\varepsilon_l = 0.105 + 0.073 = 0.178 \qquad \Rightarrow \quad r = \frac{\varepsilon_w}{\varepsilon_t} = \frac{0.105}{0.073} = 1.44$$

Problem 5.8

(a) no necking will occur, strain is interrupted prior to UTS.

(b) for alloy A: $\varepsilon_l = \ln\frac{60}{50} = 0.18$ $\qquad \varepsilon_w = \ln\frac{5.2}{6} = -0.14$

$$\varepsilon_l + \varepsilon_t + \varepsilon_w = 0$$
$$\varepsilon_t = -0.04 \qquad \Rightarrow \quad r = \frac{\varepsilon_w}{\varepsilon_t} = \frac{-0.14}{-0.04} = 3.5$$

for alloy B: $\varepsilon_l = \ln\frac{62}{50} = 0.22$ $\qquad \varepsilon_w = \ln\frac{5.4}{6} = -0.11$

$$\varepsilon_l + \varepsilon_t + \varepsilon_w = 0$$
$$\varepsilon_t = -0.11 \qquad \Rightarrow \quad r = \frac{\varepsilon_w}{\varepsilon_t} = \frac{-0.11}{-0.11} = 1$$

choose alloy A, because r value is higher.

Problem 5.9

'Ears' are caused by the development of a preferred crystallographic orientation of grains during cold rolling (or other deformation processes), because slip systems active during deformation will tend to become oriented parallel to the rolling direction. Therefore, properties of sheet will vary with respect to the rolling direction, causing ears when drawn into cylindrical cup shapes.

Problem 5.10

$r_o = 1.5$, $w_o = 10$ mm, $t_o = 1$ mm, $l_o = 20$ cm, $e = 15\%$

$l_1 = 20+(0.15)20 = 23$ cm $\qquad \varepsilon_l = \ln\frac{23}{20} = 0.14$

from volume constancy: $\varepsilon_t + \varepsilon_w + \varepsilon_l = 0$

and so, also: $\dfrac{\varepsilon_t}{\varepsilon_t} + \dfrac{\varepsilon_w}{\varepsilon_t} + \dfrac{\varepsilon_l}{\varepsilon_t} = 0$

$$\frac{\varepsilon_t}{\varepsilon_t} = 1, \qquad \varepsilon_l = 0.14, \qquad \frac{\varepsilon_w}{\varepsilon_t} = r_o = 1.5$$

$$\therefore \quad 1 + 1.5 + \frac{0.14}{\varepsilon_t} = 0 \qquad \Rightarrow \qquad \varepsilon_t = -0.056$$

change in thickness: $\varepsilon_t = -0.056 = \ln\dfrac{t_1}{t_o}, \qquad t_o = 1\ mm$

$$t_1 = \exp(-0.056) = 0.95\ mm$$

change in width: $r_o = 1.5 = \dfrac{\varepsilon_w}{\varepsilon_t} = \dfrac{\varepsilon_w}{-0.056} \qquad \Rightarrow \qquad \varepsilon_w = -0.084$

$$\varepsilon_w = -0.084 = \ln\frac{w_1}{w_o} = \ln\frac{w_1}{10} \qquad \Rightarrow \qquad w_1 = 9.19\ mm$$

new width = 9.19 mm, new thickness = 0.95 mm, new length = 23 cm.

Problem 5.11

(a) r = 20 mm, h = 60 mm, t = 0.5mm
assume volume constancy during deep drawing:
volume of bottom of deep drawn cup = $\pi r^2 t = \pi \cdot 20^2 \cdot 0.5 = 628.3\ mm^3$
volume of sides of deep drawn cup = $2\pi rth = 2\pi \cdot 20 \cdot 0.5 \cdot 60 = 3770\ mm^3$
total volume = $4398.3\ mm^2$

since thickness of bottom of cup is not altered during deep drawing (see section 5.6), the thickness of the blank from which the cup is drawn is 0.5 mm. Therefore the blank diameter r_b is:

$$r_b^2 = \frac{V}{\pi \cdot t} = \frac{4398.3}{\pi \cdot 0.5} \qquad \Rightarrow \qquad r_b = 52.9\ mm$$

diameter of the blank is about 106 mm.

(b) use equation 5.13 with d_o = 106 mm, d_p = 40 mm, UTS = 150 MPa, t = 0.5 mm

$$F_d = \pi d_p t_{avg} \cdot UTS \left[\frac{d_o}{d_p} - 0.7\right] = \pi \cdot 40 \cdot 0.5 \cdot 150 \left[\frac{106}{40} - 0.7\right] = 18.4 kN$$

(c) use equation 5.2 with: t = 0.5 mm, $l = \pi \cdot 106 = 333$ mm.

$$F_S = 0.7 \cdot UTS \cdot t \cdot l = 0.7 \cdot 150 \cdot 0.5 \cdot 333 = 17.5\ kN$$

(d) For HCP crystal structures with a high c/a ratio slip will occur on the basal plane (0001). Since there are very few slip systems, the metal will developa strong preferred orientation, leading to a low limiting draw ratio (LDR) and high earing. Consequently, the Zn deep drawing operation may not be successful.

(e) diameter of blank for impact extrusion = 40+2·0.5 = 41 mm.

area of punch - $A_P = \pi \frac{40^2}{4} = 1256\ mm^2$

inital blank area - $A_o = \pi \frac{41^2}{4} = 1320\ mm^2$

final cross sectional area - $A_f = \pi \frac{41^2 - 40^2}{4} = 63.6\ mm^2$

$$Re = \frac{A_o}{A_f} = \frac{1320}{63.6} = 20.7 \qquad \varepsilon = \ln 20.7 = 3.03$$

from question: K = 200 MPa and n = 0.15

$$\sigma_{tm} = K\frac{\varepsilon^n}{n+1} = 200 \cdot \frac{3.03^{0.15}}{1.15} = 205\ MPa$$

$$Q_e = 0.8 + 1.2 \ln Re = 4.44$$

$$p_e = Q_e \cdot \sigma_{tm} = 4.44 \times 205 = 909\ MPa$$

$$F_P = p_e \times A_P = 909 \times 1256 = 1142\ kN$$

(f) d = 41 mm, from part (a) volume, V = 4398 mm^3

$$V = \pi \frac{d^2}{4} \cdot t \quad \Rightarrow \quad t = \frac{4V}{\pi d^2} = \frac{4 \cdot 4398}{\pi \cdot 41^2} = 3.3\ mm$$

(g) again using equation 5.2 with t = 3.3 mm and l = π·41 = 129 mm

$$F_S = 0.7 \cdot UTS \cdot t \cdot l = 0.7 \cdot 150 \cdot 3.3 \cdot 129 = 44.7\ kN$$

(h) The force for impacting is much greater than for deep drawing since impacting is a bulk deformation process (whole volume is deformed), while for deep drawing only local deformation occurs in a small volume of the workpiece.

Problem 5.12

(a) To calculate force to shear one slug use equation 5.2. To calculate UTS:
at UTS ε = n and for AA-1100 from Table 3.1: K = 140 MPa and n = 0.25.
True stress at UTS: $\sigma_{t_{UTS}} = 140 \cdot 0.25^{0.25} = 99\ MPa$

from equation 3.28: $UTS = \frac{\sigma_{t_{UTS}}}{\exp(n)} = \frac{99}{\exp(0.25} = 77.1\ MPa$

Force to shear one slug: F_S = 0.7·77.1·π·5·7 = 59 kN

Force to shear 6 slugs is 59·6 = 354 kN, so press has capacity to make slugs.

Calculate force for extrusion:

$$A_o = \pi \frac{d_o^2}{4} = \pi \frac{0.05^2}{4} = 0.00196 \ m^2$$

$$A_f = \pi \frac{d_o^2 - d_i^2}{4} = \pi \frac{0.05^2 - 0.048^2}{4} = 0.000154 \ m^2$$

$$A_P = \pi \frac{d_i^2}{4} = \pi \frac{0.048^2}{4} = 0.0018 \ m^2$$

extrusion ratio: $Re = \frac{A_o}{A_f} = \frac{0.00196}{0.000154} = 12.7$ $\quad\quad \varepsilon = \ln Re = 2.54$

$$Q_e = 0.8 + 1.2 \ln Re = 3.85$$

$$\sigma_{tm} = K \frac{\varepsilon^n}{n+1} = 140 \cdot \frac{2.54^{0.25}}{1.25} = 141 \ MPa$$

$$p_e = \sigma_{tm} \times Q_e = 141 \times 3.85 = 543 \ MPa$$

$$F_e = p_e \times A_P = 543 \times 0.0018 = 982 \ kN$$

Press has sufficient capacity for both operations.

(b) volume of workpiece, V = 0.00196·0.007 = 1.37×10^{-5} m^3

$$Work \ Done = KV \cdot \frac{\varepsilon^{n+1}}{n+1} = 140 \cdot 1.37 \times 10^{-5} \cdot \frac{2.54^{1.25}}{1.25} = 4920 \ N{\cdot}m$$

$$Power = 4920 \times \frac{120 \ strokes}{60 \ seconds} = 9.8 \ kW$$

Chapter 7 - Machining Processes

Problem 7.1

P = 1 kW, f = 0.3 mm, η = 0.7, d_m = 250 mm, $\kappa_r = 70°$, N = 100 rpm (1.67 /s)

Use equation 7.4 to calculate undeformed chip thickness, u_c:

$$u_c = f \sin\kappa_r = 0.3 \sin 70° = 0.28 \ mm$$

From Figure 7.20, estimate for u_c = 0.28 mm and cast iron, that specific power, $p_s \approx 1.6$ GJ/m^3.
Calculate material removal rate using equation 7.12:

$$MRR = \frac{P\eta}{p_s} = \frac{1000 \ J/s \cdot 0.7}{1.6\times10^9 \ J/m^3} = 437.5 \ mm/s^3$$

Use equation 7.2 to calculate cutting depth, which is controlled by the maximum workpiece diameter of 250 mm:

$$d_c = \frac{MRR}{\pi f N d_m} = \frac{437.5 \ mm/s^3}{\pi \cdot 0.3 \, mm \cdot 1.67 \, /s \cdot 250 \, mm} = 1.1 \ mm$$

The estimated maximum cutting depth is 1.1 mm. At the 250 mm diameter this cutting depth will require the full 1 kW of power available.

Problem 7.2

d_c = 2 mm, f = 0.25 mm, $\kappa_r = 30°$, width = 150 mm, length = 200 mm

50 strokes per minute of 250 mm length, cutting speed is:

$$v = 50 \, /\text{min} \times 250 \, mm \times 2 = 417 \ mm/s$$

calculate metal removal rate according to equation 7.5:

$$MRR = v f d_c = 417 \, mm/s \times 0.25 \, mm \times 4 \, mm = 417 \ mm^3/s$$
$$u_c = f \sin\kappa_r = 0.25 \sin 30 = 0.125 \ mm$$

Estimate specific power as 4 GJ/m^3 from Figure 7.20 and from Table 7.1 η = 0.7, use equation 7.12 to calculate power required:

$$P = \frac{p_S \cdot MRR}{\eta} = \frac{4\times10^9 \, J/m \times 417 \times 10^{-9} m^3/s}{0.7} = 2.4 \ kW$$

During each stroke feed is 0.25 mm, so width of 150 mm requires 600 strokes, which at 50 strokes per minute takes 12 minutes.

Problem 7.3

f = 0.25 mm, $\kappa_r = 60°$, P = 1 kW, N = 600 rpm

Using equation 7.7 calculate undeformed chip thickness:

$$u_c = \frac{f}{2}\sin\kappa_r = \frac{0.25}{2}\sin 60° = 0.11 \ mm$$

From Figure 7.20 estimate specific power as 1.2 GJ/m^3 and from Table 7.1 η = 0.75.
Use equation 7.12 to calculate metal removal rate:

$$MRR = \frac{P\eta}{p_S} = \frac{1000 \ J/s \times 0.75}{1.2\times10^9 \ J/m^3} = 625 \ mm^3/s$$

Use equation 7.6 to calculate size of hole - for N = 600 rpm (10 /s):

$$D_{600} = \sqrt{\frac{4\cdot MMR}{\pi N f}} = \sqrt{\frac{4\times625 \ mm^3/s}{\pi\times10/s\times0.25 \ mm}} = 17.8 \ mm$$

$$D_{300} = \sqrt{\frac{4\cdot MMR}{\pi N f}} = \sqrt{\frac{4\times625 \ mm^3/s}{\pi\times5/s\times0.25 \ mm}} = 25.2 \ mm$$

Problem 7.4

For machining: d_w = 12.5 mm, d_m = 12 mm, N = 600 rpm, L = 150 mm,
f = 1 mm

Undeformed chip thickness, from equation 7.4: $u_c = f\sin\kappa_r = 1\sin 60° = 0.86 \ mm$

From Figure 7.20 estimate specific power as 2.2 GJ/m^3.

Calculate metal removal rate, equation 7.2:

$$MRR = \pi f d_c N d_m = \pi \ 1 \ mm\times0.25 \ mm\times10/s\times12 \ mm = 94.2 \ mm^3/s$$

Volume of metal to be removed: $V = \frac{\pi}{4}L\left(d_w^2 - d_m^2\right) = \frac{\pi}{4}150\left(12.5^2 - 12^2\right) = 1442 \ mm^3$

Time for Machining, $t = \frac{1442 \ mm^3}{94.2 \ mm^3/s} = 15.3 \ s$

$$Energy = 2.2\times10^9 \ J/m^3\times94.2\times10^{-9} \ m^3/s\times15.3 \ s = 3171 \ J$$

For uniaxial tension: From Table 3.1 for 302 stainless steel: K = 1300 MPa, n = 0.3

Strain required: $\varepsilon = \ln\frac{12.5^2}{12^2} = 0.082$ which is less than the limit for homogeneous strain.

Volume of workpiece:

$$V = \frac{\pi d^2 l}{4} = \frac{\pi 12.5^2 150}{4} = 1.84\times10^{-5}\ m^3$$

Ideal work of deformation, equation 3.20:

$$W_i = \frac{K\varepsilon^{n+1}}{n+1}\cdot V = \frac{1300\times10^6 N/m^2\ 0.082^{1.3}}{1.3}\cdot 1.84\times10^{-5} = 713\ J$$

The reduction in diameter requires much less energy if the rod is pulled in tension rather than machined. The major reason for this is the considerable friction during machining (recall example E7.3).

Problem 7.5

For Grinding: v_{ts} = 1 mm/s, f_s = 25 mm, d_c = 4 mm,

Calculate metal removal rate using equation 7.11:

$$MRR = f_s d_c v_{ts} = 25\ mm\times4\ mm\times1\ mm/s = 100\ mm^3/s$$

Estimate undeformed chip thickness as 0.005 mm and use Figure 7.20 to get a specific energy of 7 GJ/m^3. Estimate cutting efficiency as 0.85 from Table 7.1. Then use equation 7.12 to estimate power required as:

$$P = \frac{p_S\cdot MRR}{\eta} = \frac{7\times10^9\ J/m^3\times100\times10^{-9} m^3/s}{0.85} = 823\ W$$

For Milling: v_f = 1 mm/s, N = 40 rpm, n_t = 16,w = 25 mm, d_c = 4 mm, D = 100mm

Use equation 7.10 to calculate the undeformed chip thickness:

$$u_c = \frac{v_f}{Nn_t}\sqrt{\frac{d_c}{D}} = \frac{1\ mm/s}{0.66\ /s\cdot16}\sqrt{\frac{4\,mm}{100\,mm}} = 0.019\ mm$$

From Figure 7.20 estimate specific power as 3.8 GJ/m^3 and from Table 7.1 η = 0.5. Calculate metal removal rate from equation 7.9:

$$MRR = v_f d_c w = 1\ mm/s\times4\ mm\times25\ mm = 100\ mm^3/s$$

and use equation 7.12 to calculate power:

$$P = \frac{p_S\cdot MRR}{\eta} = \frac{3.8\times10^9\ J/m^3\times100\times10^{-9}\ m^3/s}{0.5} = 760\ W$$

Problem 7.6

$\alpha = 10°$, $d_c = 0.25$ mm, $d'_c = 0.35$ mm, $F_c = 1900$ N, w = 5 mm.
From equation 7.18:

$$\tan\phi = \frac{d_c}{d'_c} \cdot \frac{\cos\alpha}{1 - \frac{d_c}{d'_c}\sin\alpha} = 0.71 \frac{\cos 10°}{1 - 0.71 \sin 10°}$$

$$\phi = 38.5°$$

Using Merchant's analysis, equation 7.26:

$$\begin{aligned} 2\phi &= 90 - (\beta - \alpha) \\ 2 \cdot 38.5° &= 90 - (\beta - 10) \\ \beta &= 23° \end{aligned}$$

From Figure 7.24 and example E7.3,
coefficient of friction: $\mu = \tan\beta = \tan 23° = 0.42$

Since the cutting force is 1900 N, calculate the resultant force using equation 7.20:

$$\begin{aligned} F_c &= R\cos(\beta - \alpha) \\ 1900\,N &= R\cos(23 - 10) \\ R &= 1950\,N \end{aligned}$$

Then use equation 7.21 to calculate the shearing force:

$$\begin{aligned} F_S &= R\cos(\phi + \beta - \alpha) \\ F_S &= 1950\,N\cos(38.5 + 23 - 10) \\ F_S &= 1214\,N \end{aligned}$$

Finally use equations 7.22 and 7.23 to estimate shear strength τ_s.

$$\tau_S = \frac{F_S \sin\phi}{w d_c} = \frac{1214\,N \sin 38.5}{5\,mm \cdot 0.25\,mm} = 605\ MPa$$

The estimated shear strength of the workpiece is 605 MPa.

Problem 7.7

(a) $\alpha = 7°$, $d_c = 1$ mm, $d'_c = 1.5$ mm.

Use equation 7.18 to calculate the shear angle:

$$\tan\phi = \frac{d_c}{d'_c} \cdot \frac{\cos\alpha}{1 - \frac{d_c}{d'_c}\sin\alpha} = \frac{1}{1.5} \cdot \frac{\cos 7°}{1 - \frac{1}{1.5}\sin 7°}$$

$$\phi = 35.7°$$

Use Merchant's equation 7.26 to calculate the friction angle β:

$$2\phi = 90 - (\beta - \alpha) \quad \Rightarrow \quad \beta = 90 + \alpha - 2\phi = 90 + 7 - 2(35.7) = 25.6°$$

Calculate coefficient of friction from $\mu = \tan\beta = \tan 25.6° = 0.48$.

(b) If the friction coefficient remains the same then $\beta = 25.6°$, also know that $\alpha = 17°$. Calculate new shear angle from Merchant's equation 7.26:

$$2\phi = 90 - (\beta - \alpha) = 90 - (25.6° - 17°)$$
$$\phi = 40.7°$$

Calculate new chip thickness from equation 7.16:

$$\sin\phi = \frac{d_c}{d'_c}\cos(\phi - \alpha) \Rightarrow d'_c = \frac{d_c \cos(\phi - \alpha)}{\sin\phi} = \frac{1\cos(40.7° - 17°)}{\sin 40.7°} = 1.4 \; mm$$

The new chip thickness is 1.4 mm.

Problem 7.8

C´ = 400, n = 0.5, so Taylor tool life equation is $vt^{0.5} = 400$.
Given that $v_2 = ½v_1$, so that:

$$½\, v_1 t_2^{0.5} = 400 = v_1 t_1^{0.5}$$
$$½\, t_2^{0.5} = t_1^{0.5}$$
$$\frac{t_2^{0.5}}{t_1^{0.5}} = 2$$
$$\frac{t_2}{t_1} = 4$$

Increase tool life by a factor of 4 or: $\frac{4-1}{1} \times 100 = 300\%$

Problem 7.9

(a) if speed is doubled: $t^{0.25} = \frac{C'}{(2v) f^{0.75} d_c^{0.8}}$

since C´, f and d_c remain constant, then: $t^{0.25} \propto \frac{1}{2v} \Rightarrow t \propto \frac{1}{2^4} = 0.0625$

since original life is 40 min, then life after speed is doubled is 2.5 min.

(b) if feed is doubled: $t^{0.25} = \frac{C'}{v(2f)^{0.75} d_c^{0.8}}$

since C´, v and d_c are constant, then: $t^{0.25} \propto \frac{1}{2f^{0.75}} \Rightarrow t \propto \left(\frac{1}{2^{0.75}}\right)^4 = 0.125$

since original life is 40 min, then after feed is doubled the life will be 5 min.

Choosing to double the feed will cut the cutting time in half and provide twice the tool life as compared to doubling the cutting speed. However, the tool now needs to be changed every 5 min, compared to previously when the tool needed to be changed every 40 min. Therefore, whether the actual production rate can be increased depends on the time required to change the tool.

Chapter 8 - Joining Processes

Problem 8.1

The important difference during MIG welding compared to TIG or shielded (manual) metal arc welding, is that MIG involves the transfer of metal droplets from the electrode to the base metal. If reverse polarity is used during MIG welding the positive ions of the plasma exert considerable force to the metal droplets from the electrode, increasing the energy with which they strike the base metal, causing greater weld depth. For TIG and shielded (manual) metal arc welding there is much less or no transfer of mass from the electrode to the base metal. Therefore, greater weld depth is achieved with straight polarity which become negative ions (with higher kinetic energy than the positive ions and this releases comparatively more energy) to strike the base metal.

Problem 8.2

The flux used to protect the weld pool during submerged arc welding also serves to prevent radiative heat losses from the welding arc, so that less of the heat produced escapes to the surrounding atmosphere, thereby improving the heat transfer efficiency.

Problem 8.3

GOLDSCHMIDT PROCESS $Cr_2O_3 + 2Al \rightarrow Al_2O_3 + 2Cr + heat$

Problem 8.4

Weld 2 should be done first. Since weld 1 is further from the neutral axis of the beam, it could be expected that it would cause greater distortion. However, since it distorts against the tensile stresses introduced by weld 1, it has proportionately less effect. Yet it is further from the nuetral axis, thereby providing a greater effect than weld 1. Therefore, weld 1 can be arranged to remove the distortion introduced by weld 2. Additionally, the cross sectional areas of the two welds can be used to balance the distortion caused by both welds taken together.

Problem 8.5

By clamping each component of the beam prior to welding, the thermal stresses created during welding cannot be relieved by distortion. Therefore assuming that there is no annealing or other stress relief treatment, local plastic yielding of the base metal in areas surrounding the welded joints will occur and the local residual stress level will be approximately equal to the yield stress of the base metal. This residual stress exists as a result of welding and is present even when no external loads are applied.

Problem 8.6

Since the beam has the same cross-section as the beam of example E8.1, then the neutral axis, moment of inertia will be the same. Therefore, the total beam deflection will be:

$$\delta = 0.005 \frac{27.25\ mm \times 3000^2\ mm^2 \times 36\ mm}{3.5 \times 10^6\ mm^4} = 12.6\ mm$$

From equation 8.9 the bending moment causing the longitudinal bowing is $M=F\bar{y}$ and this quantity can be calculated from equation 8.10 (assume E=205 GPa for steel):

$$F\bar{y} = \frac{8\,\delta\, E\, \hat{I}}{l^2} = \frac{8 \times 12.6\ mm \times 205 \times 10^3\ N/mm^2 \times 3.5 \times 10^6\ mm^4}{3000^2\ mm^2} = 8 \times 10^6\ N{\cdot}mm$$

This is the bending moment applied at the end of the beam, as shown in Figure 8.17. The straightening force, S_F, required will be:

$$S_F = \frac{F\bar{y}}{l} = \frac{8 \times 10^6\ N{\cdot}mm}{3000\ mm} = 2.7 \times 10^3\ N$$ about 623 lb_f.

Problem 8.7

The coefficient of thermal expansion of the base metal is the major indicator of distortion during welding. Since aluminium has a higher thermal expansion coefficient than steel and in addition its thermal conductivity is greater, so that heat will 'spread out' over a mucher larger volume of the base material. Thus a greater volume of metal around a weld is affected and contributes to distortion.

Problem 8.8

Since the plate thickness is only 5 mm, start by assuming that thin plate conditions exist.
Use equation 8.15 to calculate heat input rate possible.
From Figure 8.27, the cooling time, $\tau_{8/5}$, required to avoid martensite completely is about 1.8 s.
From Table 8.2, for low alloy the constants are: k = 0.4 W/cm·K
ρ = 7.8 g/cm^3
C = 0.50 kJ/kg·K

Therefore, using equation 8.15:

$$\tau_{8/5} = \frac{(Q/h)^2}{4\pi k \rho C}\left[\frac{1}{(500 - T_o)^2} - \frac{1}{(800 - T_o)^2}\right]$$

$$\left(\frac{Q}{h}\right)^2 = \frac{1.8 \times 4\pi \times 0.04 \times 0.0078 \times 0.50}{2.8 \times 10^{-6}} = 1259$$

$$Q = 5\sqrt{1259} = 177\ J/mm$$

Now check if thin plate assumption is satisfied, equation 8.16:

$$\tau = h\sqrt{\frac{\rho C\left(T_c - T_p\right)}{Q}} = 5\ mm\sqrt{\frac{0.0078\,g/mm^3 \times 0.50\ J/g{\cdot}K\left(550°C - 20°C\right)}{177\ J/mm}} = 0.54$$

since τ<0.75, thin plate assumption is valid.
Calculate welding speed using equation 8.17, from Table 8.3, use η = 0.8:

$$v = \eta\frac{VI}{Q} = 0.8\frac{25\ V\cdot 100\ A}{177\ J/mm} = 11.3\ \ mm/s$$

The maximum welding speed is 11.3 mm/s.

Problem 8.9

Steel composition is 0.15%C, 0.37%Si, 1.42%Mn
From example problem E8.2 we know that $\tau_{8/5}$=6.7 seconds
The carbon equivalent using equation 8.18 is:

$$C_{eq} = 0.15 + \frac{1.42}{3} = 0.62\%$$

calculate the cooling time for martensite formation from equation 8.17:

$$\log \tau_{8/5}]_M = 2.5 \times 0.62 - 1.27 = 0.28$$
$$\tau_{8/5}]_M = 1.9\ \ seconds$$

since $\tau_{8/5}]_M < \tau_{8/5}$, we use equation 8.20, but first must calculate the Vickers pyramid hardness number for the base steel according to equation 8.21:

$$VPN_0 = 164\left(0.15 + \frac{0.37}{2}\right) + 153 = 208\ \ VPN$$

and equation 8.19 to calculate the martensite hardness:

$$VPN_M = 812 \times 0.15 + 293 = 415\ \ VPN$$

then from equation 8.20:

$$VPN_{HAZ} = 208 + (415 - 208)\exp\left[-0.2\left(\frac{6.7}{1.9} - 1\right)\right] = 333\ \ VPN$$

If the preheating is applied as in example E8.3 it will not change the HAZ hardness, because the result of the example was that the welding speed and heat input were also increased compared to example E8.2. The same critical cooling time of 6.7 seconds was used for these calculations - it is seen from equation 8.20 that in this case the HAZ hardness depends only on the steel composition (VPN of martensite and base metal) and the critical cooling time. If the preheating were applied, without increasing the heat input or welding speed, then a lower HAZ hardness would result, since preheating effectively increases $\tau_{8/5}$.

Problem 8.10

During brazing the filler metal will dissolve a small amount of the base metal. In this manner the filler becomes alloyed with the base metal, improving it's strength.

Chapter 9 - Surface Modification

Problem 9.1

Use equation 9.15 to calculate time required, since x_p is the same for both temperatures.

$$\begin{aligned} at\ 1000^\circ C \quad x_p &= \sqrt{D_{1000} t_{1000}} \\ at\ 900^\circ C \quad x_p &= \sqrt{D_{900} t_{900}} \\ so \quad \sqrt{D_{1000} t_{1000}} &= \sqrt{D_{900} t_{900}} \\ \therefore\ t_{900} &= \frac{3.5 \times 10^{-7}}{1 \times 10^{-7}} \times 10 = 35\ \text{min} \end{aligned}$$

Problem 9.2

Carburizing relies on diffusional flow of additional carbon into surface of steel. The increased carbon concentration at the surface increases the hardness after quenching. Induction hardening is a heat treatment process that transforms the surface region to martensite. Induction hardening relies on hardening caused by the base steel carbon concentration. Therefore, maximum hardness can only be achieved by induction hardening for steels containing greater than about 0.5% carbon.

Problem 9.3

For the original carbon profile c_s = 1.4 and c_o = 0.2 and using equation 9.14 the carbon concentration at the case penetration depth for the original profile is:

$$c(x_p) = \frac{c_s + c_o}{2} = \frac{0.2 + 1.4}{2} = 0.8$$

so the case depth x_p = 0.5 mm. Now calculate the diffusion constant using equation 9.15:

$$x_p = \sqrt{Dt} \Rightarrow D = \frac{x_p^2}{t} = \frac{0.05^2\ cm^2}{9900\ s} = 2.5 \times 10^{-7}\ cm^2/s$$

Use equation 9.13 to calculate the frequency factor (T = 970°C = 1243 K, R = 8.314 J/mole·K and from Table 9.1 Q = 142 kJ/mole):

$$D = D_o \exp(-Q/RT) \Rightarrow D_o = \frac{2.5 \times 10^{-7}}{\exp(-142{,}000/8.314 \times 1243)} = 0.23\ cm^2/s$$

This frequency factor is close to the 0.21 cm^2/s listed in Table 9.1 for carbon diffusion in austenite.

For the new carbon profile: $c(x_p) = \frac{1.6 + 0.2}{2} = 0.9$

so from plot x_p = 0.7 mm.

and $$D = \frac{0.07^2}{9900} = 5.0\times10^{-7}\ cm^2/s$$

Calculate new temperature required from equation 9.13:

$$D = D_o \exp(-Q/RT)$$
$$5\times10^{-7} = 0.23\exp\left(\frac{-142{,}000}{8.314\times T}\right)$$
$$T = 1310\ K = 1037°C$$

The new temperature required is about 1037°C.

Problem 9.4

(a) answer may be found be reviewing section 9.3.1.2

(b) From the graph in question c_o = 0.2 and c_s = 1.4%
Using equation 9.14:

$$c(x_p) = \frac{c_o + c_s}{2} = \frac{0.2+1.4}{2} = 0.8\%\ carbon$$

so from graph: for HHP x_p = 0.5 mm,
for FHFP x_p = 1.0 mm

using equation 9.15: $$x_p = \sqrt{Dt} \Rightarrow t = \frac{x_p^2}{D}$$

where from equation 9.13: $$D = D_o \exp(-Q/RT)$$

from Table 9.1 for carbon diffusing in austenite (fcc-Fe), Q = 142 kJ/mole
D_o = 0.21 cm^2/s

universal gas constant R = 8.31 kJ/mole·K

so for HHP: $$D = 0.21\exp\left(\frac{-142{,}000}{8.31\times1173}\right) = 1\times10^{-7}\ cm^2/s$$

and for FHFP: $$D = 0.21\exp\left(\frac{-142{,}000}{8.31\times1373}\right) = 8.26\times10^{-7}\ cm^2/s$$

time for HHP: $$HHP_{time} = \frac{0.05^2}{1\times10^{-7}} = 6.9\ hours$$

time for FHFP: $$FHFP_{time} = \frac{0.1^2}{8.26\times10^{-7}} = 3.4\ hours$$

Problem 9.5

From the Fe-C phase diagram of Figure 2.32 estimate that at 1000°C the saturated carbon content of austenite is about 1.5%.
C_o = 0.2%, so:

$$c(x_P) = \frac{c_s + c_o}{2} = \frac{1.5 + 0.2}{2} = 0.85$$

at 1000°C (1273K) calculate diffusion constant, D:
(from Table 9.1, D_o = 0.21 cm^2/s and Q = 142 kJ/mole)

$$D = D_o \exp(-Q/RT) = 0.21 \exp\left(\frac{-142{,}000}{8.314 \cdot 1273}\right) = 3.1 \times 10^{-7} \ cm^2/s$$

so now solve equation 9.12 for x_P for t = 8 hours = 28,800 s.

$$0.85 = 1.5 + (0.2 - 1.5)\, erf\left(\frac{x_P}{2\sqrt{3.1 \times 10^{-7} \cdot 28800}}\right)$$

$$\frac{0.65}{1.3} = erf\left(\frac{x_P}{0.194}\right)$$ use assumption stated at end of question.

$$\frac{x_p}{0.194} = 0.5 \Rightarrow x_P = 0.097 \ cm$$

to meet condition that carbon concentration at 1000°C case depth be maintained means that at 900°C must get c(0.097) = 0.85.
Calculate diffusion coefficient for 900°C:

$$D = D_o \exp(-Q/RT) = 0.21 \exp\left(\frac{-142{,}000}{8.314 \cdot 1173}\right) = 1 \times 10^{-7} \ cm^2/s$$

From Fe-C phase diagram at 900°C, c_s = 1.3%
Then solve equation 9.12 to calculate time at 900°C:

$$0.85 = 1.3 + (0.2 - 1.3)\, erf\left(\frac{0.097}{2\sqrt{1 \times 10^{-7} \cdot t}}\right)$$

$$\frac{0.45}{1.1} = erf\left(\frac{0.049}{\sqrt{1 \times 10^{-7} \cdot t}}\right)$$

$$0.41 = \frac{0.049}{\sqrt{1 \times 10^{-7} \cdot t}} \Rightarrow t = 39.4 \ hours$$

Time required for carburizing at 900°C is 39.4 hours.

Problem 9.6

Process that involve cementation: pack carburizing, gas carburizing, liquid carburizing, nitriding, carbonitriding, (sometimes) aluminizing, siliconizing, chromizing

Problem 9.7

Induction hardening will probably not be satisfactory for surface hardening of low carbon steel gears. This is evident from Figure 9.6 which indicates that to attain maximum hardness requires a carbon content of at least 0.5%. Low carbon steels typically contain less than 0.5% carbon, but during gas carburizing additional carbon is diffused into the surface layers. However, induction hardening utilizes only the carbon already present in the steel and relies only on the transformation to martensite to achieve hardness. At carbon levels <0.5% this hardness will be less than the maximum obtained when additional carbon is introduced to raise the carbon level to >0.5%.

Problem 9.8

From equation 9.16 it is clear that depth of penetration of the induced current, and therefore the hardened dpeth, is proportional to the inverse square root of the frequency, or:

$$d \propto \sqrt{\frac{1}{f}}$$

$$\frac{d_d}{d_i} = \frac{2^{-½}}{1^{-½}}$$ so the relationship between the initial and doubled frequency is:

$$d_d = 0.707\, d_i$$

Problem 9.9

First calculate the weight of the silver deposit:
volume of silver deposited = 50 cm^2 x 25 x 10^{-4} cm = 0.125 cm^3
weight of silver deposit = 0.125 cm^3 x 10.5 g/cm^3 = 1.31 g

The current applied is 50 cm^2 x 0.02 A/cm^2 = 1 A

Using Faraday's equation (equation 9.17):

$$t = \frac{WvF}{\eta w I} = \frac{1.31\ g \cdot 1 \cdot 96{,}500\, A{\cdot}s/mole}{1 \cdot 107.9\ g/mole \cdot 1\ A} = 1171\ seconds\ (19.5\ \text{min})$$

(Assuming that silver is costs about $6 per once (≈£3), the value of the silver on one knife is about 25¢ (≈12p). Remember this the next time you buy a silver plated knife!)