PMATH 340 **Assignment 4, due Wednesday March 19**

1. Let p be a prime congruent to 1 mod 4. Show that $(1|p)+2(2|p)+3(3|p)+\ldots(p-1)((p-1)|p)=0$.
2. Use quadratic reciprocity and properties of the Legendre symbol to evaluate $(254|3001)$. You can use the fact that 3001 is prime.
3. Use quadratic reciprocity to give a table of values for $(5|p)$, where p is an odd prime. The lines of your table should depend on the relevant values of $p \mod 20$.
4. Find all primitive Pythagorean triples with $y=60$, i.e. find all positive integer solutions x, z to the equation: $x^2+60^2=z^2$ with $\gcd(x,60,z)=1$.
5. Let n be a positive integer. Show that n and $2n$ have the same number of representations as a sum of two squares (of non-negative integers). Hint: $(a^2+b^2)(c^2+d^2)=(ac+bd)^2+(ad-bc)^2$.
6. Let x, y, z be a Pythagorean triple, i.e. an integer solution to $x^2+y^2=z^2$. Show that: a) at least one of x and y is divisible by 3. b) at least one of x, y, z, is divisible by 5. c) xyz is divisible by 60.
7. Show that $3, 4, 5$ is the only primitive Pythagorean triple consisting of positive integers whose terms are in arithmetic progression.
8. Illustrate Euler's method for expressing primes of the form $4k+1$ as a sum of two squares of integers, using the prime $p=521$, starting with the solution $z=235$ to $z^2 \equiv -1 \mod 521$.