PMATH 340 **Assignment 2, due Feb 7**

1. Let $f_n = 2^{2^n} + 1$. Show for $n \geq 1$, that $f_n = f_{n-1}^2 - 2f_{n-1} + 2$.
2. Prove that $f_n \equiv 7 \mod 10$, for $n \geq 2$. Hint: use the previous problem.
3. List all the elements of $Z_{20} = \{0, 1, 2, \ldots, 19\}$ that are invertible under multiplication modulo 20, and also their inverses in Z_{20}.
4. Use the Euclidean algorithm to find $75351^{-1} \mod 981623$.
5. Let $n \geq 2$. Find all integers n such that $\phi(n) + \sigma(n) = 2n$. Hint: $\sigma(n) = \sum_{d|n} d$, and $\phi(n) = \sum_{d|n} \mu(n/d)d$.
6. Let p, q be two distinct primes. Show that $p^{q-1} + q^{p-1} \equiv 1 \mod pq$.