PMATH 340 — Assignment 1, due Friday Jan 24

1. Use the Euclidean algorithm to find $\gcd(2159, 4063)$. Apply back substitution to find integers x, y such that: $2159x + 4063y = \gcd(2159, 4063)$.

2. Use induction to prove that $n^7 - n$ is a multiple of 7 for every positive integer n.

3. Show that the only positive integer n for which $n^3 + 1$ is prime is $n = 1$.

4. Show that if k is a positive integer, then $\gcd(5k + 2, 13k + 5) = 1$.

5. Let $f_1 = 1$,$f_2 = 1$, $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$ denote the Fibonacci numbers. Show that the gcd of any two consecutive Fibonacci numbers equals 1.

6. Let $N > 1$, be an integer. Prove that
$$\sum_{j=1}^{N} \frac{1}{j} \notin \mathbb{Z}.$$