MATH 1007F **Term test #1 – solution** **28 January 2013**

This test was written in room: ________________ Total marks: 40 marks

This test is closed book. No calculators or electronic aids are permitted. **Please supply your answers on this sheet.**

PLEASE PRINT ______________________________ ____________

First name Last name Student number

Please show your work where appropriate! TA's have extra paper if you need it. Test duration: 50 minutes.

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1. **Simplify** as much as possible

   a. [1] $(62)^x \cdot (62)^{1-x} =$ $62^{[x+(1-x)]} = 62$

   b. [2] $\left(\dfrac{3x\,y^{3/2}}{2x^{-1}y}\right)^{-2} =$ $\left(\dfrac{2x^{-1}y}{3xy^{3/2}}\right)^2 = \dfrac{4}{9x^4y}$

2. Fill in the blanks:

   a. [1] $\cos\,(x - \frac{\pi}{2}) = \sin x$ ... TRUE ✓ FALSE ___

   b. [1] $\sin\,(\frac{5\pi}{3}) =$ $-\frac{\sqrt{3}}{2}$

   c. [1] If $f(x) = \sin(x) + 2$ then the **range** of f is $[1, 3]$

   d. [1] If $g(x) = 3\cos\,(x-3)$. The **domain** of g is $\mathbb{R}$

   e. [1] The function $f(x) = \left(\frac{2}{5}\right)^x$ is decreasing ... TRUE ✓ FALSE ___

3. Let $f(x) = \dfrac{3x+1}{2x-3}$ and $g(x) = \sqrt{3x+5}$

   a. [2] What are the domains of each function, f and g?

   $dom(f) = \mathbb{R} \setminus \{3/2\} = \{x \in \mathbb{R} \mid x \neq 3/2\} = (-\infty, 3/2) \cup (3/2, +\infty)$

   $dom(g) = [-5/3, +\infty) = \{x \in \mathbb{R} \mid x \geq -5/3\}$

   b. [3] What is the domain $f + g$?

   $dom(f+g) = dom(f) \cap dom(g)$

   $= [-5/3, 3/2) \cup (3/2, +\infty) = \{x \in \mathbb{R} \mid x \geq -5/3,\ x \neq 3/2\}$

   c. [3] What is the rule of $f \circ f\,(x)$? (*simplify as much as possible*)

   $f \circ f(x) = f(f(x)) = \dfrac{3(f(x)) + 1}{2(f(x)) - 3} = \dfrac{\left(\frac{9x+3}{2x-3}\right) + 1}{\left(\frac{6x+2}{2x-3}\right) - 3} = \dfrac{9x+3+2x-3}{6x+2-6x+9}$ $(x \neq 3/2)$

   $\ldots = x$

   d. [3] What is the domain of $f \circ f\,(x)$?

   $dom(f \circ f) = \{x \in dom(f) \mid f(x) \in dom(f)\} = \mathbb{R} \setminus \{3/2\}$

4. [3] Determine the equation of the line that passes through the points (1, –1) and (–3, 1).

$m = \frac{1-(-1)}{-3-1} = -1/2 \Rightarrow y = -\frac{1}{2}x + b$ (1,−1) on curve $\Rightarrow -1 = -\frac{1}{2}(1) + b$

$\Rightarrow b = -1/2$

$\boxed{\therefore\ y = -\frac{1}{2}x - \frac{1}{2}} \Rightarrow \boxed{2y + x + 1 = 0}$

5. Solve the following, expressing the solution in the form of an interval or a union of sub-intervals.

a. [2] $3x + 5 > 10x + 26 \Rightarrow 3x - 10x > 26 - 5$

$-7x > 21$

$x < -3 \Rightarrow x \in (-\infty, -3)$

OR $\{x \in \mathbb{R} \mid x < -3\}$

b. [3] $\underline{|x-2| \geq 3}$ this means:

$x - 2 \geq 3$ OR $x - 2 \leq -3$

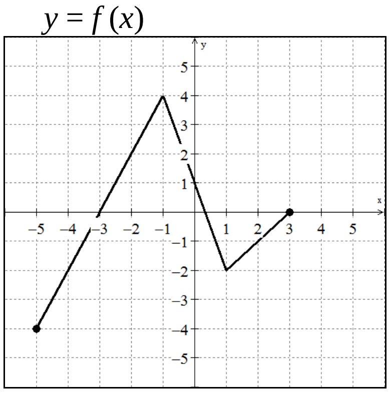
$x \geq 5$ $\quad x \leq -1$

$\therefore\ x \in (-\infty, -1] \cup [5, +\infty)$

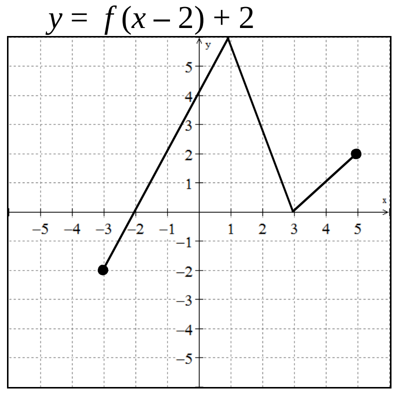
OR $\{x \in \mathbb{R} \mid x \geq 5 \text{ OR } x \leq -1\}$

6. [4] The graph of f is plotted. Plot the graphs of $y = f(x-2) + 2$ and $y = f(1-x)$

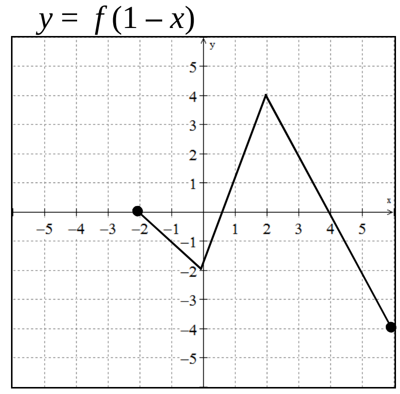
$y = f(x)$



$y = f(x-2) + 2$



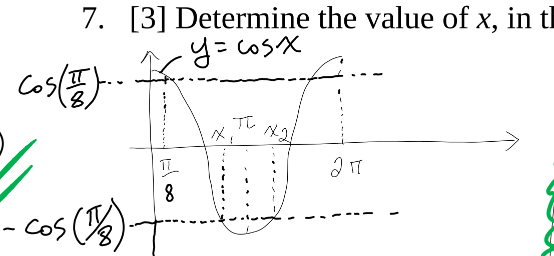
$y = f(1-x)$



[1] What is the range of f ? ANSWER: $[-4, 4]$

7. [3] Determine the value of x, in the interval $0 \leq x \leq 2\pi$, for which $\cos x = -\cos(\pi/8)$

METHOD ①



$\boxed{x_1 = \pi - \pi/8 = 7\pi/8}$ and $\boxed{x_2 = \pi + \pi/8 = 9\pi/8}$

OR $\left\{\begin{array}{l} -\cos x = \cos(\pi - x) \\ -\cos x = \cos(\pi + x) \end{array}\right\}$ ... using addition/subtraction laws for angles ...

Indeed: $\cos(\pi - x) = (\cos\pi)(\cos x) - (\cancel{\sin\pi})(\sin x)$ $= -\cos x$ ... the same for $\cos(\pi + x)$ ...

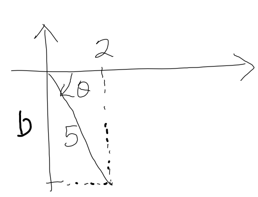
METHOD ②

$\therefore$ if $x = \pi/8$, then

$-\cos\pi/8 = \underbrace{\cos(\pi + \pi/8)}_{x_2} = \underbrace{\cos(\pi - \pi/8)}_{x_1}$

8. Let $\cos\theta = 2/5$ and $3\pi/2 \leq \theta \leq 2\pi$. Determine:
(*You may use a graphical technique or one involving identities*)

Q4



a. [3] $\sin\theta$

Method ① $\left\{\begin{array}{l} \therefore\ b = -\sqrt{5^2 - 2^2} = -\sqrt{21} \\ \therefore\ \sin\theta = -\sqrt{21}/5 \end{array}\right.$

Method ② $\left\{\begin{array}{l} \sin\theta = -\sqrt{1 - \cos^2\theta} \quad (\text{in Q4 } \sin\theta < 0) \\ = -\sqrt{1 - (2/5)^2} = -\sqrt{\frac{21}{25}} = -\sqrt{21}/5 \end{array}\right.$

b. [2] $\tan\theta$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{21}/5}{2/5} = -\frac{\sqrt{21}}{2}$$