MATH 1007A **Practice Term test #1 – solution** **25 May 2013**

This test is closed book. No calculators or electronic aids are permitted. **Please supply your answers on this sheet.**

PLEASE PRINT	First name	Last name	Student number

Please show your work where appropriate! TA's have extra paper if you need it. Test duration: 50 minutes.

1. **Simplify** as much as possible

   a. $(3x^{3/2}y^2)^{-2}(3x^2y^{-1})^3 =$ $\dfrac{27x^6y^{-3}}{9x^3y^4} = \dfrac{3x^3}{y^7}$

   b. $\dfrac{\frac{a^2}{b}-b}{\frac{a+b}{ab}} =$ $\dfrac{\frac{a^2-b^2}{b}}{\frac{a+b}{ab}} = \dfrac{(a-b)(a+b)}{b}\cdot\dfrac{ab}{a+b} = (a-b)a = a^2-ab$

2. Fill in the blanks:

   a. $\cos\left(\frac{2\pi}{3}\right) =$ $-\frac{1}{2}$

   b. Let $f(x) = 2\sin(x+\pi/3)$. Then *dom* (f) = $\mathbb{R}$ (do you see why? .... this is a horizontally-shifted version of the regular sine function ... it is also vertically stretched by a factor of 2 by the way, but that does not affect the domain of the function)

   c. $\sin^2 3 + \cos^2 3 =$ 1

3. Let $f(x) = \dfrac{2}{4-2x}$ and $g(x) = \sqrt{x+3}$

   a. What are the domains of each function, f and g?

   $dom(f) \Rightarrow 4-2x \neq 0$
   $\Rightarrow x \neq 2$
   $dom(f) = \mathbb{R}\setminus\{2\}$

   $dom(g) \Rightarrow x+3 \geq 0$
   $x \geq -3$
   $dom(g) = [-3, +\infty)$

   b. What is the domain of $f-g$?

   $f-g(x) = \dfrac{2}{4-2x} - \sqrt{x+3} \Rightarrow$ both conditions apply, so:
   $dom(f-g) = [-3,2)\cup(2,+\infty)$

   c. What is the rule for $f\circ f(x)$?

   $f\circ f(x) = \dfrac{2}{4-2(f(x))} = \dfrac{2}{4-2\left(\frac{2}{4-2x}\right)} = \dfrac{2}{\frac{16-8x-4}{4-2x}} = \dfrac{8-4x}{12-8x} = \dfrac{2-x}{3-2x}$

   (By the way, what would the domain of this composition be? ... answer: all x except 3/2 and 2, so $dom(f\circ f) = \mathbb{R}\setminus\{3/2, 2\}$ ... do you see why? If x=3/2, then $4-2f(x) = 0$ and therefore $f\circ f$ is undefined, and if x=2, then $f(x)$ is undefined)

4. Determine the equation of the line that is parallel to the line $3x - y - 12 = 0$, and which passes through the point $(-1, -1)$

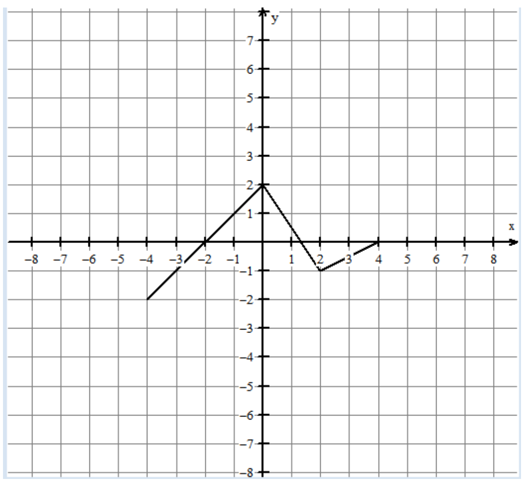
   Slope of the line $\Rightarrow 3x-y-12=0 \Rightarrow y = 3x-12$
   $= 3$ b.c.

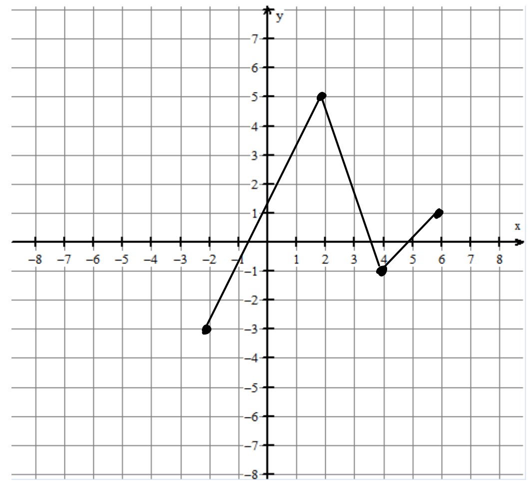
   $\therefore\ y = 3x+b \Rightarrow (-1,-1)$ is on the line; so.
   $-1 = 3(-1)+b \Rightarrow b = 2$
   $\therefore\ y = 3x+2$

5. Prove the following identity: $\sec x - \cos x = \tan x \sin x$. (hint: work on the left side)

$\frac{1}{\cos x} - \cos x = \frac{1-\cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} = \left(\frac{\sin x}{\cos x}\right)\cdot \sin x = \tan x \sin x$

6. The graph of a function *f* is given.

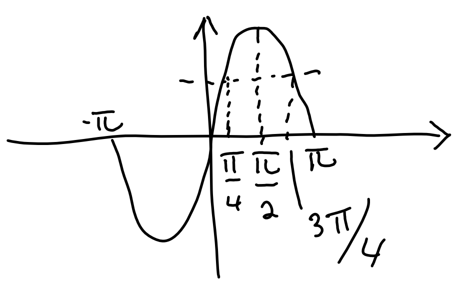




a. Plot the graph of $h(x) = 2f(x-2)+1$

b. What is the range of *h* ? $ran(h) = [-3, 5]$

7. On the interval $[-\pi, \pi]$, for which values of *t* is $\sin t = -\sin(-\pi/4)$? $= \sin(\pi/4)$



$t = \pi/4,\ 3\pi/4$

8. Let *f* and *g* be 2 functions. It is known that the domain of *f* is $[-2, 4]$. The rule for *g* (*x*) is given by $g(x) = \sqrt{x-2}$. Determine the domain of $f \circ g$

$dom(g) = [2, \infty)\ i.e. \{x \in \Re \mid x \geq 2\}$
$dom(f \circ g) = \{x \in dom(g) \mid g(x) \in dom(f)\} = [2, \infty) \cap -2 \leq g(x) \leq 4$

Now *g* (*x*) is never negative (because $g(x) = \sqrt{x-2}$ is a positive root), so the set of inequalities $-2 \leq g(x) \leq 4$ becomes a new set of inequalities, namely: $0 \leq g(x) \leq 4 \ldots$

$\sqrt{x-2} \geq 0 \quad \Rightarrow x-2 \geq 0 \quad \Rightarrow x \geq 2$

AND

$\sqrt{x-2} \leq 4 \quad \Rightarrow x-2 \leq 16 \quad \Rightarrow x \leq 18$

So the domain of the composition is thus:
$dom(f \circ g) = \{x \in dom(g) \mid g(x) \in dom(f)\} = [2, \infty) \cap -2 \leq g(x) \leq 4 = [2, 18]$