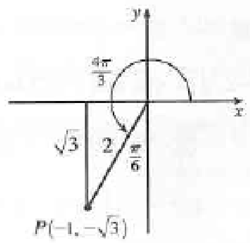
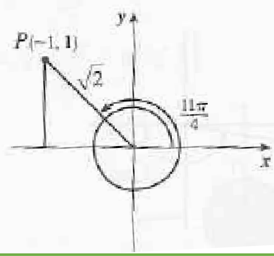
Calculus 1000B (section 002) 2014 Assignment 1-5 Solutions

Assignment 1

Appendix D # 24, 28, 34

24. 

From the diagram and Figure 8, we see that a point on the terminal side is $P(-1, -\sqrt{3})$. Therefore, taking $x = -1$, $y = -\sqrt{3}$, $r = 2$ in the definitions of the trigonometric ratios, we have $\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$, $\cos\frac{4\pi}{3} = -\frac{1}{2}$, $\tan\frac{4\pi}{3} = \sqrt{3}$, $\csc\frac{4\pi}{3} = -\frac{2}{\sqrt{3}}$, $\sec\frac{4\pi}{3} = -2$, and $\cot\frac{4\pi}{3} = \frac{1}{\sqrt{3}}$.

28. 

From the diagram, we see that a point on the terminal side is $P(-1, 1)$. Therefore taking $x = -1$, $y = 1$, $r = \sqrt{2}$ in the definitions of the trigonometric ratios we have $\sin\frac{11\pi}{4} = \frac{1}{\sqrt{2}}$, $\cos\frac{11\pi}{4} = -\frac{1}{\sqrt{2}}$, $\tan\frac{11\pi}{4} = -1$, $\csc\frac{11\pi}{4} = \sqrt{2}$, $\sec\frac{11\pi}{4} = -\sqrt{2}$, and $\cot\frac{11\pi}{4} = -1$.

34. Since $\frac{3\pi}{2} < \theta < 2\pi$, θ is in the fourth quadrant where x is positive and y is negative. Therefore $\csc\theta = r/y = -\frac{4}{3} \Rightarrow r = 4$, $y = -3$, and $x = \sqrt{r^2 - y^2} = \sqrt{7}$. Taking $x = \sqrt{7}$, $y = -3$, and $r = 4$ in the definitions of the trigonometric ratios, we have $\sin\theta = -\frac{3}{4}$, $\cos\theta = \frac{\sqrt{7}}{4}$, $\tan\theta = -\frac{3}{\sqrt{7}}$, $\sec\theta = \frac{4}{\sqrt{7}}$, and $\cot\theta = -\frac{\sqrt{7}}{3}$.

Assignment 2

Section 1.6   #26, 52, 64, 70

26:

$y = f(x) = \dfrac{e^x}{1+2e^x} \Rightarrow y + 2ye^x = e^x \Rightarrow y = e^x - 2ye^x \Rightarrow y = e^x(1-2y) \Rightarrow e^x = \dfrac{y}{1-2y} \Rightarrow$ $x = \ln\left(\dfrac{y}{1-2y}\right)$. Interchange x and y: $y = \ln\left(\dfrac{x}{1-2x}\right)$. So $f^{-1}(x) = \ln\left(\dfrac{x}{1-2x}\right)$. Note that the range of f and the domain of f^{-1} is $(0, \frac{1}{2})$.

52. (a) $\ln(x^2-1) = 3 \Leftrightarrow x^2 - 1 = e^3 \Leftrightarrow x^2 = 1 + e^3 \Leftrightarrow x = \pm\sqrt{1+e^3}$.

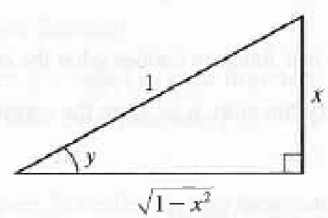
(b) $e^{2x} - 3e^x + 2 = 0 \Leftrightarrow (e^x - 1)(e^x - 2) = 0 \Leftrightarrow e^x = 1$ or $e^x = 2 \Leftrightarrow x = \ln 1$ or $x = \ln 2$, so $x = 0$ or $\ln 2$.

64. (a) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ since $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ and $\frac{\pi}{6}$ is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(b) $\sec^{-1} 2 = \frac{\pi}{3}$ since $\sec\frac{\pi}{3} = 2$ and $\frac{\pi}{3}$ is in $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$.

70. Let $y = \sin^{-1} x$. Then $\sin y = x$, so from the triangle we see that

$\tan(\sin^{-1} x) = \tan y = \dfrac{x}{\sqrt{1-x^2}}$.



Assignment 3

Section 2.2 # 32, 34

32. $\lim_{x \to 5^-} \frac{e^x}{(x-5)^3} = -\infty$ since the numerator is positive and the denominator approaches 0 from the negative side as $x \to 5^-$.

34. $\lim_{x \to \pi^-} \cot x = \lim_{x \to \pi^-} \frac{\cos x}{\sin x} = -\infty$ since the numerator is negative and the denominator approaches 0 through positive values as $x \to \pi^-$.

Section 2.3 # 16, 22, 26, 39

16. $\lim_{x \to -1} \frac{2x^2 + 3x + 1}{x^2 - 2x - 3} = \lim_{x \to -1} \frac{(2x+1)(x+1)}{(x-3)(x+1)} = \lim_{x \to -1} \frac{2x+1}{x-3} = \frac{2(-1)+1}{-1-3} = \frac{-1}{-4} = \frac{1}{4}$

22. $$\lim_{u \to 2} \frac{\sqrt{4u+1} - 3}{u - 2} = \lim_{u \to 2} \frac{\sqrt{4u+1} - 3}{u-2} \cdot \frac{\sqrt{4u+1} + 3}{\sqrt{4u+1} + 3} = \lim_{u \to 2} \frac{\left(\sqrt{4u+1}\right)^2 - 3^2}{(u-2)\left(\sqrt{4u+1} + 3\right)}$$

$$= \lim_{u \to 2} \frac{4u + 1 - 9}{(u-2)\left(\sqrt{4u+1} + 3\right)} = \lim_{u \to 2} \frac{4(u-2)}{(u-2)\left(\sqrt{4u+1} + 3\right)}$$

$$= \lim_{u \to 2} \frac{4}{\sqrt{4u+1} + 3} = \frac{4}{\sqrt{9} + 3} = \frac{2}{3}$$

26. $\lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t^2 + t} \right) = \lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t(t+1)} \right) = \lim_{t \to 0} \frac{t + 1 - 1}{t(t+1)} = \lim_{t \to 0} \frac{1}{t+1} = \frac{1}{0+1} = 1$

39. $-1 \le \cos(2/x) \le 1 \quad \Rightarrow \quad -x^4 \le x^4 \cos(2/x) \le x^4$. Since $\lim_{x \to 0} (-x^4) = 0$ and $\lim_{x \to 0} x^4 = 0$, we have $\lim_{x \to 0} \left[x^4 \cos(2/x)\right] = 0$ by the Squeeze Theorem.

Assignment 4

Section 2.5 # 46

46. $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x - a + b & \text{if } x \ge 3 \end{cases}$

At $x = 2$: $\displaystyle\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x^2-4}{x-2} = \lim_{x\to 2^-} \frac{(x+2)(x-2)}{x-2} = \lim_{x\to 2^-}(x+2) = 2+2 = 4$

$\displaystyle\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(ax^2 - bx + 3) = 4a - 2b + 3$

We must have $4a - 2b + 3 = 4$, or $\mathbf{4a - 2b = 1}$ **(1)**.

At $x = 3$: $\displaystyle\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(ax^2 - bx + 3) = 9a - 3b + 3$

$\displaystyle\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(2x - a + b) = 6 - a + b$

We must have $9a - 3b + 3 = 6 - a + b$, or $\mathbf{10a - 4b = 3}$ **(2)**.

Now solve the system of equations by adding -2 times equation **(1)** to equation **(2)**.

$$\begin{aligned} -8a + 4b &= -2 \\ 10a - 4b &= 3 \\ \hline 2a \quad &= 1 \end{aligned}$$

So $a = \frac{1}{2}$. Substituting $\frac{1}{2}$ for a in **(1)** gives us $-2b = -1$, so $b = \frac{1}{2}$ as well. Thus, for f to be continuous on $(-\infty, \infty)$, $a = b = \frac{1}{2}$.

Section 2.6 # 24, 36

24. $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{9x^6 - x}}{x^3+1} = \lim_{x\to-\infty} \frac{\sqrt{9x^6 - x}\,/x^3}{(x^3+1)/x^3} = \frac{\lim_{x\to-\infty} -\sqrt{(9x^6 - x)/x^6}}{\lim_{x\to-\infty}(1 + 1/x^3)}$ [since $x^3 = -\sqrt{x^6}$ for $x < 0$]

$$= \frac{\lim_{x\to-\infty} -\sqrt{9 - 1/x^5}}{\lim_{x\to-\infty} 1 + \lim_{x\to-\infty}(1/x^3)} = \frac{-\sqrt{\lim_{x\to-\infty} 9 - \lim_{x\to-\infty}(1/x^5)}}{1+0} = -\sqrt{9-0} = -3$$

36. Since $0 \le \sin^2 x \le 1$, we have $0 \le \dfrac{\sin^2 x}{x^2+1} \le \dfrac{1}{x^2+1}$. We know that $\displaystyle\lim_{x\to\infty} 0 = 0$ and $\displaystyle\lim_{x\to\infty} \frac{1}{x^2+1} = 0$, so by the Squeeze Theorem, $\displaystyle\lim_{x\to\infty} \frac{\sin^2 x}{x^2+1} = 0$.

Section 2.8# 26

26. $g'(t) = \lim_{h \to 0} \frac{g(t+h) - g(t)}{h} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h} = \lim_{h \to 0} \frac{\frac{\sqrt{t} - \sqrt{t+h}}{\sqrt{t+h}\sqrt{t}}}{h} = \lim_{h \to 0} \left(\frac{\sqrt{t} - \sqrt{t+h}}{h\sqrt{t+h}\sqrt{t}} \cdot \frac{\sqrt{t} + \sqrt{t+h}}{\sqrt{t} + \sqrt{t+h}} \right)$

$= \lim_{h \to 0} \frac{t - (t+h)}{h\sqrt{t+h}\sqrt{t}\,(\sqrt{t} + \sqrt{t+h})} = \lim_{h \to 0} \frac{-h}{h\sqrt{t+h}\sqrt{t}\,(\sqrt{t} + \sqrt{t+h})} = \lim_{h \to 0} \frac{-1}{\sqrt{t+h}\sqrt{t}\,(\sqrt{t} + \sqrt{t+h})}$

$= \frac{-1}{\sqrt{t}\sqrt{t}\,(\sqrt{t} + \sqrt{t})} = \frac{-1}{t\,(2\sqrt{t})} = -\frac{1}{2t^{3/2}}$

Domain of g = domain of $g' = (0, \infty)$.

Assignment 5

Section 3.1 # 18

18. $y = \sqrt{x}(x-1) = x^{3/2} - x^{1/2} \Rightarrow y' = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2} = \frac{1}{2}x^{-1/2}(3x-1)$ [factor out $\frac{1}{2}x^{-1/2}$] or $y' = \dfrac{3x-1}{2\sqrt{x}}$.

Section 3.2 # 28

28. $f(x) = x^{5/2}e^x \Rightarrow f'(x) = x^{5/2}e^x + e^x \cdot \frac{5}{2}x^{3/2} = \left(x^{5/2} + \frac{5}{2}x^{3/2}\right)e^x$ $\left[\text{or } \frac{1}{2}x^{3/2}e^x(2x+5)\right] \Rightarrow$

$f''(x) = \left(x^{5/2} + \frac{5}{2}x^{3/2}\right)e^x + e^x\left(\frac{5}{2}x^{3/2} + \frac{15}{4}x^{1/2}\right) = \left(x^{5/2} + 5x^{3/2} + \frac{15}{4}x^{1/2}\right)e^x$ $\left[\text{or } \frac{1}{4}x^{1/2}e^x(4x^2+20x+15)\right]$

Section 3.3 # 14

14. $y = \dfrac{1-\sec x}{\tan x} \Rightarrow$

$$y' = \frac{\tan x\,(-\sec x\,\tan x) - (1-\sec x)(\sec^2 x)}{(\tan x)^2} = \frac{\sec x\,(-\tan^2 x - \sec x + \sec^2 x)}{\tan^2 x} = \frac{\sec x\,(1-\sec x)}{\tan^2 x}$$

Section 3.4 # 22, 44

22. $f(s) = \sqrt{\dfrac{s^2+1}{s^2+4}} \Rightarrow$

$$f'(s) = \frac{1}{2}\left(\frac{s^2+1}{s^2+4}\right)^{-1/2}\frac{(s^2+4)(2s)-(s^2+1)(2s)}{(s^2+4)^2} = \frac{1}{2}\left(\frac{s^2+4}{s^2+1}\right)^{1/2}\frac{2s[(s^2+4)-(s^2+1)]}{(s^2+4)^2}$$

$$= \frac{(s^2+4)^{1/2}(2s)(3)}{2(s^2+1)^{1/2}(s^2+4)^2} = \frac{3s}{(s^2+1)^{1/2}(s^2+4)^{3/2}}$$

44. $y = 2^{3^{x^2}} \Rightarrow y' = 2^{3^{x^2}}(\ln 2)\dfrac{d}{dx}\left(3^{x^2}\right) = 2^{3^{x^2}}(\ln 2)3^{x^2}(\ln 3)(2x)$

Section 3.5 #12

12. $\dfrac{d}{dx}\cos(xy) = \dfrac{d}{dx}(1+\sin y) \Rightarrow -\sin(xy)(xy' + y\cdot 1) = \cos y \cdot y' \Rightarrow -xy'\sin(xy) - \cos y \cdot y' = y\sin(xy)$

$y'[-x\sin(xy) - \cos y] = y\sin(xy) \Rightarrow y' = \dfrac{y\sin(xy)}{-x\sin(xy)-\cos y} = -\dfrac{y\sin(xy)}{x\sin(xy)+\cos y}$