

MAT 1348 — First Homework Assignment — Due Jan. 25 at 11:30am

1. Write each of the following statements as a compound proposition using correct logical connectives. You must clearly **define the propositional variables** you used in your compound propositions.
 - (a) If the file system is locked or the system is not functioning normally, new messages are not sent to the message buffer.
 - (b) The system not being locked and new messages being queued is a necessary and sufficient condition for the system to be functioning normally.
 - (c) New messages are not queued only if the file system is locked.
 - (d) The system not functioning normally is a necessary condition for new messages not to be sent to the message buffer.
 - (e) New messages are queued unless the system is not functioning normally.
2. Use a **truth table** to determine whether the given set of propositions is **consistent**. Clearly explain what feature of the truth table suggests your answer.

$$\{p \rightarrow \neg(q \wedge r), (\neg p \leftrightarrow r) \vee q, (p \rightarrow \neg q) \wedge \neg(p \vee r)\}$$

3. The table below is a truth table for two mystery compound propositions A and B . Each of these consists of atomic propositions p and q , and logical connectives.

p	q	A	B
T	T	F	T
T	F	T	F
F	T	F	T
F	F	T	T

- (a) Write a **DNF formula** (Disjunctive Normal Form) for each of the compound propositions A and B .
 - (b) Write a formula for each of the compound propositions A and B **using only logical connectives \neg and \vee** .
 - (c) Write a formula for each of the compound propositions A and B **using only logical connectives \neg and \rightarrow** .
4. Consider the following compound propositions:
 - (i) $(\neg x \rightarrow z) \vee (y \rightarrow z)$
 - (ii) $(\neg x \wedge y) \rightarrow z$
 - (iii) $((x \vee y) \wedge (x \rightarrow z) \wedge (y \rightarrow z)) \rightarrow z$

- (a) Determine for each of the compound propositions above whether it is a tautology, a contradiction, or a contingency.
 - (b) For each contingency, give all truth assignments of the propositional variables for which the compound proposition is **false**.
 - (c) Which pairs of compound propositions (if any) in the list above are **logically equivalent**?
5. On the Island of Knights and Knaves you meet two natives, A and B. Given their statements below, what can you determine about their identities? Fully explain your reasoning.
- (a) A says: “I am a knight only if B is a knave” and B says nothing.
 - (b) A says: “I am a knight or B is a knight” and B says “Either I am a knave or A is a knave.”