Université d'Ottawa · University of Ottawa

Faculté de génie
Génie mécanique

Faculty of Engineering
Mechanical Engineering

MCG 3340 FLUID MECHANICS I

Final Examination **Duration: 3 hrs**
December 11, 2009
Professors B. Greenhalgh and C. Mavriplis

Closed Book. No notes permitted. Calculators of All Kinds Permitted.

Marks

1. **18%**

The power requirement **W** of a centrifugal pump is known to depend on:

- Volumetric discharge rate, **Q**
- Fluid density, **ρ**
- Angular speed of the impeller **ω**
- Impeller diameter, **D** and
- Fluid viscosity, **μ.**

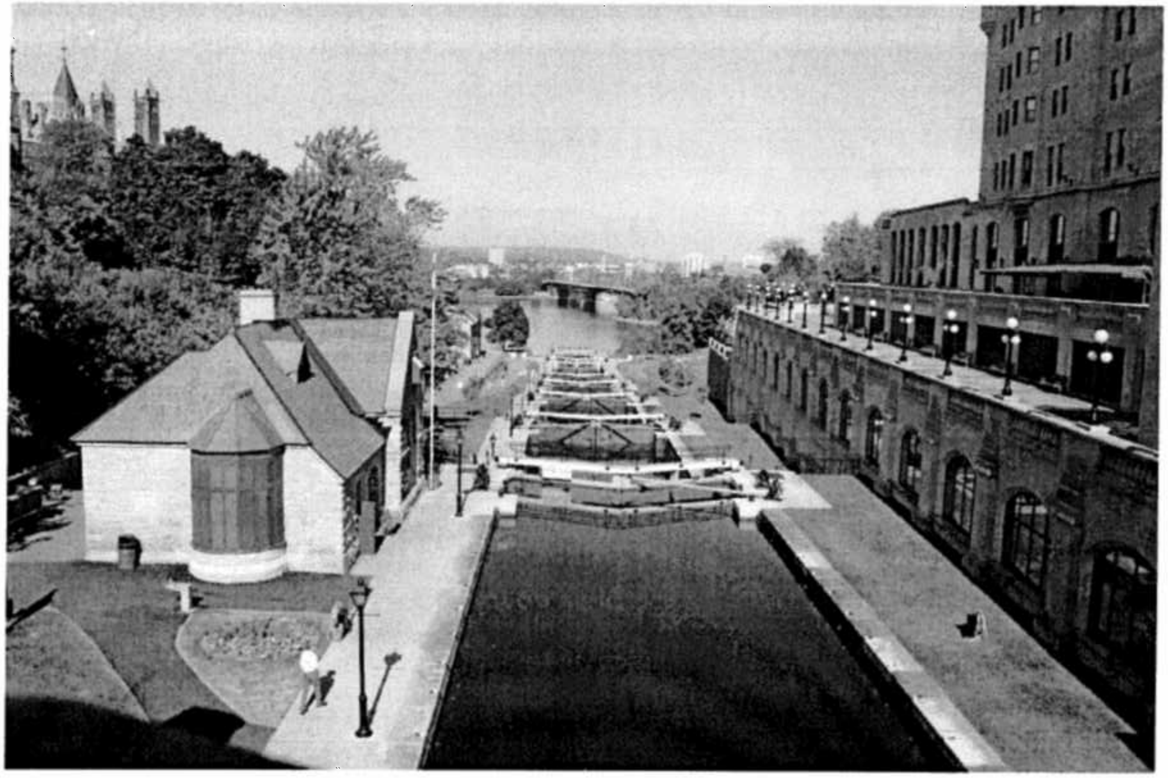
a) Use the Buckingham π theorem to determine the relationship between W, Q, ρ, ω, D and μ.

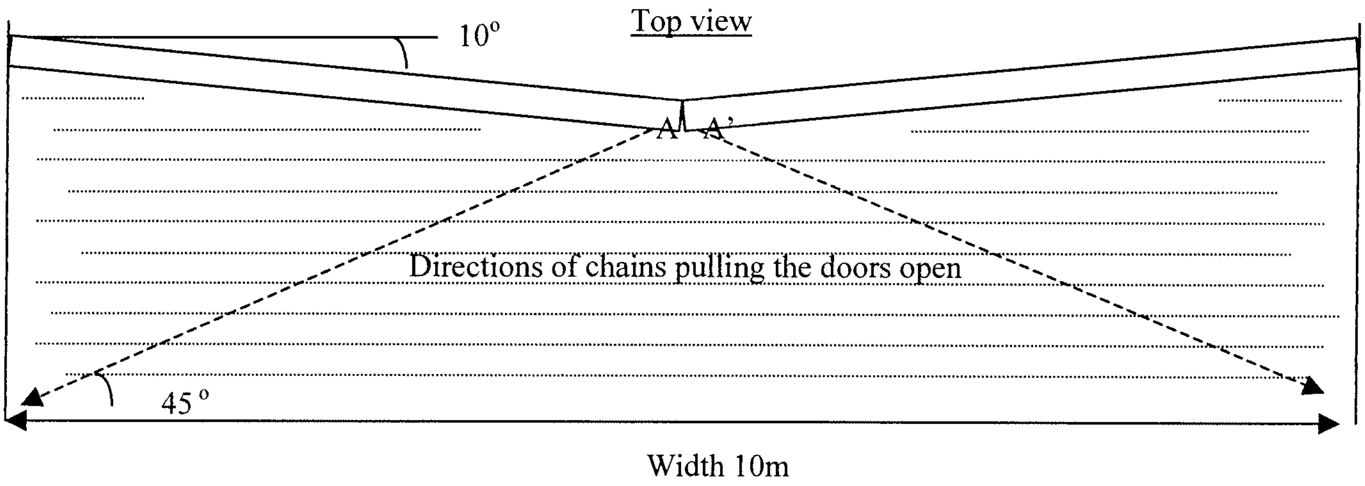
b) A centrifugal pump with a 25cm impeller rotating at 1800 rpm is to discharge water at a rate of 30 L/min. Explain using calculations how you would estimate the power requirements of this pump using a 1:10 scale pump.

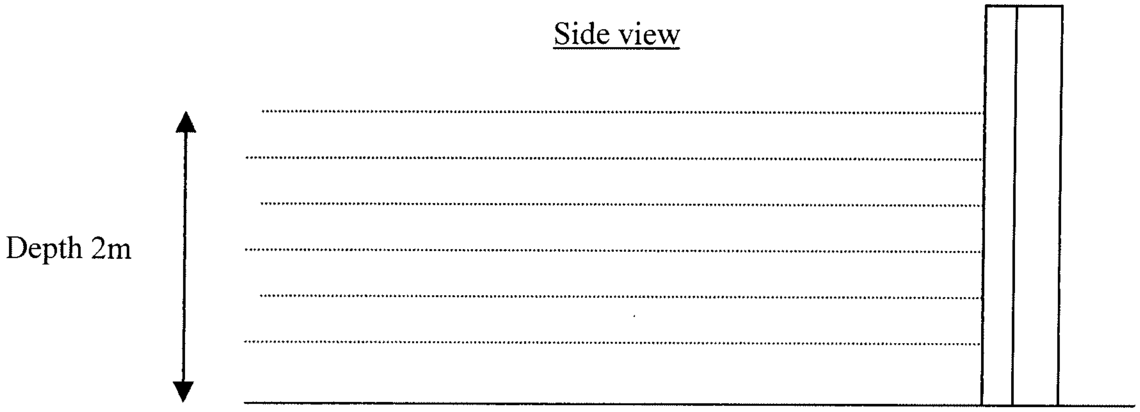
Marks

2. **20%**

Perhaps you recognize the neighbouring Rideau Canal locks alongside the Château Laurier in the picture below. Note that the 1st lock we see is made up of two doors in a "V" formation in the closed position, retaining the water of the canal. The schematic on the next page gives some dimensions for the sake of this exercise. Calculate the force needed to crack open the gate(s) by pulling a chain attached to points A and A' on the schematic.




10°
Top view
A
A'
Directions of chains pulling the doors open
45°
Width 10m

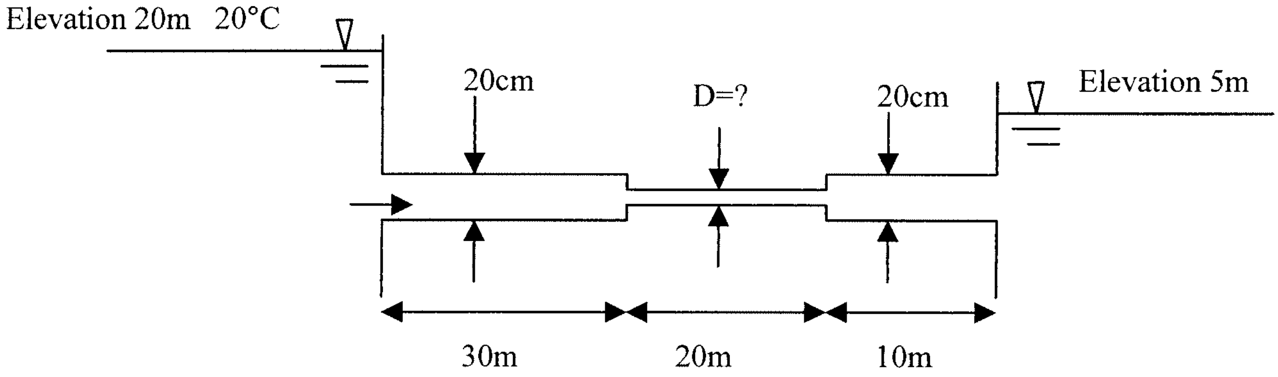

Side view
Depth 2m

Marks

3. **30%**

Your choice of a) or b) (extra points if you do both):

a) Find the minimum allowable diameter D, that avoids cavitation in the water piping system below.

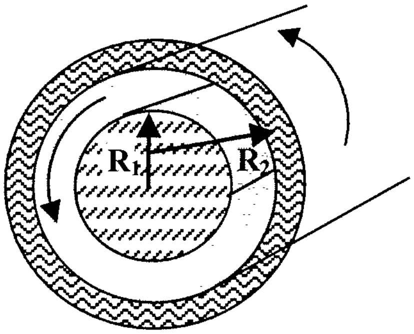


Note that the pipes are made of cast iron and that the ground provides a cooling rate of 1 W/m length of pipe.

What does the flow profile look like in the smallest pipe section?

<u>**NOTE TO 2011 students (we did not cover this material in b) you could do b-iv) and -v) if I gave you the velocity profile)**</u>

b) The viscosity of a fluid can be measured in the following device:



$$\frac{\partial \theta}{\partial t} = \omega(rad/time)$$

The torque required to rotate the outer cylinder at a steady speed of ω (rad/time) is measured, and is related to the fluid viscosity.

b-i) In cylindrical coordinates, continuity is given by:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r V_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho V_\theta)}{\partial \theta} + \frac{\partial(\rho V_z)}{\partial z} = 0$$

Show that for steady, incompressible fluid, that the flow is fully developed.

b-ii) The Navier – Stokes equation for Newtonian laminar flow in the θ direction is given by:

$$\rho\left(\frac{\partial V_\theta}{\partial t}+V_r\frac{\partial V_\theta}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta}+\frac{V_rV_\theta}{r}+V_z\frac{\partial V_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial[rV_\theta]}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial V_r}{\partial \theta}+\frac{\partial^2 V_r}{\partial z^2}\right\}+\rho g_\theta$$

Neglecting gravity and assuming steady incompressible fluid, simplify this equation.

(Hint: you will need to make an argument that $\frac{\partial p}{\partial \theta}=0$. Simply stating "symmetry" is not sufficient. Make a physical argument.)

b-iii) Write down the boundary conditions for this problem.
(Hint: the inner cylinder is held fixed.)

b-iv) Calculate the shear stress which acts on the larger cylinder's inner surface.

b-v) What is the torque required to rotate the outer cylinder at an angular velocity of ω (rad/time)?

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4. **20%**

Below is a photo of a fellow field engineer testing firehoses at a plant. Calculate the force needed to hold the hose. Estimate the parameters from what you see in the photo. Clearly state your assumptions and parameter values at the start.

Marks

5. **22%**

Fundamentals

A) What is a Reynolds Number?

B) What is Laminar Flow?

C) What is Turbulent Flow?

D) How are Reynolds Numbers, Laminar Flow and Turbulent Flow related?

E) Suppose a velocity field is given by u= (2y+f(x)), v=t.

E-i) For incompressible flow, what is f(x)?

E-ii) What if the flow were compressible?

Now assume f(x) = 0 and that f is incompressible.

E-iii) Determine the equation of the streamline through (0,0).

E-iv) Determine the pathline of a particle passing through (1,2) at a time t =2.

E-v) How do streamlines and pathlines differ under steady flow? Explain your answer using a physical argument and/or a simple sketch.

E-vi) Determine the stream function corresponding to your answer in c).

Recall: $\frac{\partial \Psi}{\partial x} = -v$, $\frac{\partial \Psi}{\partial y} = u$ for incompressible flow.

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F) A pitot tube (shown in the picture below) is a device which measures the speed of an aircraft. Using the Bernoulli equation, explain how such a device can be used to calculate fluid velocity.

Total (including a 10% bonus) **110%**

TABLE A.3 Properties of the U.S. Standard Atmosphere (Data from [7])

Geometric Altitude (m)	Temperature (K)	p/p_{SL} (—)	ρ/ρ_{SL} (—)
−500	291.4	1.061	1.049
0	288.2	1.000[a]	1.000[b]
500	284.9	0.9421	0.9529
1,000	281.7	0.8870	0.9075
1,500	278.4	0.8345	0.8638
2,000	275.2	0.7846	0.8217
2,500	271.9	0.7372	0.7812
3,000	268.7	0.6920	0.7423
3,500	265.4	0.6492	0.7048
4,000	262.2	0.6085	0.6689
4,500	258.9	0.5700	0.6343
5,000	255.7	0.5334	0.6012
6,000	249.2	0.4660	0.5389
7,000	242.7	0.4057	0.4817
8,000	236.2	0.3519	0.4292
9,000	229.7	0.3040	0.3813
10,000	223.3	0.2615	0.3376
11,000	216.8	0.2240	0.2978
12,000	216.7	0.1915	0.2546
13,000	216.7	0.1636	0.2176
14,000	216.7	0.1399	0.1860
15,000	216.7	0.1195	0.1590
16,000	216.7	0.1022	0.1359
17,000	216.7	0.08734	0.1162
18,000	216.7	0.07466	0.09930
19,000	216.7	0.06383	0.08489
20,000	216.7	0.05457	0.07258
22,000	218.6	0.03995	0.05266
24,000	220.6	0.02933	0.03832
26,000	222.5	0.02160	0.02797
28,000	224.5	0.01595	0.02047
30,000	226.5	0.01181	0.01503
40,000	250.4	0.002834	0.003262
50,000	270.7	0.0007874	0.0008383
60,000	255.8	0.0002217	0.0002497
70,000	219.7	0.00005448	0.00007146
80,000	180.7	0.00001023	0.00001632
90,000	180.7	0.000001622	0.000002588

[a] $p_{SL} = 1.01325 \times 10^5$ N/m^2 (abs) (= 14.696 psia).

[b] $\rho_{SL} = 1.2250$ kg/m^3 (= 0.002377 slug/ft^3).

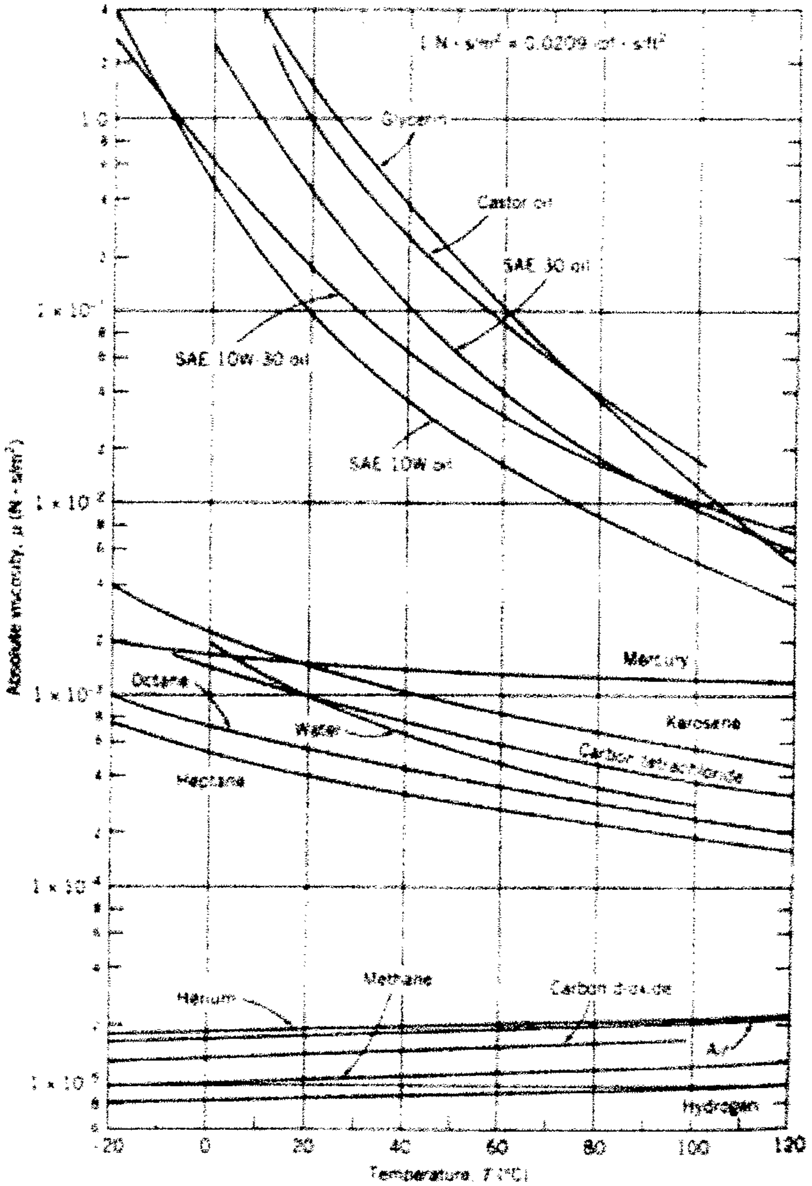

Fig. A.2 Dynamic (absolute) viscosity of common fluids as a function of temperature. (Data from [1, 6, and 10].)

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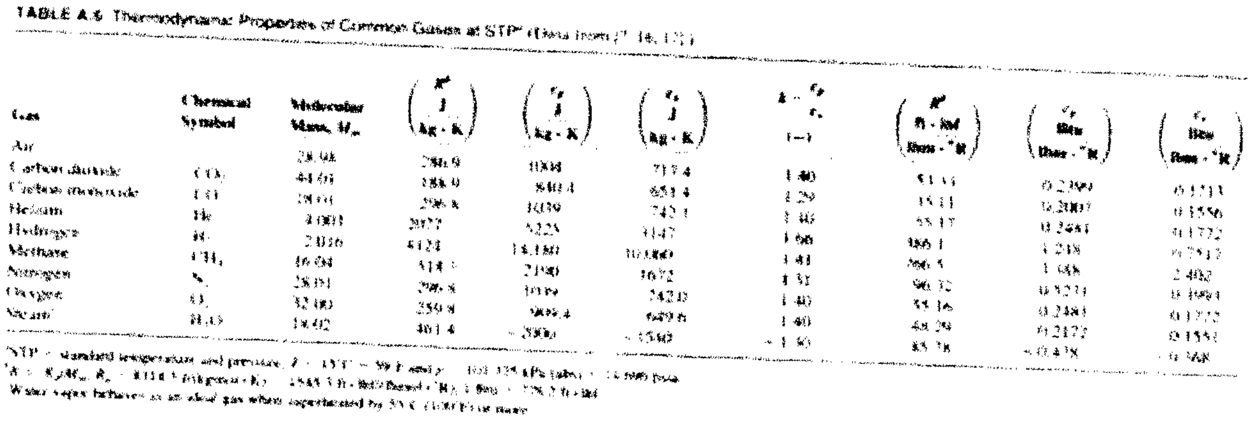

TABLE A.6 Thermodynamic Properties of Common Gases at STP^a (Data from [7, 16, 17])

Gas	Chemical Symbol	Molecular Mass, M_m	$R^b\left(\frac{J}{kg \cdot K}\right)$	$c_p\left(\frac{J}{kg \cdot K}\right)$	$c_v\left(\frac{J}{kg \cdot K}\right)$	$k = \frac{c_p}{c_v}$ (—)	$R^b\left(\frac{ft \cdot lbf}{lbm \cdot {}^\circ R}\right)$	$c_p\left(\frac{Btu}{lbm \cdot {}^\circ R}\right)$	$c_v\left(\frac{Btu}{lbm \cdot {}^\circ R}\right)$
Air		28.98	286.9	1004	717.4	1.40	53.33	0.2399	0.1713
Carbon dioxide	CO_2	44.01	188.9	840.4	651.4	1.29	35.11	0.2007	0.1556
Carbon monoxide	CO	28.01	296.8	1039	742.1	1.40	55.17	0.2481	0.1772
Helium	He	4.003	2077	5225	3147	1.66	386.1	1.248	0.7517
Hydrogen	H_2	2.016	4124	14,180	10,060	1.41	766.5	3.388	2.402
Methane	CH_4	16.04	518.3	2190	1672	1.31	96.32	0.5231	0.3993
Nitrogen	N_2	28.01	296.8	1039	742.0	1.40	55.16	0.2481	0.1772
Oxygen	O_2	32.00	259.8	909.4	649.6	1.40	48.29	0.2172	0.1551
Steam^c	H_2O	18.02	461.4	~2000	~1540	~1.30	85.78	~0.478	~0.368

^{a}STP = standard temperature and pressure, $T = 15°C = 59°F$ and $p = 101.325$ kPa (abs) = 14.696 psia.
$^b R \equiv R_u/M_m$; $R_u = 8314.3$ J/(kgmole · K) = 1545.3 ft · lbf/(lbmole · °R); 1 Btu = 778.2 ft · lbf.
^{c}Water vapor behaves as an ideal gas when superheated by 55°C (100°F) or more.

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TABLE A.8 Properties of Water (SI Units)

Temperature, T (°C)	Density, ρ (kg/m^3)	Dynamic Viscosity, μ (N · s/m^2)	Kinematic Viscosity, ν (m^2/s)	Surface Tension, σ (N/m)	Vapor Pressure, p_v (kPa)	Bulk Modulus, E_v (GPa)
0	1000	1.76E-03	1.76E-06	0.0757	0.661	2.01
5	1000	1.51E-03	1.51E-06	0.0749	0.872	
10	1000	1.30E-03	1.30E-06	0.0742	1.23	
15	999	1.14E-03	1.14E-06	0.0735	1.71	
20	998	1.01E-03	1.01E-06	0.0727	2.34	2.21
25	997	8.93E-04	8.96E-07	0.0720	3.17	
30	996	8.00E-04	8.03E-07	0.0712	4.25	
35	994	7.21E-04	7.25E-07	0.0704	5.63	
40	992	6.53E-04	6.59E-07	0.0696	7.38	
45	990	5.95E-04	6.02E-07	0.0688	9.59	
50	988	5.46E-04	5.52E-07	0.0679	12.4	2.29
55	986	5.02E-04	5.09E-07	0.0671	15.8	
60	983	4.64E-04	4.72E-07	0.0662	19.9	
65	980	4.31E-04	4.40E-07	0.0654	25.0	
70	978	4.01E-04	4.10E-07	0.0645	31.2	
75	975	3.75E-04	3.85E-07	0.0636	38.6	
80	972	3.52E-04	3.62E-07	0.0627	47.4	
85	969	3.31E-04	3.41E-07	0.0618	57.8	
90	965	3.12E-04	3.23E-07	0.0608	70.1	2.12
95	962	2.95E-04	3.06E-07	0.0599	84.6	
100	958	2.79E-04	2.92E-07	0.0589	101	

TABLE A.10 Properties of Air at Atmospheric Pressure (SI Units)

Temperature, T (°C)	Density, ρ (kg/m^3)	Dynamic Viscosity, μ (N · s/m^2)	Kinematic Viscosity, ν (m^2/s)
0	1.29	1.72E-05	1.33E-05
5	1.27	1.74E-05	1.37E-05
10	1.25	1.76E-05	1.41E-05
15	1.23	1.79E-05	1.45E-05
20	1.21	1.81E-05	1.50E-05
25	1.19	1.84E-05	1.54E-05
30	1.17	1.86E-05	1.59E-05
35	1.15	1.88E-05	1.64E-05
40	1.13	1.91E-05	1.69E-05
45	1.11	1.93E-05	1.74E-05
50	1.09	1.95E-05	1.79E-05
55	1.08	1.98E-05	1.83E-05
60	1.06	2.00E-05	1.89E-05
65	1.04	2.02E-05	1.94E-05
70	1.03	2.04E-05	1.98E-05
75	1.01	2.06E-05	2.04E-05
80	1.00	2.09E-05	2.09E-05
85	0.987	2.11E-05	2.14E-05
90	0.973	2.13E-05	2.19E-05
95	0.960	2.15E-05	2.24E-05
100	0.947	2.17E-05	2.29E-05

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Appendix B

Equations of Motion in Cylindrical Coordinates

The continuity equation in cylindrical coordinates for constant density is

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0 \qquad \text{(B.1)}$$

Normal and shear stresses in cylindrical coordinates for constant density and viscosity are

$$\sigma_{rr} = -p + 2\mu\frac{\partial v_r}{\partial r} \qquad \tau_{r\theta} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$\sigma_{\theta\theta} = -p + 2\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) \qquad \tau_{\theta z} = \mu\left(\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right)$$

$$\sigma_{zz} = -p + 2\mu\frac{\partial v_z}{\partial z} \qquad \tau_{zr} = \mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right) \qquad \text{(B.2)}$$

The Navier-Stokes equations in cylindrical coordinates for constant density and viscosity are

r component:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right)$$

$$= \rho g_r - \frac{\partial p}{\partial r} + \mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right\} \qquad \text{(B.3a)}$$

θ component:

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right)$$

$$= \rho g_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right\} \qquad \text{(B.3b)}$$

z component:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right)$$

$$= \rho g_z - \frac{\partial p}{\partial z} + \mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right\} \qquad \text{(B.3c)}$$

TABLE 8.1 Roughness for Pipes of Common Engineering Materials (Data from [8])

Pipe	Roughness, *e* Feet	Roughness, *e* Millimeters
Riveted steel	0.003–0.03	0.9–9
Concrete	0.001–0.01	0.3–3
Wood stave	0.0006–0.003	0.2–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015

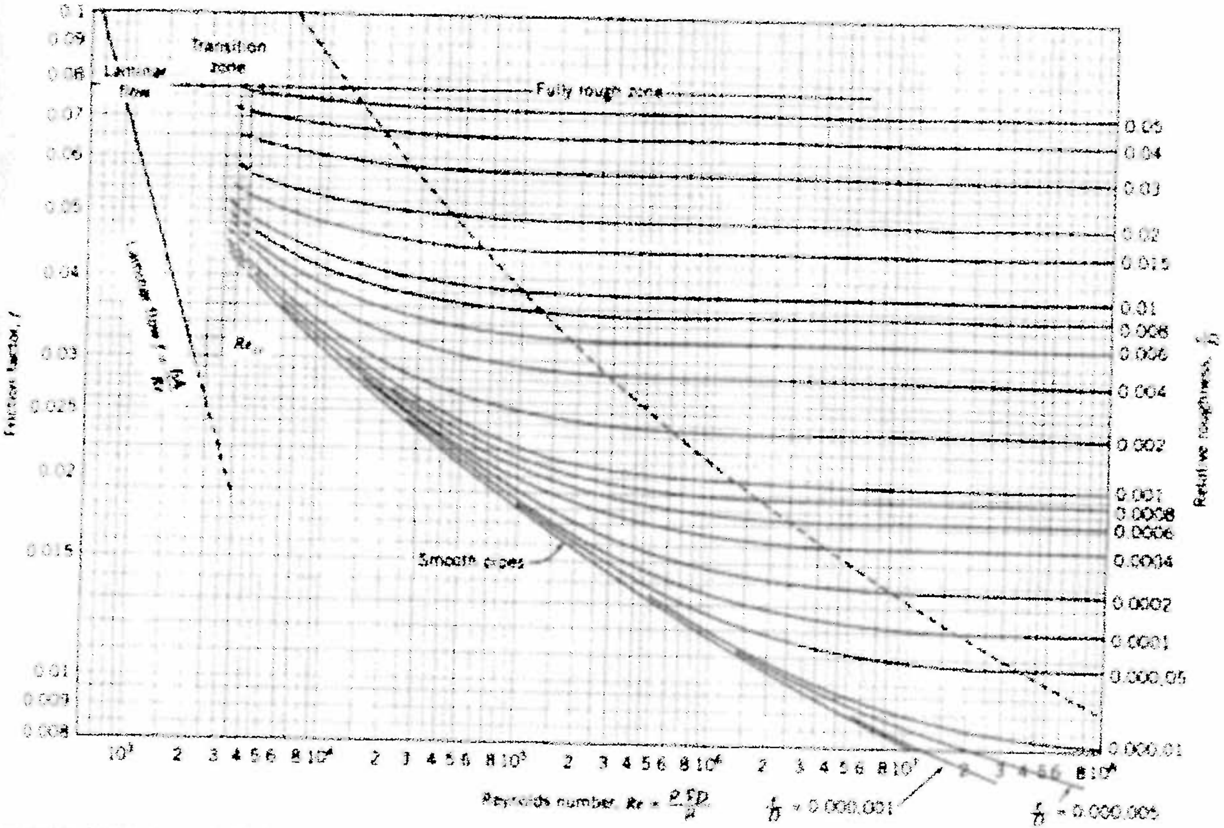

Fig. 8.13 Friction factor for fully developed flow in circular pipes. (Data from [8], used by permission.)