Physics 201: Discussion 04

57. A stone at the end of a sling is whirled in a vertical circle of radius 1.20 m at a constant speed $v_0 = 1.50$ m/s as in Figure P4.57. The center of the sling is 1.50 m above the ground. What is the range of the stone if it is released when the sling is inclined at 30.0° with the horizontal (a) at A? (b) at B? What is the acceleration of the stone (c) just before it is released at A? (d) just after it is released at A?

P4.57 Choose upward as the positive y-direction and leftward as the positive x-direction. The vertical height of the stone when released from A or B is

$$y_i = (1.50 + 1.20\sin 30.0°)\text{ m} = 2.10\text{ m}$$

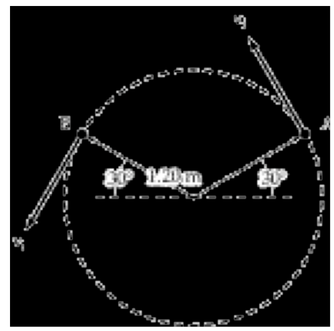


FIG. P4.57

(a) The equations of motion after release at A are

$$v_y = v_i \sin 60.0° - gt = (1.30 - 9.80t)\text{ m/s}$$
$$v_x = v_i \cos 60.0° = 0.750\text{ m/s}$$
$$y = (2.10 + 1.30t - 4.90t^2)\text{ m}$$
$$\Delta x_A = (0.750t)\text{ m}$$

When $y = 0$, $t = \dfrac{-1.30 \pm \sqrt{(1.30)^2 + 41.2}}{-9.80} = 0.800\text{ s}$. Then, $\Delta x_A = (0.750)(0.800)\text{ m} = \boxed{0.600\text{ m}}$.

(b) The equations of motion after release at point B are

$$v_y = v_i(-\sin 60.0°) - gt = (-1.30 - 9.80t)\text{ m/s}$$
$$v_x = v_i \cos 60.0 = 0.750\text{ m/s}$$
$$y_i = (2.10 - 1.30t - 4.90t^2)\text{ m}.$$

When $y = 0$, $t = \dfrac{+1.30 \pm \sqrt{(-1.30)^2 + 41.2}}{-9.80} = 0.536\text{ s}$. Then, $\Delta x_B = (0.750)(0.536)\text{ m} = \boxed{0.402\text{ m}}$.

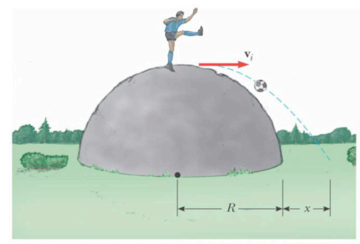
(c) $a_r = \dfrac{v^2}{r} = \dfrac{(1.50\text{ m/s})^2}{1.20\text{ m}} = \boxed{1.87\text{ m/s}^2\text{ toward the center}}$

(d) After release, $\mathbf{a} = -g\hat{\mathbf{j}} = \boxed{9.80\text{ m/s}^2\text{ downward}}$

62. A person standing at the top of a hemispherical rock of radius R kicks a ball (initially at rest on the top of the rock) to give it horizontal velocity $\mathbf{v}_i$ as in Figure P4.62. (a) What must be its minimum initial speed if the ball is never to hit the rock after it is kicked? (b) With this initial speed, how far from the base of the rock does the ball hit the ground?

P4.62 Measure heights above the level ground. The elevation y_b of the ball follows

$$y_b = R + 0 - \frac{1}{2}gt^2$$



with $x = v_i t$ so $y_b = R - \dfrac{gx^2}{2v_i^2}$.

(a) The elevation y_r of points on the rock is described by

$$y_r^2 + x^2 = R^2.$$

We will have $y_b = y_r$ at $x = 0$, but for all other x we require the ball to be above the rock surface as in $y_b > y_r$. Then $y_b^2 + x^2 > R^2$

$$\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 > R^2$$

$$R^2 - \frac{gx^2R}{v_i^2} + \frac{g^2x^4}{4v_i^4} + x^2 > R^2$$

$$\frac{g^2x^4}{4v_i^4} + x^2 > \frac{gx^2R}{v_i^2}.$$

If this inequality is satisfied for x approaching zero, it will be true for all x. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock: $1 > \dfrac{gR}{v_i^2}$

$$\boxed{v_i > \sqrt{gR}}.$$

(b) With $v_i = \sqrt{gR}$ and $y_b = 0$, we have $0 = R - \dfrac{gx^2}{2gR}$

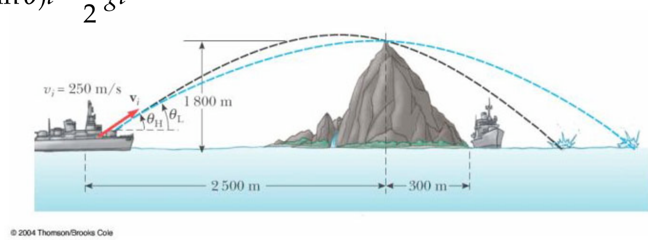
or $x = R\sqrt{2}$.

The distance from the rock's base is $x - R = \boxed{\left(\sqrt{2} - 1\right)R}$

71. An enemy ship is on the east side of a mountain island, as shown in Figure P4.71. The enemy ship has maneuvered to within 2 500 m of the 1 800-m-high mountain peak and can shoot projectiles with an initial speed of 250 m/s. If the western shoreline is horizontally 300 m from the peak, what are the distances from the western shore at which a ship can be safe from the bombardment of the enemy ship?

***P4.71** Find the highest firing angle θ_H for which the projectile will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest firing angle; this will yield the maximum range under these conditions if both θ_H and θ_L are $> 45°$; $x = 2500\text{ m}$, $y = 1800\text{ m}$, $v_i = 250\text{ m/s}$.

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = v_i(\sin\theta)t - \frac{1}{2}gt^2$$
$$x_f = v_{xi}t = v_i(\cos\theta)t$$



Thus $t = \dfrac{x_f}{v_i\cos\theta}$.

Substitute into the expression for y_f

$$y_f = v_i(\sin\theta)\frac{x_f}{v_i\cos\theta} - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta}\right)^2 = x_f\tan\theta - \frac{gx_f^2}{2v_i^2\cos^2\theta}$$

but $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$ so $y_f = x_f\tan\theta - \dfrac{gx_f^2}{2v_i^2}\left(\tan^2\theta + 1\right)$ and

$$0 = \frac{gx_f^2}{2v_i^2}\tan^2\theta - x_f\tan\theta + \frac{gx_f^2}{2v_i^2} + y_f.$$

Substitute values, use the quadratic formula and find

$\tan\theta = 3.905$ or 1.197, which gives $\theta_H = 75.6°$ and $\theta_L = 50.1°$.

Range $(\text{at }\theta_H) = \dfrac{v_i^2\sin 2\theta_H}{g} = 3.07\times10^3\text{ m}$ from enemy ship

$3.07\times10^3 - 2\,500 - 300 = 270\text{ m}$ from shore.

Range $(\text{at }\theta_L) = \dfrac{v_i^2\sin 2\theta_L}{g} = 6.28\times10^3\text{ m}$ from enemy ship

$6.28\times10^3 - 2\,500 - 300 = 3.48\times10^3$ from shore.

Therefore, safe distance is $\boxed{< 270\text{ m}}$ or $\boxed{> 3.48\times10^3\text{ m}}$ from the shore.

72. In the **What If**? section of Example 4.7, it was claimed that the maximum range of a ski jumper occurs for a launch angle θ given by

$$\theta = 45° - \frac{\phi}{2}$$

where is the angle that the hill makes with the horizontal in Figure 4.16. Prove this claim by deriving the equation above.

***P4.72** We follow the steps outlined in Example 4.7, eliminating $t = \frac{d\cos\phi}{v_i\cos\theta}$ to find

$$\frac{v_i\sin\theta\, d\cos\phi}{v_i\cos\theta} - \frac{gd^2\cos^2\phi}{2v_i^2\cos^2\theta} = -d\sin\phi\,.$$

Clearing of fractions,

$$2v_i^2\cos\theta\sin\theta\cos\phi - gd\cos^2\phi = -2v_i^2\cos^2\theta\sin\phi\,.$$

To maximize d as a function of θ, we differentiate through with respect to θ and set $\frac{dd}{d\theta} = 0$:

$$2v_i^2\cos\theta\cos\theta\cos\phi + 2v_i^2\sin\theta(-\sin\theta)\cos\phi - g\frac{dd}{d\theta}\cos^2\phi = -2v_i^2 2\cos\theta(-\sin\theta)\sin\phi\,.$$

We use the trigonometric identities from Appendix B4 $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to find $\cos\phi\cos 2\theta = \sin 2\theta\sin\phi$. Next, $\frac{\sin\phi}{\cos\phi} = \tan\phi$ and $\cot 2\theta = \frac{1}{\tan 2\theta}$ give $\cot 2\phi = \tan\phi = \tan(90° - 2\theta)$ so $\phi = 90° - 2\theta$ and $\theta = 45° - \frac{\phi}{2}$.