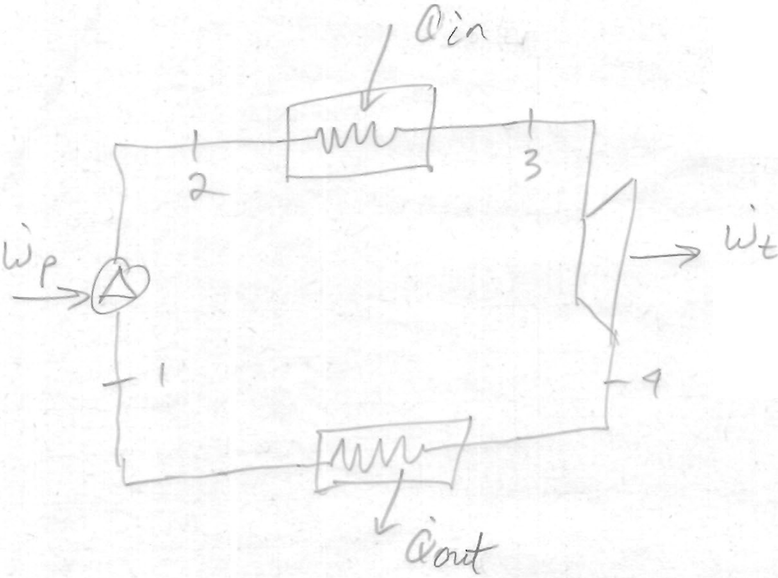
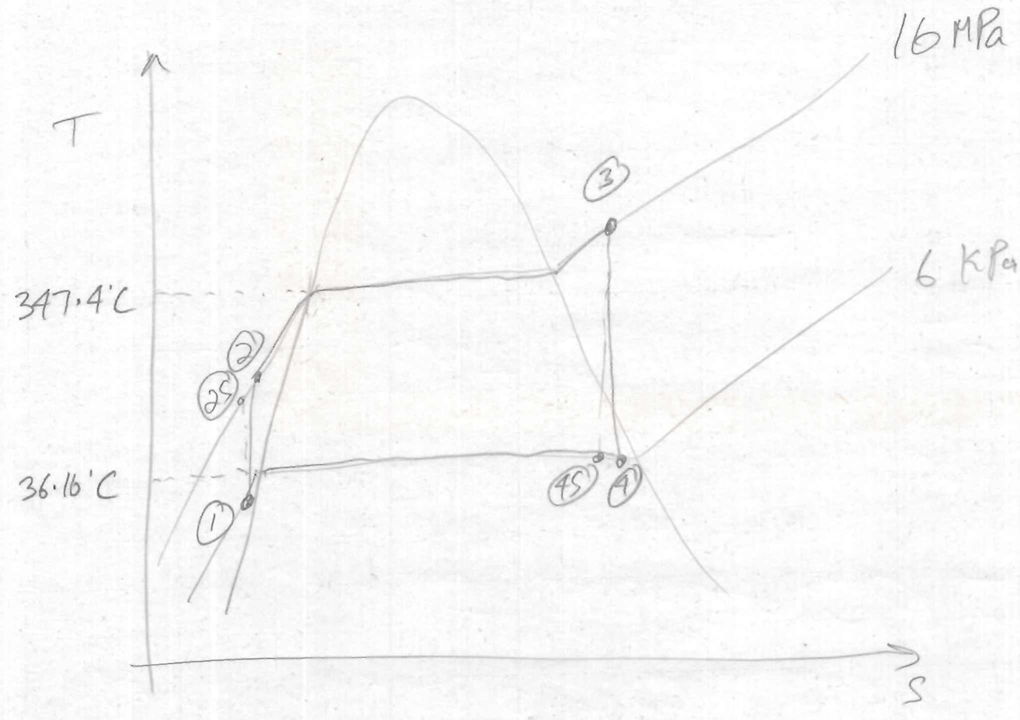
Rankine Cycle Question



a)



b)

$$\left.\begin{aligned} T_{sat}\,(16\,MPa) &= 347.4°C \\ T_{sat}\,(6\,KPa) &= 36.16°C \end{aligned}\right\}\ \text{Table A-3}$$

Given: * State ① 6 KPa and 30°C $\Rightarrow T_1 < T_{sat}(6\,KPa)$ so subcooled liquid.

* $\dot{Q}_{in} = 1.5\,GW$

* $\dot{m} = 440\ \frac{kg}{s}$

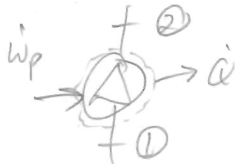
* $P_2 = P_3 = 16\ MPa$

* $P_1 = P_4 = 6\ KPa$

* $\eta_t = 0.91$

* $\eta_p = 0.65$

<u>pump</u>



$$\frac{d\cancel{E_{cv}}}{dt} = \dot{W}_p - \cancel{\dot{Q}} + \dot{m}\left(h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right) - \dot{m}\left(h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{gz_2}\right)$$

SS assume adiabatic ignore ΔKE & ΔPE

$$\frac{\dot{W}_p}{\dot{m}} = h_2 - h_1$$

$$\frac{\dot{W}_{ps}}{\dot{m}} = h_{2s} - h_1$$

$$T\cancel{ds} = dh - v\,dP$$

$$\int_1^{2s} dh = \int_1^{2s} v\,dP$$

$$h_{2s} - h_1 \approx v_1 (P_2 - P_1)$$ since can treat subcooled liquid as incompressible

$$v_1 \approx v_f(T_1) = 1.0043 \cdot 10^{-3}\ \frac{m^3}{kg}$$ (Table A-2).

$$h_1 \approx h_f(T_1) = 125.79\ \frac{kJ}{kg}$$ (Table A-2).

$$\frac{\dot{W}_{ps}}{\dot{m}} = h_{2s} - h_1 \approx v_1 (P_2 - P_1)$$

$$= \left(1.0043 \cdot 10^{-3} \frac{m^3}{kg}\right)(16\,000 - 6)\ kPa \left(\frac{1\ kN/m^2}{kPa}\right)\left(\frac{1\ kJ}{kN \cdot m}\right)$$

$$= 16.06\ \frac{kJ}{kg}$$

$$\eta_p = \frac{\dot{W}_{ps}/\dot{m}}{\dot{W}_p/\dot{m}} \Rightarrow \frac{\dot{W}_p}{\dot{m}} = \frac{16.06\ \frac{kJ}{kg}}{0.65}$$

$$\frac{\dot{W}_p}{\dot{m}} = 24.71\ \frac{kJ}{kg}$$

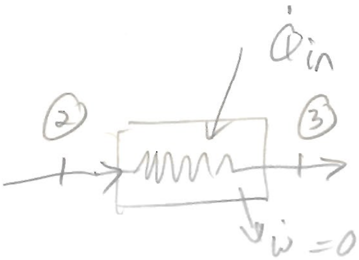
$$\frac{\dot{W}_p}{\dot{m}} = h_2 - h_1 \Rightarrow h_2 = 125.79 + 24.71\ \left(\frac{kJ}{kg}\right)$$

d) $\boxed{h_2 = 150.5\ \frac{kJ}{kg}}$

$$\dot{W}_p = \left(440\ \frac{kg}{s}\right)\left(24.71\ \frac{kJ}{kg}\right)\left(\frac{1\ kW}{kJ/s}\right)\left(\frac{1\ MW}{1000\ kW}\right)$$

c) $\boxed{\dot{W}_p = 10.9\ MW}$

boiler



$$\underbrace{\frac{dE_{cv}}{dt}}_{SS} = \dot{Q}_{in} - \underbrace{\dot{W}}_{\text{no work}} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_2 - \dot{m}\left(h + \frac{V^2}{2} + gz\right)_3$$

ignore ΔKE & ΔPE effects

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_3 - h_2)$$

$$h_3 = h_2 + \frac{\dot{Q}_{in}}{\dot{m}}$$

$$= 150.5 \ \frac{kJ}{kg} + \frac{(1.5 \ GW)\left(\frac{10^6 \ kW}{GW}\right)\left(\frac{1 \ kJ/s}{kW}\right)}{440 \ \frac{kg}{s}}$$

$$= 3559.6 \ \frac{kJ}{kg}$$ Superheated steam by Table A-4.

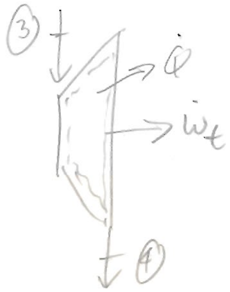
Table A-4 @ 16 MPa: 560°C $3465.4 \ \frac{kJ}{kg}$
600°C $3573.5 \ \frac{kJ}{kg}$ } Table A-4

$$\frac{T_3 - 560}{600 - 560} = \frac{3559.6 - 3465.4}{3573.5 - 3465.4}$$

$$\boxed{\Rightarrow T_3 = 595°C}$$

e)

turbine



SS   assume adiabatic   neglect ΔKE & ΔPE effects

$$\cancel{\frac{dE_{cv}}{dt}} = -\dot{W}_t - \cancel{\dot{Q}} + \dot{m}\left(h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right) - \dot{m}\left(h_4 + \cancel{\frac{V_4^2}{2}} + \cancel{gz_4}\right)$$

$$\frac{\dot{W}_t}{\dot{m}} = (h_3 - h_4)$$

$$\frac{\dot{W}_{ts}}{\dot{m}} = h_3 - h_{4s}$$

(3) 16 MPa, 595°C

$h_3 = 3559.6 \frac{KJ}{Kg}$ (from boiler)

$S_3 = 6.6236 \frac{KJ}{KgK}$ (interpolated from A-4)

(4s) 6 KPa

$S_{4s} = S_3 = 6.6236 \frac{KJ}{KgK}$

$\left.\begin{array}{l} S_f = 0.5210 \frac{KJ}{KgK} \\ S_g = 8.3304 \frac{KJ}{KgK} \end{array}\right\}$ Table A-3

$$X_{4s} = \frac{S_{4s} - S_f}{S_g - S_f} = 0.781$$

$\left.\begin{array}{l} h_f = 151.53 \frac{KJ}{Kg} \\ h_{fg} = 2415.9 \frac{KJ}{Kg} \end{array}\right\}$ Table A-3

$$\therefore h_{4s} = 151.53 \frac{kJ}{kg} + (0.781)\left(2415.9 \frac{kJ}{kg}\right)$$

$$= 2039.4 \frac{kJ}{kg}$$

$$\eta_t = \frac{\dot{W}_t/\dot{m}}{\dot{W}_{ts}/\dot{m}} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$\Rightarrow h_4 = 3559.6 \frac{kJ}{kg} - (0.91)(3559.6 - 2039.4)\frac{kJ}{kg}$$

$$= 2176.2 \frac{kJ}{kg}$$

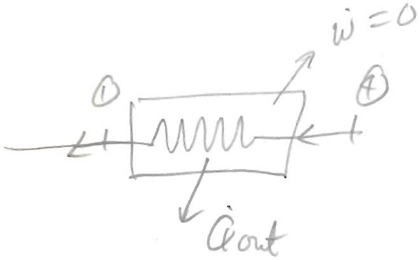
$$\dot{W}_t = \dot{m}(h_3 - h_4) = \left(440 \frac{kg}{s}\right)(3559.6 - 2176.2)\frac{kJ}{\cancel{kg}}$$

g) $\boxed{\dot{W}_t = 608.7 \text{ MW}}$

$$X_4 = \frac{h_4 - h_f}{h_{fg}} = \frac{2176.2 - 151.53}{2415.9}$$

f) $\boxed{X_4 = 0.838}$

condenser



ss | no work | neglect ΔKE & ΔPE effects

$$\cancel{\frac{dE_{cv}}{dt}} = -\dot{Q}_{out} - \cancel{\dot{W}} + \dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)_4 - \dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)_1$$

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_4 - h_1$$

$$\dot{Q}_{out} = \left(440\ \frac{kg}{s}\right)(2176.2 - 125.79)\ \frac{kJ}{kg}$$

h) $$\boxed{\dot{Q}_{out} = 902.2\ MW}$$

check EB on entire cycle

$$\dot{W}_p + \dot{Q}_{in} \overset{?}{=} \dot{W}_t + \dot{Q}_{out}$$

$$10.9\ MW + 1500\ MW \overset{?}{=} 608.7\ MW + 902.2\ MW \ \checkmark$$

bwr

$$bwr = \frac{\dot{W}_p}{\dot{W}_t} = \frac{10.9\ MW}{608.7\ MW}$$

i) $\boxed{bwr = 0.018}$

cycle efficiency

$$\eta_{cycle} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_t - \dot{W}_p}{\dot{Q}_{in}} = \frac{608.7 - 10.9}{1500} \frac{(MW)}{(MW)}$$

j) $\boxed{\eta_{cycle} = 0.399}$

k) $X_4 = 0.838$. This is far too long. This much liquid will cause material and structural degradation to the turbine.

The quality at state 4 could be raised by further superheating state 3 (i.e. raise its temperature).

another method to raise the quality at state 4 would be to have two-stage expansion in the turbine with reheat between the stages.