**MAT 2379 A, Final Examination (with Solutions)**

**December 18, 2012** **Professor Raluca Balan**
**Time: 3 hours**

**Student Number:**____________________ **Seat Number:** ______________

**Family Name:** ______________________ **First Name:** ________________

- **This is a closed book examination. Only TI 30 and Casio calculators are permitted.**
- **Record your answer to each question in the table below. Each question is worth 1 mark.**
- **At the end of the examination, hand in only this page.**

| Question | Answer | Question | Answer |
|---|---|---|---|
| 1 | D | 14 | B |
| 2 | E | 15 | B |
| 3 | B | 16 | D |
| 4 | D | 17 | B |
| 5 | E | 18 | B |
| 6 | A | 19 | B |
| 7 | A | 20 | A |
| 8 | E | 21 | E |
| 9 | B | 22 | B |
| 10 | A | 23 | B |
| 11 | E | 24 | C |
| 12 | C | 25 | B |
| 13 | C | | |

**Professor's use only:** Grade=______/25

1. Consider the following R output:

```
> pbinom(35,125,0.35)
[1] 0.05909366
> pbinom(36,125,0.35)
[1] 0.08553514
> pbinom(37,125,0.35)
[1] 0.1197826
> pbinom(42,125,0.35)
[1] 0.4108588
> pbinom(43,125,0.35)
[1] 0.4850622
> pbinom(44,125,0.35)
[1] 0.559525
```

Let $X$ be a binomial random variable with $n = 125$ and $p = 0.35$. Using the R output above, calculate $P(37 < X < 43)$.

A) 0.3653 B) 0.5152 C) 0.3885 D) 0.2911 E) 0.3015

*Solution:* This problem is based on Section 6.2. We have:

$$P(37 < X < 43) = P(38 \leq X \leq 42) = P(X \leq 42) - P(X \leq 37) = 0.4108588 - 0.1197826 = 0.$$

The answer is D. The wrong answer A is obtained computing

$$P(X \leq 43) - P(X \leq 37) = 0.4850622 - 0.1197826 = 0.3652796$$

2. Hydrocarbons emitted by the exhaust systems of automobiles are some of the major contributors to air pollution. Let $X$ be the number of grams of hydrocarbons emitted by an automobile per mile. Assume that $X$ is normally distributed with a mean of 1 g and a standard deviation of 0.25 g. We claim that 89.97% of automobiles emit more than $x_0$ grams of hydrocarbons per mile. What is the value of $x_0$?

A) 0.75 B) 1.25 C) 0.59 D) 1.95
E) 0.68

*Solution:* This problem is based on Section 7.2. We would like to find $x_0$ such that $P(X > x_0) = 0.8997$. This means that $P(X \leq x_0) = 0.1003$.

By standardization,

$$0.1003 = P(X \leq x_0) = P\left(\frac{X-1}{0.25} \leq \frac{x_0-1}{0.25}\right) = P\left(Z \leq \frac{x_0-1}{0.25}\right).$$

From Table 17.2, we find $\frac{x_0-1}{0.25} = -1.28$ Hence

$$x_0 = 1 - (1.28)(0.25) = 0.68.$$

The answer is E.

3. It is known that in Canada, the blood types have the following distribution: 46% O, 42% A, 9% B, 3% AB. What is the probability that in a randomly chosen Canadian couple, the man and woman do not have the same blood type? Assume that the man's blood type is independent of the woman's blood type.

   A) 0.397 B) 0.603 C) 0.2116 D) 0.1764 E) 0.7512

   *Solution:* This problem is using material from Sections 3.2 and 5.1. Since the event that the woman has type O blood is independent of the event that the man has type O blood, the probability that both have blood type O is $(0.46)^2 = 0.2116$ (see Section 5.1). The probability that both have blood type A is $(0.42)^2 = 0.1764$. The probability that both have blood type B is $(0.09)^2 = 0.0081$. The probability that both have blood type AB is $(0.03)^2 = 0.0009$. Hence, by the addition rule (Section 3.2), the probability that both man and woman have the same blood type is:

   $$0.2116 + 0.1764 + 0.0081 + 0.0009 = 0.397$$

   The probability that they do not have the same blood type is $1 - 0.397 = 0.603$. The answer is B.

4. Recent data suggests that the number of heart attacks are higher in the days following a severe winter storm, due to snow shoveling. The maximum heart rate $\text{HR}_{\text{max}}$, the highest heart rate an individual can achieve without severe problems through exercise stress, depends on age and is calculated using the formula $\text{HR}_{\text{max}} = 220 - \text{age}$. Using this formula, we may assume that the average $\text{HR}_{\text{max}}$ for the population of

healthy adults of age 45 is $\mu = 175$. Assume that the standard deviation of the $HR_{max}$ in the same population is $\sigma = 20$. Let $\bar{X}$ be the average $HR_{max}$ for a sample of $n = 64$ individuals of age 45. Give an approximation for $P(\bar{X} > 180)$.

A) 0.6255 B) 0.3745 C) 0.0101 D) 0.0228
E) 0.9964

*Solution:* This problem is based on Section 9.2. By the Central Limit Theorem,

$$\frac{\bar{X} - 175}{20/\sqrt{64}} \quad \text{has approximately a N(0,1) distribution.}$$

Hence,

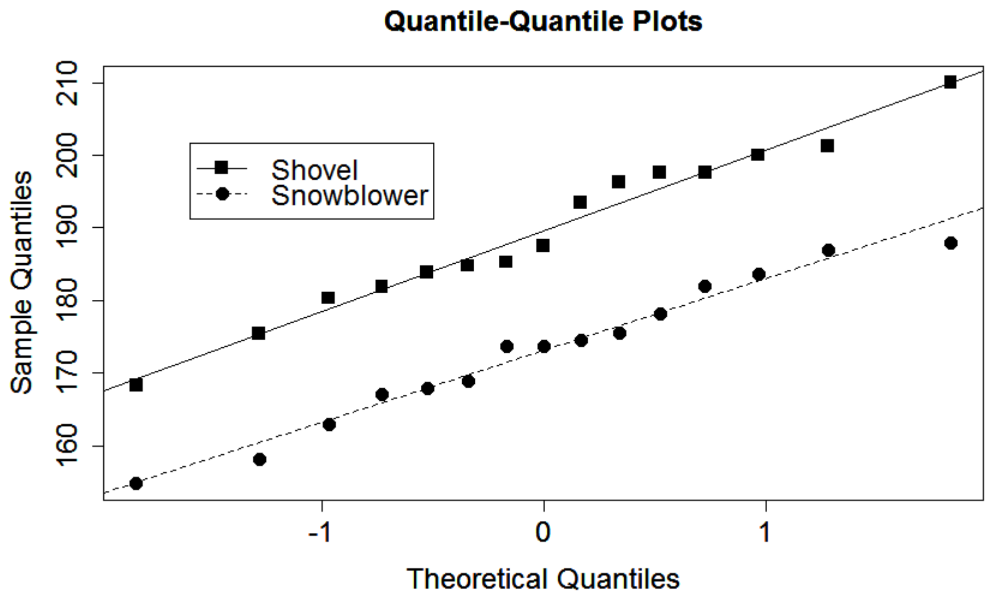
$$P(\bar{X} > 180) = P\left(\frac{\bar{X} - 175}{20/\sqrt{64}} > \frac{180 - 175}{20/\sqrt{64}}\right) \approx P(Z > 2.00) = 1 - 0.9772 = 0.0228.$$

The answer is D.

5. We continue with the situation in Question 4. We consider a group of 30 volunteers of age 45, which is split randomly into two sub-groups of size 15 each. The 15 persons in the first group are asked to snow shovel for 20 minutes. The values of $HR_{max}$ for these persons are recorded in R in the variable $x$. The other 15 volunteers are asked to use an electric snowblower for 20 minutes; their $HR_{max}$ is recorded in R in the variable $y$. Below is the summary of these data.

```
> mean(x)
[1] 189.5593
> sd(x)
[1] 11.08633
> mean(y)
[1] 173.0736
> sd(y)
[1] 9.895271
```

The picture below gives the overlayed QQ-plots for $x$ and $y$.




Which one of the following statements is correct? (Only one statement is correct.)

A) Using these QQ-plots, we cannot infer that the two populations are normally distributed, but we can infer that two populations have the same variance.

B) Using these QQ-plots, we can infer that the two populations are normally distributed, but we cannot infer that the two populations have the same variance.

C) We cannot use these QQ-plots because our data are paired measurements. Hence, we should calculate instead the differences between $x$ and $y$, and then produce a QQ-plot for these differences to verify that they are normally distributed.

D) We cannot use these QQ-plots because our data are paired measurements. For paired data, it is not necessary to verify that the differences between the $x$ and $y$ measurements are normally distributed.

E) Using these QQ-plots, we can infer that the two populations are normally distributed and have the same variance.

*Solution:* This problem is based on Sections 12.2 and Chapter 13. The plots seem to be linear and the lines parallel. The correct answer is E.

6. In a study of angina in rats, 18 animals with a history of angina were given an experimental drug, which might affect their oxygen intake, measured in milliliters per minute. A normal value for the oxygen intake in these rats is around 1600 ml/min. The research hypothesis is that, on average, the drug will affect the oxygen intake, i.e. the mean will change. For the 18 rats, the sample mean was $\bar{x} = 1702$ ml/min, with a sample standard deviation $s = 181$ ml/min. Assuming that the data is normally distributed, is there enough evidence that the oxygen intake is affected significantly by this drug? Report the observed value of the test statistic ($t_0$) and the range of the $p$-value. Use the significance level $\alpha = 0.05$.

   A) $t_0 = 2.391$; $p$-value is between 0.02 and 0.05; the drug affects significantly the oxygen intake
   B) $t_0 = 2.391$; $p$-value is between 0.01 and 0.025; the drug affects significantly the oxygen intake
   C) $t_0 = 0.136$; $p$-value is between 0.01 and 0.05; the drug affects significantly the oxygen intake
   D) $t_0 = 0.136$; $p$-value is between 0.02 and 0.10; we cannot draw any conclusion
   E) $t_0 = 2.391$; $p$-value is between 0.05 and 0.10; the drug does not affect significantly the oxygen intake

   *Solution:* This problem is based on Section 11.2. We would like to test $H_0 : \mu = 1600$ against $H_1 : \mu \neq 1600$. The observed value of the test statistic is:

$$t_0 = \frac{1702 - 1600}{181/\sqrt{18}} = 2.391$$

   Since this is a *two-tailed test*, we infer that the $p$-value is $2P(T > 2.391)$ From Table 17.4 (row 17), we see that $P(T > 2.391)$ is between 0.01 and 0.025. Hence, the $p$-value is between 0.02 and 0.05. Since the $p$-value is smaller than $\alpha = 0.05$, we reject $H_0$ in favor of $H_1$. We infer that the drug affects significantly the oxygen intake. The answer is A. The wrong answer B is obtained when we compute the $p$-value as $P(T > 2.391)$. The incorrect value $t_0 = 0.136$ in B,C is obtained as $(1702 - 1600)/(181\sqrt{18})$.

7. The following table shows the distribution of blood types in the Aus-

tralian population:

| Blood Type | A | B | AB | O |
|---|---|---|---|---|
| Proportion | 7% | 41% | 3% | 49% |

A sample of 10 Australians is randomly selected. Find the probability that in this sample, at most 2 persons have blood type A or AB.

A) 0.93 B) 0.07 C) 0.01 D) 0.54
E) 0.38

*Solution:* This problem is based on Section 6.2. Let $X$ be the number of persons in this sample who have blood type A or AB. Then $X$ has a binomial distribution with $n = 10$ trials and probability of success $p = 0.07 + 0.03 = 0.1$. The probability that at most 2 persons have blood of type A or AB is:

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) =$$
$$\binom{10}{0}(0.1)^0(0.9)^{10} + \binom{10}{1}(0.1)^1(0.9)^9 + \binom{10}{2}(0.1)^2(0.9)^8 =$$
$$0.34868 + 0.38742 + 0.19371 = 0.92981.$$

The answer is A.

8. A diagnostic test for a certain disease has a sensitivity of 0.95 and a specificity of 0.85. Assume that the incidence rate of this disease is 0.2%. What is the positive predictive value of this test?

A) 0.95 B) 0.0329 C) 0.9670 D) 0.9875
E) 0.0125

*Solution*: This problem is based on Section 4.3. We select randomly an individual from this population. Let True+ be the event that this individual has the disease (true positive) and True− be the event that this individual does not have the disease (true negative). Let Test+, Test− be the events that the individual has a positive test result, respectively a negative test result. We know that $P(\text{Test+}|\text{True+}) = 0.95$; $P(\text{Test}-|\text{True}-) = 0.85$ and $P(\text{True+}) = 0.002$. By the total probability rule,

$$\begin{aligned} P(\text{Test+}) &= P(\text{Test+}|\text{True}-)P(\text{True}-) + P(\text{Test+}|\text{True+})P(\text{True+}) \\ &= (0.15)(0.998) + (0.95)(0.002) = 0.1497 + 0.0019 = 0.1516. \end{aligned}$$

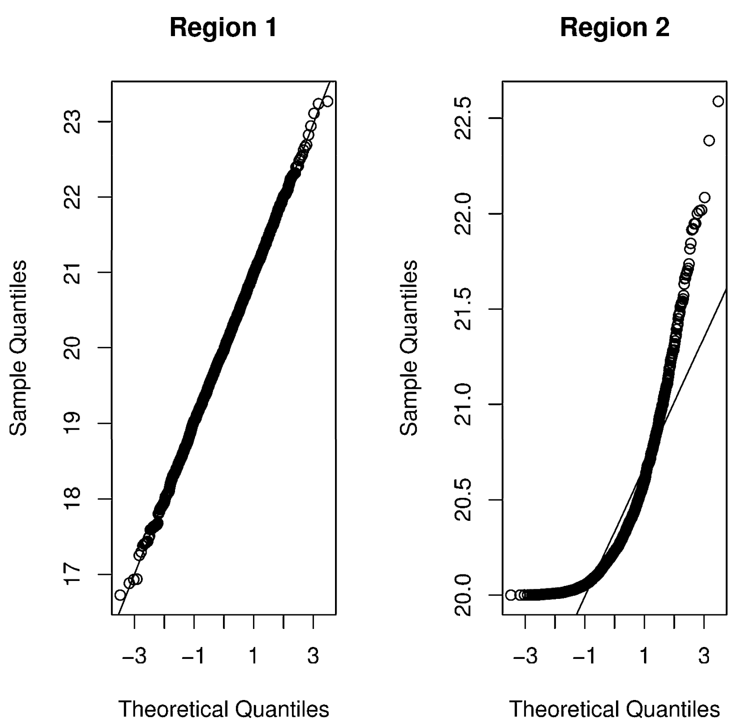
Using Bayes' formula,

$$\text{PPV} = P(\text{True}+|\text{Test}+) = \frac{P(\text{Test}+|\text{True}+)P(\text{True}+)}{P(\text{Test}+)}$$

$$= \frac{(0.95)(0.002)}{0.1516} = \frac{0.0019}{0.1516} = 0.0125.$$

The answer is E.

9. We would like to compare the average length of snakes in Forest Valley (Region 1) and Dry Valley (Region 2). Samples from both valleys have been studied and the following QQ-plots have been generated to assess the normality of the length of snakes in both valleys. Which one of the




following statements is correct? (Only one statement is correct.)

A) The lengths are normally distributed in both regions.
B) The length is normally distributed in Region 1, but is not normally distributed in Region 2.

C) The length is normally distributed in Region 2, but is not normally distributed in Region 1.
D) The length is not normally distributed in either one of the two regions.

*Solution:* This problem is based on Section 9.3. The plot for Region 1 is linear, so this population is normally distributed. The plot for Region 2 deviates significantly from the linear tendency, so this population is not normally distributed. The answer is B.

10. Consider a variable x which gives the yield of corn cob, in kg per hectare. We have created the variable x in R, for which we have used the commands "quantile" and "mean". Below is the R output.

```
> x=c(2134,2170,2142,2799,2364,2199,2310,1620,1808,1476,1695)
> quantile(x,type=6)
  0%  25%  50%  75% 100%
1476 1695 2142 2310 2799
> mean(x)
[1] 2065.182
```

Which one of the following statements is correct? (Only one statement is correct.)
A) There are no outliers.
B) The value 1476 is an outlier.
C) The value 2799 is an outlier.
D) The values 1476 and 2799 are outliers.
E) The mean is larger than the median.

*Solution:* This problem is based on Section 9.1. We have $IQR = Q3 - Q1 = 2310 - 1695 = 615$.

$$\text{Fence1} = 1695 - (1.5)(615) = 1695 - 922.5 = 772.5$$

$$\text{Fence2} = 2310 + (1.5)(615) = 2310 + 922.5 = 3232.5$$

Since the minimum value (1476) and the maximum value (2799) are within the two fences, we conclude that there are no outliers. The answer is A.

11. Before preparation for microscopic study, tissue cells are exposed to 20 minutes of anoxia (an abnormally low oxygen supply). These cells are then graded for the extent of damage as follows: 0=undamaged, 1=slightly damaged, 2=moderately damaged, 3=extensively damaged. Let $X$ be the classification value for the extent of damage in a randomly chosen cell. Below is the table of the cumulative distribution function of $X$:

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $F(x)$ | 0.15 | 0.40 | 0.90 | 1.00 |

Find the expected value of $X$ and the variance of $X$.

A) $E(X) = 5.20, \text{Var}(X) = 14.041$   B) $E(X) = 1.55, \text{Var}(X) = 3.15$
C) $E(X) = 5.20, \text{Var}(X) = 0.1573$   D) $E(X) = 2.45, \text{Var}(X) = 1.52$
E) $E(X) = 1.55, \text{Var}(X) = 0.7475$

*Solution:* This problem is based on Section 6.1. Below is the table of the probability mass function of $X$:

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$ | 0.15 | 0.25 | 0.50 | 0.10 |

The mean of $X$ is:

$$\mu = E(X) = (0)(0.15) + (1)(0.25) + (2)(0.50) + (3)(0.10) = 1.55$$

The variance of $X$ is:

$$\sigma^2 = \text{Var}(X) = (0^2)(0.15) + (1^2)(0.25) + (2^2)(0.50) + (3^2)(0.10) - (1.55)^2$$

$$= 3.15 - 2.4025 = 0.7475$$

The answer is E.

12. Among the patients with lung cancer, at most 10% will survive for three years after the diagnostic. Roche Holding, one of the world's largest manufacturers of cancer drugs claims that its Tarceva medicine can extend the lifetime of patients with non-small-cell lung cancer. Assume that in a study of 150 patients with lung cancer treated with Tarceva, 22 survived for 3 years. Compute a 85% confidence interval

for the proportion $p$ of lung cancer patients treated with Tarceva, who will survive for 3 years after the diagnostic. Based on this interval, can we conclude that the proportion $p$ is higher than 10%?

A) [9.0%; 20.3%]; we cannot conclude that $p$ is higher than 10%
B) [9.9%; 19.4%]; we cannot conclude that $p$ is higher than 10%
C) [10.5%; 18.8%]; the proportion $p$ is higher than 10%
D) [11.3%; 17.5%]; the proportion $p$ is higher than 10%
E) [8.7%; 23.1%]; we cannot conclude that $p$ is higher than 10%

*Solution:* This problem is based on Section 10.3. The number of surviving patients in this sample is 22. The estimate for $p$ is:

$$\hat{p} = \frac{22}{150} = 0.1467 \quad \text{(or 14.67\%)}$$

We need to find $z$ such that $P(Z < z) = 0.85 + (1 - 0.85)/2 = 0.85 + 0.075 = 0.925$. From Table 17.3, we find $P(Z < 1.44) = 0.9251$. We take $z = 1.44$. The 85% confidence interval for $p$ is:

$$0.1476 \pm 1.44\sqrt{\frac{(0.1467)(0.8534)}{150}} = 0.1467 \pm 0.0416 = [0.1051; 0.1883]$$

The answer is C. The incorrect answers A and B are obtained when working with $z = 1.96$, $z = 1.645$.

13. We continue with the situation in Question 12. Using a test of hypothesis of level $\alpha = 0.05$, is there enough evidence that the proportion $p$ is higher than 10%? Report the observed value of the test statistic and the $p$-value.

A) $z_0 = 1.41$, $p$-value=0.0793; there is not enough evidence that $p$ is higher than 10%
B) $z_0 = 2.53$, $p$-value=0.0057; there is enough evidence that $p$ is higher than 10%
C) $z_0 = 1.91$, $p$-value=0.0281; there is enough evidence that $p$ is higher than 10%
D) $z_0 = 1.62$, $p$-value=0.0526; there is not enough evidence that $p$ is higher than 10%

E) $z_0 = 1.04$, $p$-value=0.1492; there is not enough evidence that $p$ is higher than 10%

*Solution:* This problem is based on Section 11.3. We would like to test $H_0 : p = 0.10$ against $H_1 : p > 0.10$. The observed value of the test statistic is:

$$z_0 = \frac{0.1467 - 0.10}{\sqrt{(0.1)(0.9)/150}} = 1.91$$

The $p$-value is $P(Z > 1.91) = 1 - 0.9719 = 0.0281$ (using Table 17.3). Since the $p$-value is smaller than 0.05, we reject $H_0$ in favor of $H_1$. There is enough evidence that $p > 0.10$. The answer is C. The wrong answer D is obtained by computing:

$$z_0 = \frac{0.1467 - 0.10}{\sqrt{(0.1476)(0.8533)/150}} = 1.62$$

14. A pharmaceutical company has produced a new drug and claims that it helps reducing the systolic blood pressure. To test this claim, a physician prescribes this drug to 10 of his patients. The patients' systolic blood pressure, before and after the drug treatment, are recorded in the variables "Before" and "After" given below:

```
Before=c(140,135,122,150,126,138,141,155,128,130)
After=c(135,136,120,148,122,136,140,153,120,128)
```

The physician types into the `R` console:

```
 t.test(Before,After,var.equal=TRUE,alternative="greater")
```

and obtains the following output:

```
data:  Before and After
t = 0.5499, df=18, p-value = 0.2946
alternative hypothesis: true difference in means is greater than 0
sample estimates: mean of x mean of y
    136.5     133.8
```

The physician's assistant types into the `R` console:

```
t.test(Before,After,paired=TRUE,alternative="greater")
```

and obtains the following output:

```
data: Before and After
t = 3.4825, df = 9, p-value = 0.003456
alternative hypothesis: true difference in means is greater than 0
sample estimates: mean of the differences
                    2.7
```

Which one of the following statements is correct? (Only one statement is correct.)

A) The assistant uses the correct command. There is not enough evidence that the new drug reduces the systolic blood pressure.

B) The assistant uses the correct command. There is enough evidence that the new drug reduces the systolic blood pressure.

C) The physician uses the correct command. There is not enough evidence that the new drug reduces the systolic blood pressure.

D) The physician uses the correct command. There is enough evidence that the new drug reduces the systolic blood pressure.

E) Neither the physician, nor the assistant are using the correct command. The $t$-test should not be used in this situation.

*Solution:* This problem is based on the material in Section 12.2 and Chapter 13. Since the measurements are made on the same patients, these are paired samples. The assistant has used the correct command. We would like to test $H_0 : \mu_D = 0$ versus $H_1 : \mu_D > 0$, where $D = X - Y$. Since the $p$-value is very small, we reject $H_0$ in favor of $H_1$. We conclude that there is enough evidence that the new drug reduces the blood pressure. The answer is B.

15. Assume that among the students in a certain college, 55% are overweight, 20% have high blood pressure, and 60% are overweight or have high blood pressure. A student is randomly selected from this college. Let $O$ be the event that the student is overweight and $H$ be the event that the student has high blood pressure. Compute $P(O \cap H)$. Is the

fact that a student is overweight independent of the fact that this student has high blood pressure?

A) The events are independent, and $P(O \cap H) = 0.15$
B) The events are dependent, and $P(O \cap H) = 0.15$
C) The events are independent, and $P(O \cap H) = 0.6$
D) The events are dependent, and $P(O \cap H) = 0.6$
E) The events are independent, and $P(O \cap H) = 0.11$

*Solution:* This problem is based on the material in Section 3.2 and Section 5.1. We know that $P(O) = 0.55$, $P(H) = 0.20$ and $P(O \cup H) = 0.60$. By the addition rule (Section 3.2),

$$P(O \cap H) = P(O) + P(H) - P(O \cup H) = 0.55 + 0.20 - 0.60 = 0.15$$

Since $P(O)P(H) = (0.55)(0.2) = 0.11 \neq P(O \cap H) = 0.15$, the two events are not independent (Section 5.1). The answer is B.

16. Let $X$ be the amount of calcium (in miligrams) per deciliter of blood for a randomly chosen healthy adult. In a sample of 25 individuals, it was found that the average amount of calcium was 9.8 mg, with a standard deviation of 0.5 mg. Assuming that the data is normally distributed, find a 80% confidence interval for the average amount of calcium per deciliter of blood in the general population of healthy adults.

    A) [9.6289; 9.9711] B) [9.5936; 10.0064] C) [9.1432; 9.9876]
    D) [9.6682; 9.9318] E) [8.7654; 10.1632]

    *Solution:* This problem is based on Section 10.2. Table 17.4 (row 24, column 0.9) we read $t = 1.318$. The interval is:

$$9.8 \pm (1.318)\frac{0.5}{\sqrt{25}} = 9.8 \pm 0.1318 = [9.6682; 9.9318]$$

    The answer is D. The wrong answers A, B are obtained when working with $t = 1.711$, respectively $t = 2.064$.

17. A class consists of 490 female and 510 male students. The students are classified according to their final grade in this course, as follows:

| | Passed | Did not pass |
|---|---|---|
| Female | 430 | 60 |
| Male | 410 | 100 |

A student is randomly selected from this class. What is the probability that the student did not pass the course, given that this student is a female?

A) 0.06 B) 0.12 C) 0.41 D) 0.81
E) 0.88

*Solution:* This problem is based on the material in Section 4.1. Let $A$ be the event that the student passed the course, and $F$ be the event that the student is a female. We have

$$P(A'|F) = \frac{P(A' \cap F)}{P(F)} = \frac{60/1000}{490/1000} = 0.12$$

The answer is B.

18. A sample of 20 fish was catched in lake $A$ and their length (in cm) was measured. Another sample of 12 fish was catched in lake $B$ and their length (in cm) was also measured. The data are summarized in the table below:

| | sample size | sample mean | sample standard deviation |
|---|---|---|---|
| Lake A | 20 | 11.09 | 1.222 |
| Lake B | 12 | 10.29 | 1.186 |

Let $\mu_1$ and $\mu_2$ be the average fish length in lake $A$, respectively lake $B$. We would like to test $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$. Assume that the two populations are normally distributed with equal variances. Find the range of the $p$-value for this test.

A) the $p$ value is between 0.025 and 0.05
B) the $p$-value is between 0.05 and 0.1
C) the $p$-value is between 0.95 and 0.975
D) the $p$-value is between 0.1 and 0.2
E) the $p$-value is between 0.25 and 0.5

*Solution:* This problem is based on Section 12.2. The pooled variance is

$$s_p^2 = \frac{(19)(1.222)^2 + (11)(1.186)^2}{30} = 1.461.$$

The observed value of the test statistic is:

$$t_0 = \frac{11.09 - 10.29}{\sqrt{(1.461)(1/20 + 1/12)}} = 1.812$$

$p$-value is $2P(T_{30} > 1.812)$ From Table 17.4 (row 30) we see that $P(P(T_{30} > 1.812)$ is between 0.025 and 0.05. Hence the $p$-value is between 0.05 and 0.1. The answer is B.

19. A survey was conducted on 1000 adults, among which 48% were men. The results of the survey show that 15% of men and 30% of women are afraid of flying. Using a contingency table, test the hypothesis that sex is independent of the fact that a person is afraid of flying. Report the range of the $p$-value and your conclusion at level $\alpha = 0.05$.

    A) $p$-value $< 0.005$. The sex and the fact that a person is afraid of flying are independent.
    B) $p$-value $< 0.005$. There is an association between the sex and the fact that a person is afraid of flying.
    C) $0.005 < p$-value $< 0.01$. There is an association between the sex and the fact that a person is afraid of flying.
    D) $0.05 < p$-value $< 0.10$. The sex and the fact that a person is afraid of flying are independent.
    E) $p$-value $> 0.10$. The sex and the fact that a person is afraid of flying are independent.

    *Solution:* This problem is based on Section 14.1. We would like to test $H_0$: "the sex and the fact that a person is afraid of flying are independent" against $H_1$: "there is an association between the sex and the fact that a person is afraid of flying". Below is the contingency table.

| | Not Afraid of Flying | Afraid of Flying | Total |
|---|---|---|---|
| Men | 408 (370.56) | 72 (109.44) | 480 |
| Women | 364 (401.44) | 156 (118.56) | 520 |
| Total | 772 | 228 | 1000 |

The observed value of the test statistic is:

$$u_0 = \frac{(408-370.56)^2}{370.56} + \frac{(72-109.44)^2}{109.44} + \frac{(364-401.44)^2}{401.44} + \frac{(156-118.56)^2}{118.56} = 31.91.$$

The $p$-value is $P(U > 31.91)$, where $U$ has a $\chi^2(1)$ distribution. Using Table 17.5, we see that the $p$-value is smaller than 0.005. Since the $p$-value is smaller than $\alpha = 0.05$, we reject $H_0$ in favor of $H_1$. We conclude that there is an association between the sex and the fact that a person is afraid of flying. The answer is B.

20. A scientist's job is to detect a situation in which the mean bacteria count has risen above the maximum safe level of 70 in Ottawa River. If the bacteria count exceeds 70, water is considered unsafe. To test whether the water is unsafe, the following sample of size 9 is drawn:

    69 74 75 70 72 73 71 73 68.

    Note that the sample standard deviation is $s = 2.35$. Assume that the data is normally distributed. Is there enough evidence that the water is unsafe? Determine the $p$-value and use $\alpha = 0.05$.

    A) The water is unsafe and $0.025 < p - \text{value} < 0.05$.
    B) The water is unsafe and $p - \text{value} < 0.025$.
    C) The water is safe and $0.025 < p - \text{value} < 0.05$.
    D) The water is safe and $p - \text{value} < 0.025$.
    E) The water is unsafe and $p - \text{value} > 0.95$.

    *Solution:* This problem is based on Section 11.2. We would like to test $H_0 : \mu = 70$ versus $H_1 : \mu > 70$. Note that $\bar{x} = 71.67$. The observed value of the test statistic is:

    $$t_0 = \frac{71.67 - 70}{(2.35)/\sqrt{9}} = 2.132$$

    Hence $p - \text{value} = P(T_8 > 2.132)$. From Table 17.4 (row 8), we see that $0.025 < p - \text{value} < 0.05$. Because the $p - \text{value}$ is smaller than 0.05, we reject the null hypothesis. There is enough evidence that the water is unsafe. The answer is A.

21. Consider the following data sets:

```
x=c(1,4,6,12,10,16,20)
y=c(2,4,5,8,11,14)
```

The sample standard deviation for the first data set is 6.74. Find the pooled variance.
*Hint:* First compute the sample variance for the second data set.

A) 22.6 B) 11.3 C) 40.2 D) 33.1
E) 34.2

*Solution:* This problem is based on Section 12.2. The sample variance for the second data set is $s_y^2 = 20.667$. The pooled variance is:

$$s_p^2 = \frac{6(6.74)^2 + 5(20.667)}{11} = 34.2.$$

The answer is E. The incorrect answer D was obtained using the formula $(s_x^2 + s_y^2)/2$. The incorrect answer C was obtained using $7[(6.74)^2 + 6(20.667)]/11 = 40.2$.

22. Let $X_1, X_2, \ldots, X_{28}$ be a random sample from a normal population with mean $\mu = 125$. Let $\bar{X}$ be the mean of this sample and $S^2$ be its variance. Find a value $c$ such that

$$P\left(\frac{\bar{X} - 125}{S/\sqrt{28}} \leq c\right) = 0.95.$$

A) $c = 1.701$ B) $c = 1.703$ C) $c = 1.645$
D) $c = -1.701$ E) $c = -1.703$

*Solution:* This problem is based on Section 10.2. We know that

$$\frac{\overline{X} - 125}{S/\sqrt{28}}$$

has a $T$ distribution with 27 d.f. From Table 17.4, $c = 1.703$. The answer is B.

23. It is known that the height of the adults in a certain community follows a normal distribution with variance $\sigma^2 = 625$ centimeters$^2$. 25 adults were randomly selected from this community and their heights were

measured. The data yielded a sample mean of $\bar{x} = 165$ centimeters. Construct a 90% confidence interval for the average height of the adults living in this community, measured in meters.

A) [157; 173] B) [1.57; 1.73] C) [155; 175]
D) [121; 193] E) [1.55; 1.75]

*Solution:* This problem is based on Section 10.1. The population has standard deviation $\sigma = 25$ cm. This number measured in meters will be $\sigma = 0.25$ m. The sample mean is $\bar{x} = 1.65$ m. For the 90% confidence interval we use $z = 1.645$. The interval is:

$$1.65 \pm 1.645 \left( \frac{0.25}{\sqrt{25}} \right) = 1.65 \pm 0.082 = [1.57, 1.73].$$

The answer is B. The wrong answer E is obtained when working with $z = 1.96$.

24. A study is conducted to identify the major risk factors in the development of coronary-artery disease. The following data gives the cholesterol level (in miligrams per deciliter) for 7 women aged 20 to 29.

175.3 171.2 175.9 176.2 170.2 185.3 174.7

Compute: (a) a point estimate for the average cholesterol level of women aged 20 to 29; (b) the estimated standard error of the mean.

A) (a) 175.54; (b) 4.898 B) (a) 170.31; (b) 4.898
C) (a) 175.54; (b) 1.851 D) (a) 170.31; (b) 3.253
E) (a) 173.25; (b) 1.851

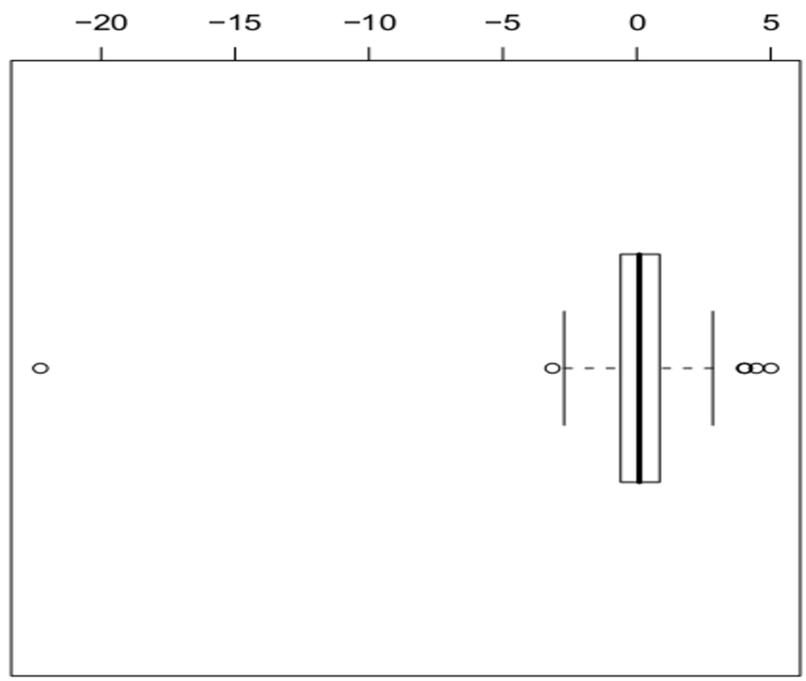
*Solution:* This problem is based on Section 10.2. The point estimate is $\bar{x} = 175.54$. The sample standard deviation is $s = 4.898$. The estimated standard error of the mean is:

$$s\{\bar{X}\} = \frac{s}{\sqrt{n}} = \frac{4.898}{\sqrt{7}} = 1.851$$

The answer is C.

25. We include below the summary and the boxplot of data produced by R for a particular data set.

```
      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-22.27000  -0.58500   0.09504   0.02696   0.84570   5.01100
```




Which one of the following statements is correct? (Only one statement is correct.)

A) The IQR is 27.281.
B) There are three outliers greater than 0, two outliers smaller than 0 and the extreme values −22.27 and 5.011 are among the outliers.
C) The range of the data is 1.4307.
D) The distance between the two fences is 1.4307.
E) −22.27 is an outlier, but 5.011 is not an outlier.

*Solution:* This problem is based on Section 9.1. From the picture we see that there are 3 outliers greater than 0 and 2 outliers smaller than 0. The IQR is $Q_3 - Q1 = 0.84570 - (-0.585) = 1.4307$.

$$\text{Fence1} = Q1 - (1.5)IQR = -0.585 - (1.5)(1.4307) = -0.585 - 2.14605 = -2.73105$$

Fence2 $= Q3 + (1.5)IQR = 0.8457 + (1.5)(1.4307) = 0.8457 + 2.14605 = 2.99175$

The distance between the two fences is 5.7228. The range of the data is $5.011 - (-22.27) = 27.281$. The values -22.27 and 5.011 fall outside the fences, so they are outliers. The answer is B.