MATH 1119B: Test 2

Monday, October 17, 2011, 09:35-10:25

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Total marks: / 40

Authorized Devices: Non-programmable calculators, digeridoos. Absolutely no hammocks allowed.

Name: Student number:

[13] 1. Identify if the following represent vector equations or matrix equations. If it is a vector equation, re-write it as a matrix equation. If it is a matrix equation, re-write it as a vector equation. If the equation is pre-solved for you, check that it is true. If the equation contains unknowns, solve it. If there are free variables, put it in parametric vector form (e.g., let $x_2 = s \in \mathbb{R} \ldots$).

2

(a) $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 6 \end{bmatrix} = \begin{bmatrix} -4 \\ 14 \end{bmatrix}$

Matrix eq'n

$$1\begin{bmatrix} 1 \\ 2 \end{bmatrix} - 2\begin{bmatrix} 1 \\ -3 \end{bmatrix} + 6\begin{bmatrix} -1 \\ 1/2 \end{bmatrix} = \begin{bmatrix} -4 \\ 14 \end{bmatrix}$$

False: $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 6 \end{bmatrix} = \begin{bmatrix} -7 \\ 11 \end{bmatrix}$

2

(b) $2\begin{bmatrix} 1 \\ 2 \end{bmatrix} - 3\begin{bmatrix} 3 \\ -2 \end{bmatrix} + 3\begin{bmatrix} 12 \\ -3 \end{bmatrix} = \begin{bmatrix} 29 \\ 4 \end{bmatrix}$

Vector eq'n

$$\begin{bmatrix} 1 & 3 & 12 \\ 2 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix} = \begin{bmatrix} 29 \\ 4 \end{bmatrix}$$

False,

Row 2: $2(2) - 3(-2) + 3(-3) = 1$

9

(c) $\begin{bmatrix} 2 & -2 & 2 & 2 \\ -1 & 1 & 3 & 7 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \end{bmatrix}.$

Matrix equation

$$a_1\begin{bmatrix} 2 \\ -1 \end{bmatrix} + a_2\begin{bmatrix} -2 \\ 1 \end{bmatrix} + a_3\begin{bmatrix} 2 \\ 3 \end{bmatrix} + a_4\begin{bmatrix} 2 \\ 7 \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 2 & -2 & 2 & 2 & 8 \\ -1 & 1 & 3 & 7 & 12 \end{array}\right] \sim \begin{bmatrix} 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & 4 \end{bmatrix}$$

$$\begin{aligned} x_1 &= x_2 + x_4 \\ x_2 &= s \quad \in \mathbb{R} \\ x_3 &= 4 - 2x_4 \\ x_4 &= t \quad \in \mathbb{R} \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \end{bmatrix} + s\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

[5] 2. Indicate if the following statements are TRUE or FALSE.

T / F [a] The definition of a homogeneous equation is an equation $Ax = b$ having the zero vector as a solution.

T / F [b] The solution set of a linear system whose augmented matrix is $\begin{bmatrix} a_1 & a_2 & a_3 & b \end{bmatrix}$ is the same as the solution set of $Ax = b$ if $A = \begin{bmatrix} a_1 & a_2 & a_3 & \cancel{b} \end{bmatrix}$.

T / F [c] Any linear combination of vectors can always be written in the form Ax for a matrix A and a suitable vector x.

T / F [d] The weights $c_1, c_2, \ldots, c_p$ in a linear combination $c_1v_1 + c_2v_2 + \cdots + c_pv_p$ cannot all be zero.

T / F [e] A homogeneous linear system cannot contain more than 2 free variables.

[2] 3. Write the 4×4 identity matrix I_4.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_4$$

[9] 4. Let $w_1 = \begin{bmatrix}-2\\1\end{bmatrix}$, $w_2 = \begin{bmatrix}9\\3\end{bmatrix}$ and $w_3 = \begin{bmatrix}3\\-2\end{bmatrix}$ and $d = \begin{bmatrix}-3\\-6\end{bmatrix}$.

1

(i) How many vectors are in $\{w_1, w_2, w_3\}$? 3

3

(ii) Give the definition (in math symbols or in words) of Span(w_1, w_2, w_3). How many vectors are in Span(w_1, w_2, w_3)?

Definition varies, Infinitely many

↳ 2 for perfect, 1 for ok, 0 for not

(iii) Determine if d is in Span(w_1, w_2, w_3).

3

$$\begin{bmatrix}-2 & 9 & 3 & -3\\ 1 & 3 & -2 & -6\end{bmatrix} \sim \begin{bmatrix}1 & 0 & -9/5 & -3\\ 0 & 1 & -1/15 & -1\end{bmatrix}$$

consistent $\therefore$ yes, $d \in \text{Span}(w_1, w_2, w_3)$

(Note: only echelon form is needed here)

(iv) Write $d = x_1w_1 + x_2w_2 + x_3w_3$, for some scalars x_1, x_2 and x_3.

2

$$x_1 = -3 + 9/5\, x_3$$
$$x_2 = -1 + 1/15\, x_3$$
$$, \quad x_3 = 0 \rightarrow d = -3w_1 - 1w_2 + 0w_3$$

[11] 5. An economy has 3 sectors: Fuel, Military and Media. Fill in the following input-output table for the economy:

$T =$

Fuel	Military	Media	Purchased By:
.2	.4	.1	Fuel
.6	.4	.7	Military
.2	.2	.2	Media
1.0	1.0	1.0	

* b), they may write out the systems of equations.

3 (a) Fill in the table.

(b) From the filled-in table, give an **augmented** matrix A which represents the homogeneous system that solves the output problem. Do not solve the system!

$$A = \left[\, I - T \;\middle|\; \begin{matrix}0\\0\\0\end{matrix} \right] = \left[\begin{matrix}.8 & -.4 & -.1\\ -.6 & .6 & -.7\\ -.2 & -.2 & .8\end{matrix}\;\middle|\;\begin{matrix}0\\0\\0\end{matrix}\right]$$

2 1 2

(c) The reduced coefficient matrix is $\begin{bmatrix}1 & 0 & -17/12\\ 0 & 1 & -31/12\\ 0 & 0 & 0\end{bmatrix}$. Give the solution set.

2

$$\cancel{\begin{bmatrix}\text{Fuel}\\ x_2\\ x_3\end{bmatrix}} = \begin{bmatrix}\text{Fuel}\\ \text{Military}\\ \text{Media}\end{bmatrix} = \text{Media}\begin{bmatrix}17/12\\ 31/12\\ 1\end{bmatrix}$$

(d) In 2010, the total output (or price) of the Media sector was \$80 billion. Give the equilibrium prices for the economy (you can leave the answer as fractions!).

1

$$\begin{bmatrix}\text{Fuel}\\ \text{Military}\\ \text{Media}\end{bmatrix} = \begin{bmatrix}80 \cdot 17/12 \text{ billion}\\ 80 \cdot 31/12 \text{ billion}\\ 80 \text{ billion}\end{bmatrix} = \begin{bmatrix}113.33 \text{ billion}\\ 206.67 \text{ billion}\\ 80 \text{ bill}\end{bmatrix}$$