Tell the students: Show their work!
Feel free to dock marginal points for correct but terribly written solutions.

MATH 1119B: Test 1

Monday, September 26, 2011, 09:35-10:25

Instructor: David Thomson, dthomson@math.carleton.ca

Total marks: / 40

Authorized Devices: Non-programmable calculators and crayons. Absolutely no pencil crayons allowed.

Name: Student number:

[9] 1. (a) Place the following system of equations into an augmented matrix and then row reduce to *reduced* row-echelon form. Circle the pivots and indicate the pivot columns in the *reduced* matrix. Indicate whether the system is consistent or inconsistent, and list the number of solutions of the system (0, 1 or infinite).

$$\begin{array}{rcrcrcl} 3x_1 & - & 2x_2 & + & 4x_3 & = & 0 \\ 9x_1 & - & 6x_2 & + & 13x_3 & = & 0 \\ -6x_1 & + & 4x_2 & - & 8x_3 & = & 0 \end{array}$$

[5] (b) If the solution is unique, check your solution in one of the equations. If you have infinite solutions, indicate which variables are basic, and which are free. Express your solution as a column vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \cdots$.

a) $\begin{bmatrix} 3 & -2 & 4 & 0 \\ 9 & -6 & 13 & 0 \\ -6 & 4 & -8 & 0 \end{bmatrix}$ $\begin{matrix} R_2 \leftarrow R_2 - 3R_1 \\ R_3 \leftarrow R_3 + 2R_1 \end{matrix}$ $\begin{bmatrix} 3 & -2 & 4 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim$ $R_1 \leftarrow R_1 - 4R_2$ $\begin{bmatrix} 3 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_1 \leftarrow R_1/3$ $\begin{bmatrix} 1 & -2/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ consistent system w/ infinite solutions. piv -1

pivot columns

b) x_1, x_3 are basic (correspond to pivot columns), x_2 is free

$x_1 = 2/3\, x_2$
$x_2 = x_2$
$x_3 = 0$

$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 1 \\ 0 \end{bmatrix}$

[6] 3. Indicate if the following statements are TRUE or FALSE.

T / F [a] A basic variable in a linear system is a variable that corresponds to a pivot row column in the coefficient matrix.

T / F [b] Whenever a system contains free variables, the solution of the system is consistent.

T / F [c] The echelon form of a matrix is unique.

T / F [d] If every column of an augmented matrix has a pivot, the corresponding system is consistent.

T / F [e] You will obtain the same solution set of a system of linear equations whether you use substitution or the row-reduction algorithm.

T / F [f] If A is an $m \times n$ matrix and B is an $n \times p$ matrix, the product $(3A)(-2B)$ has dimensions $m \times p$.

[6] 4. Determine for which h and k the system has (a) no solution, (b) a unique solution, (c) infinite solutions.

$$\begin{array}{rcrcr} x_1 & - & (2h)x_2 & = & -1 \\ 3x_1 & + & 6x_2 & = & 3k \end{array}$$

$$\begin{bmatrix} 1 & -2h & -1 \\ 3 & 6 & 3k \end{bmatrix} \sim R_2 \leftarrow R_2 - 3R_1 \begin{bmatrix} 1 & -2h & -1 \\ 0 & 6+6h & 3k+3 \end{bmatrix}$$ 1

$6+6h = 0 \Rightarrow h = -1$

$3k+3 = 0 \Rightarrow k = -1$

a) $h = -1$ and $k \neq -1$ 2

b) $h \neq -1$ 1

c) $h = -1$ and $k = -1$ 2

[10] 2. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ -1 & -2 & 0 \end{bmatrix}$ and let $B = \begin{bmatrix} 2 & 3 & -2 \\ 0 & 1 & -3 \end{bmatrix}$. Find the following matrices:

2 (a) $2A$

$$\begin{bmatrix} 2 & 0 & 2 \\ 0 & 4 & 4 \\ -2 & -4 & 0 \end{bmatrix}$$

3

1 (b) $3B + 2A$

different sizes

$\therefore$ does not exist!

2

2 (c) AB

A is 3×3

B is 2×3

AB is undefined!

2

(d) B(2A). 5

$$\begin{bmatrix} 2 & 3 & -2 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ -1 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 8 \\ 3 & 8 & 2 \end{bmatrix}$$ 3

2

[4] 5. (a) Explain, in your own **words**, what it means to be a solution of a system of linear equations.

(Many possible answers, 2 for a good answer, 1 for sort-of, 0 if they clearly don't know).

2 (b) The following is the set of parametric solutions to a linear system.

$$\begin{bmatrix} x \\ y \\ z \\ \maltese \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 5 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, t \in \mathbb{R}.$$

Give 2 specific solutions of the system by setting the parameter t to specific values.

$t = 0 \Rightarrow \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 5 \\ 1 \end{bmatrix}$ $\quad$ $t = 1 \Rightarrow \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 6 \\ 1 \end{bmatrix}$