

1. Determine if the vector $v = \begin{bmatrix} 18 \\ 3 \\ 15 \end{bmatrix}$ can be written as a linear combination of the vectors $u_1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$, $u_3 = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$. If it is the case, find the scalars c_1, c_2, c_3 . Show all your work. (4 points)

2. Determine if the vector $v = \begin{bmatrix} 4 \\ 2 \\ -6 \end{bmatrix}$ is in $\text{span}\{u_1, u_2, u_3\}$, where $u_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$, $u_2 = \begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix}$, $u_3 = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}$. Show your work and justify your answer. (2 points)

3. Determine if the vectors $(-1, 2, 1), (3, 2, 2), (0, 1, 0)$ form a basis for \mathbb{R}^3 or not. Show all your work and justify your answer. (3 points)

4. Determine if the vectors $v_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$, $v_3 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ are linearly dependent or independent. Show your work. (3 points)

5. Use the cofactors method (and only this method) to find the determinant of $A = \begin{bmatrix} 1 & -2 & 0 & 5 \\ 3 & -1 & 4 & 2 \\ 0 & 1 & -3 & 0 \\ 2 & 0 & 2 & -1 \end{bmatrix}$. Show your work. (5 points)

6. Find a basis for $\text{Nul}(A)$ if $A = \begin{bmatrix} 1 & 3 & -2 & 4 \\ 3 & 10 & -7 & 14 \\ -2 & -6 & 4 & -8 \end{bmatrix}$. Show your work. (4 points)

7. Write the linear system $\begin{aligned} 7x_1 - 4x_2 + 5x_3 &= 7 \\ 11x_1 + 3x_2 - 2x_3 &= 2 \\ 5x_1 - 7x_2 + 6x_3 &= -8 \end{aligned}$ as a vector equation. (1 point)

8. Let A be a 4×9 matrix with $\text{rank}(A) = 3$ (that is, $\dim(\text{Col}A) = 3$). What is the nullity of A (that is, $\dim(\text{Nul}A)$)? Justify your answer. (1 point)

9. Consider the matrices $A = \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 2 & -1 & 0 & 1 & -1 \\ -3 & 2 & 1 & -2 & 1 \\ 4 & 1 & 6 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

It can be shown (you don't need to show it) that the reduced echelon form matrix of A is the matrix B

- Find a basis for $\text{Col}(A)$ (1 point)
- Find a basis for $\text{Row}(A)$ (1 point)