

# CARLETON UNIVERSITY

FINAL/DEFERRED  
**EXAMINATION**  
November 2012

**DURATION: 3 HOURS**

Department Name & Course Number: Mathematics and Statistics MATH 1009\*EF  
Course Instructor(s) : Dr E. Devdariani, Dr R. Cova.

AUTHORIZED MEMORANDA

Non programmable, non graphing, non symbolic logic calculators are required.

Students **MUST** count the number of pages in this examination question paper **before** beginning to write, and report any discrepancy immediately to a proctor. This exam consists of **9** pages, including this title page. There are two parts of the exam. Part A consists of 20 Multiple Choice questions worth 3.5 marks each. The answers to part A must be given on the **Multiple Choice Answer Sheet** which is attached to this examination paper.

This examination question paper **MAY NOT** be taken from the examination room.

This examination question paper **MAY NOT** be released to the library.

Family Name (print) : \_\_\_\_\_ First Name : \_\_\_\_\_

Student Number : \_\_\_\_\_ Section: **E** (Dr. Elena Devdariani, TR 1:00 pm)  
**F** (Dr. Ramon Cova, MW 6:00 pm)

Question	Mark
Part A (1-20)/70	
Part B:	21/10
	22/10
	23/4
	24/6
Total Exam Mark / 100	

**Multiple Choice Answer Sheet**

NAME (LAST then FIRST, in print): \_\_\_\_\_

STUDENT NUMBER: \_\_\_\_\_

Please shade in your answer to each question in Part A.

1. (a) (b) (c) (d) (e)
2. (a) (b) (c) (d) (e)
3. (a) (b) (c) (d) (e)
4. (a) (b) (c) (d) (e)
5. (a) (b) (c) (d) (e)
6. (a) (b) (c) (d) (e)
7. (a) (b) (c) (d) (e)
8. (a) (b) (c) (d) (e)
9. (a) (b) (c) (d) (e)
10. (a) (b) (c) (d) (e)
11. (a) (b) (c) (d) (e)
12. (a) (b) (c) (d) (e)
13. (a) (b) (c) (d) (e)
14. (a) (b) (c) (d) (e)
15. (a) (b) (c) (d) (e)
16. (a) (b) (c) (d) (e)
17. (a) (b) (c) (d) (e)
18. (a) (b) (c) (d) (e)
19. (a) (b) (c) (d) (e)
20. (a) (b) (c) (d) (e)

**PART A - Multiple Choice Questions**

**INSTRUCTIONS:** All answers must be given on the **Multiple Choice Answer Sheet** that is attached to this paper. Each question is worth **3.5** marks. No explanation required.

1. Evaluate  $\lim_{x \rightarrow \infty} \frac{3x^5 - x + 5}{1 + 3x^2 - x^4}$ .

- (a)  $-3$ .            (b)  $1$ .            (c)  $0$ .            (d)  $\infty$ .            (e) None of these.

2. The domain of the function  $f(x) = \frac{3x}{\sqrt{(x-4)}}$  is

- (a)  $(0, 4)$ , or equivalently,  $\{x : 0 < x < 4\}$ .  
(b)  $(-\infty, 4)$ , or equivalently,  $\{x : x < 4\}$ .  
(c)  $(4, \infty)$ , or equivalently,  $\{x : x > 4\}$ .  
(d)  $[4, \infty)$ , or equivalently,  $\{x : x \geq 4\}$ .  
(e) None of these.

3. Evaluate  $\lim_{x \rightarrow 3} \frac{3-x}{x^2-9}$ .

- (a)  $-\frac{1}{6}$             (b)  $\frac{1}{6}$             (c)  $0$             (d)  $\infty$             (e) None of the above

4. The expression  $\frac{x^{1.6} \cdot (x^2)^{-0.3}}{x^{-2}}$  simplifies to

- (a)  $x$ .            (b)  $x^{1.9}$ .            (c)  $x^{-1.92}$ .            (d)  $x^3$ .            (e) None of these.

5. Which of the following is equal to  $\log_5 \frac{1}{25}$  ?

- (a)  $-2$ .            (b)  $2$ .            (c)  $-\frac{1}{2}$ .            (d)  $\frac{1}{2}$ .            (e) None of these.

6. What is the slope of the curve  $y = 3^{2x}$  at  $x = 0$ ?
- (a) 0.            (b)  $2 \ln 3$ .            (c)  $\ln 3$ .            (d) 1.            (e) Not defined.  
(e) None of these.
7. Given that the profit function for a company is
- $$P(x) = x^2 + \frac{100}{x} + 2x - 10,$$
- find the **marginal profit** at the production level of  $x = 10$  units.
- (a) 120.            (b) 210.            (c) 21.            (d) 23.            (e) None of these
8. The graph of the function  $y = \frac{3x - 1}{x - 2}$  has
- (a) no horizontal asymptote and a vertical asymptote  $x = 2$ .  
(b) a horizontal asymptote  $y = 3$  and a vertical asymptote  $x = 2$ .  
(c) a horizontal asymptote  $y = 0$  and no vertical asymptote.  
(d) neither horizontal no vertical asymptote.  
(e) none of the these.
9. What are the critical numbers of the function  $f(x) = \frac{7}{x - 4}$ ?
- (a)  $x = -7$ .            (b)  $x = -4$ .            (c)  $x = 4$ .            (d) No critical numbers.            (e) None of these.
10. Let  $f(x) = (3 + \log_3 x)^7$ . Evaluate  $f'(x)$ , that is, find the derivative of  $f$ .
- (a)  $\frac{7(3 + \log_3 x)^6}{x}$             (b)  $\frac{7(3 + \log_3 x)^6}{x \ln 3}$             (c)  $\frac{(3 + \log_3 x)^6}{x \ln 3}$   
(d)  $7(3 + \log_3 x)^6 \ln 3$             (e) None of the above

11. Determine the interval(s) where the function  $f(x) = x^4 - 2x^2 + 1$  is INCREASING.
- (a)  $(-1, 0) \cup (1, \infty)$ , or equivalently,  $\{x : -1 < x < 0, x > 1\}$ .
  - (b)  $(-\infty, -1) \cup (0, \infty)$ , or equivalently,  $\{x : x < -1, x > 0\}$ .
  - (c)  $(-1, 1)$ , or equivalently,  $\{x : -1 < x < 1\}$ .
  - (d)  $(0, 1)$ , or equivalently,  $\{x : 0 < x < 1\}$ .
  - (e) None of these.
12. How many years would it take for the investment of \$2,000 to grow to \$6,000, if the interest is compounded semi-annually and the nominal interest rate is 4%?  $A(t) = P \left(1 + \frac{r}{m}\right)^{mt}$ .
- (a) 11.8.      (b) 17.6.      (c) 21.9.      (d) 27.7.      (e) None of these.
13. Consider  $f(x) = x^3 - 12x + 1$ , where  $x \in [-3, 3]$ . This function has
- (a) a global (absolute) minimum at  $x = 2$  and a global (absolute) maximum at  $x = -2$ .
  - (b) a global (absolute) minimum at  $x = -2$  and a global (absolute) maximum at  $x = 2$ .
  - (c) a global (absolute) minimum at  $x = 3$  and a global (absolute) maximum at  $x = -3$ .
  - (d) a global (absolute) minimum at  $x = -3$  and a global (absolute) maximum at  $x = 3$ .
  - (e) None of the above
14. The **half-life** of an exponentially decaying quantity is the time required for the quantity to be reduced by a factor of  $1/2$ . The half-life of aspirin in a human body is 5 hours. Find its decay constant. (Choose the closest approximation.)
- (a) 9.84.      (b) 4.13.      (c) 0.14.      (d) 0.05.      (e) None of these.
15. Find the inflection point(s) of the function  $f(x) = \frac{1}{6}x^4 - 9x^2 + 81.5$ .
- (a)  $(3, 5)$  and  $(-3, 5)$ .      (b)  $(3, 14)$  and  $(-3, 14)$ .      (c)  $(9, 5)$ .
  - (d)  $(9, 14)$ .      (e) None of these.

16. Let  $f(x, y) = x^3 + y^2 + 3x^2y$ . Find  $\frac{\partial f}{\partial x}$ , or  $f_x$ .
- (a)  $2y + 3x^2$ .      (b)  $3x^2 + 2y + 6xy$ .      (c)  $3x^2 + 3y$ .  
(d)  $3x^2 + 6xy$ .      (e) None of these.
17. Let  $f(x, y) = e^{xy}$ . What is  $f_{yy}(2, 1)$ ?
- (a)  $4e^2$ .      (b)  $2e^2$ .      (c)  $e^2$ .      (d)  $e$ .      (e) None of these.
18. Consider the Cobb-Douglas production function  $f(x, y) = 3x^{2/3}y^{1/3}$ , where  $x$  is the number of units of labour and  $y$  is the number of units of capital. What is the marginal productivity of **labour** at  $x = 8$ ,  $y = 27$  ?
- (a) 3.      (b) 36.      (c) 64.      (d)  $-18$ .      (e) None of these.
19. The value of  $\int_{-1}^2 (2x + 3) dx$  is
- (a) 6.      (b) 12.      (c)  $-4$ .      (d) 0.      (e) None of these.
20. Find  $\frac{d}{dx} \int_2^x \sqrt{t^2 + t + 1} dt$ .
- (a)  $\frac{t}{\sqrt{t^2 + t + 1}}$ .      (b)  $\frac{x}{\sqrt{x^2 + x + 1}} - \frac{2}{\sqrt{7}}$ .      (c)  $\sqrt{x^2 + x + 1}$ .  
(d)  $\sqrt{x^2 + x + 1} - \sqrt{7}$ .      (e) None of these.

**PART B. For full marks, be sure to show all of your steps.**

21. [10 marks] Use the method of Lagrange multipliers to find the maximum and the minimum values of the function

$$f(x, y) = (x - 1)^2 + (y - 2)^2$$

subject to the constraint

$$x^2 + y^2 = 45.$$

**22. [10 Marks]**

[4] **(a)** Find all the critical points of the function  $f(x, y) = x^2 - 4xy + 2y^2 + 4x + 8y - 1$ . (Do NOT classify them).

[6] **(b)** The function  $f(x, y) = 2x^3 - 2xy + y^2 + 7$  has two critical points  $(0, 0)$  and  $\left(\frac{1}{3}, \frac{1}{3}\right)$ . Use the Second Derivative Test to classify the nature of each point, if possible.

23. [4 marks] Consider the equation  $x^2 + 2xy^2 + y = 0$ , where  $y = y(x)$  is defined implicitly as a function of  $x$ . Find  $\frac{dy}{dx}$ .

24. [6 Marks] Evaluate the following integrals:

[3] (a)  $\int \frac{4x^4 - x + 5}{x} dx$

[3] (b)  $\int 6x^2(x^3 - 4)^7 dx$