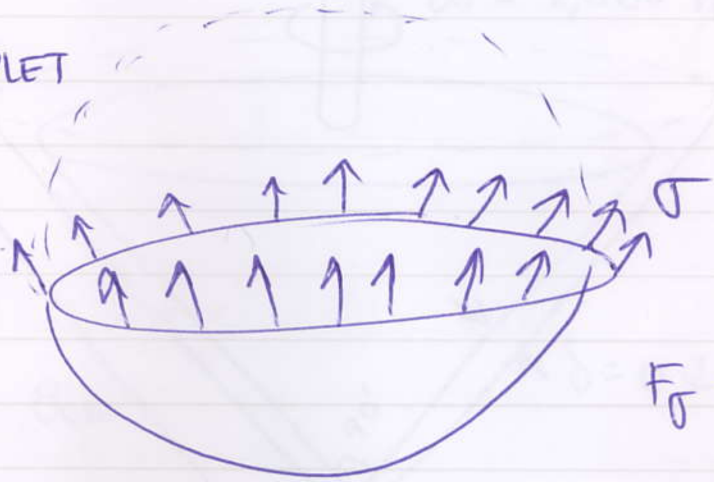
PROBLEM 1 (10 MARKS)

→ DROPLET



$\Sigma F = F_\sigma - F_p =$

$F_\sigma = \sigma L$
$= \sigma(2\pi r)$

$F_p = pA$
$= p(\pi r^2)$

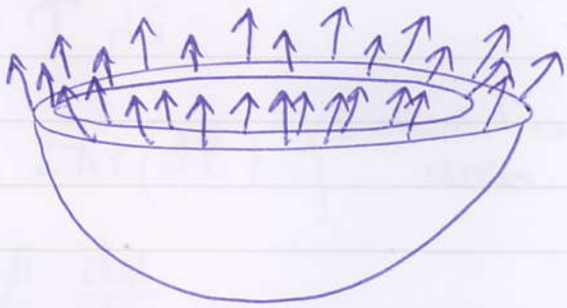
$\sigma(2\pi r) - p(\pi r^2) = 0 \Rightarrow \sigma(2\pi r) = p(\pi r^2)$

$$p = \frac{\sigma(2\cancel{\pi} r)}{\cancel{\pi} r^{\cancel{2}}} = \frac{2\sigma}{r}$$

$$p = \frac{2(0.073)}{\frac{10}{2}\times 10^{-6}} = 29{,}200\ \text{Pa} \quad \text{or} \quad 29.2\ \text{kPa}$$

$0.073 \to$ [N/m], $\frac{10}{2}\times 10^{-6} \to$ [m]

→ Bubble

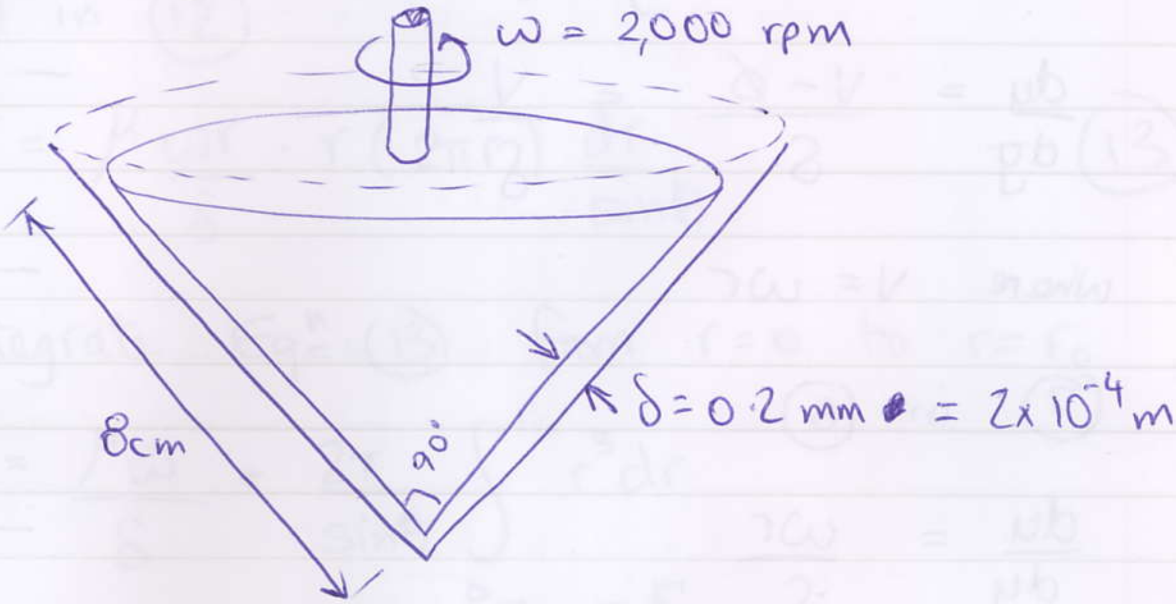


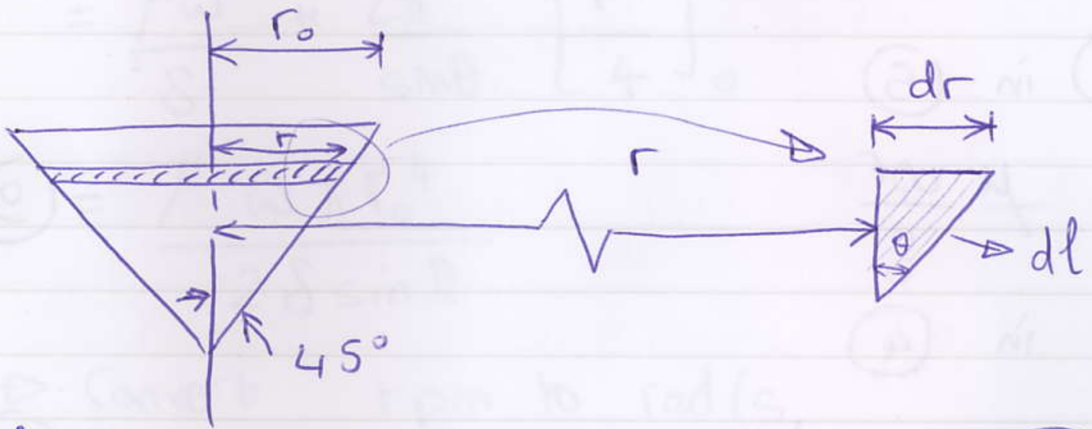
$\Sigma F = F_\sigma - F_p = 0$

$F_\sigma = \sigma L = 2\sigma(2\pi r)$
$F_p = pA = p(\pi r^2)$

$$p = \frac{4\sigma \cancel{\pi r}}{\cancel{\pi} r^{\cancel{2}}} = \frac{4\sigma}{r} = 58{,}400\ \text{Pa} \quad \text{or} \quad 58.4\ \text{kPa}$$

PROBLEM 2 (15 MARKS)





$$d\ell = \frac{dr}{\sin\theta} \quad \text{— (1)}$$

$$dT = r\,F_\tau \quad \text{— (2)}$$

$$F_\tau = \tau\, dS \quad \text{— (3)}$$

$$dS = 2\pi r (d\ell) \quad \text{— (4)}$$

{Elemental surface area

$$\tau = \mu \frac{du}{dy} \quad \text{— (5)}$$

For a linear velocity profile

$$\frac{du}{dy} = \frac{V - \emptyset}{\delta} = \frac{V}{\delta} \quad -(6)$$

where $V = \omega r \quad -(7)$

(7) in (6)

$$\frac{du}{dy} = \frac{\omega r}{\delta} \quad -(8)$$

(8) in (5)

$$\tau = \mu \frac{\omega r}{\delta} \quad -(9)$$

(1) in (4)

$$dS = 2\pi r \frac{dr}{\sin\theta} \quad -(10)$$

(10) in (3)

$$F_{\tau} = \tau \cdot 2\pi r \frac{dr}{\sin\theta} \quad -(11)$$

(11) in (2)

$$dT = r \cdot 2\pi r \cdot \tau \cdot \frac{dr}{\sin\theta} \quad -(12)$$

9 in 12

$$dT = \mu \frac{\omega r}{\delta} \cdot r(2\pi r) \frac{dr}{\sin\theta} \quad - 13$$

Integrate Eq^n 13 from $r=0$ to $r=r_0$

$$T = \frac{\mu\omega}{\delta} * \frac{2\pi}{\sin\theta} \int_0^{r_0} r^3 dr$$

$$= \frac{\mu\omega}{\delta} * \frac{2\pi}{\sin\theta} \left[\frac{r^4}{4}\right]_0^{r_0}$$

$$= \frac{\mu \omega \pi r_0^4}{2\delta \sin\theta} \quad - 14$$

→ Convert rpm to rad/s

$$\omega = 2{,}000 \frac{\cancel{rev}}{\cancel{min}} * \frac{\cancel{1min}}{60 sec.} * \frac{2\pi \text{ radians}}{\cancel{rev.}}$$

$$= 209.44 \text{ rad/sec.}$$

Given $\nu_{SAE-30} = 1.1 \times 10^{-4}\ m^2/s$

$$\mu_{SAE-30} = \nu\rho)_{SAE-30} \quad - 15$$

$S_{SAE-30} = 0.88$ (Given)

$$\mu_{SAE-30} = \nu_{SAE-30} * S_{SAE-30} * \rho_{water}$$

$$\mu_{SAE-30} = 1.1 \times 10^{-4} \left[\frac{m^2}{s}\right] * 0.88 * 999 \left[\frac{kg}{m^3}\right]$$

$$= 9.66 \times 10^{-2} \left[\frac{Ns}{m^2}\right]$$

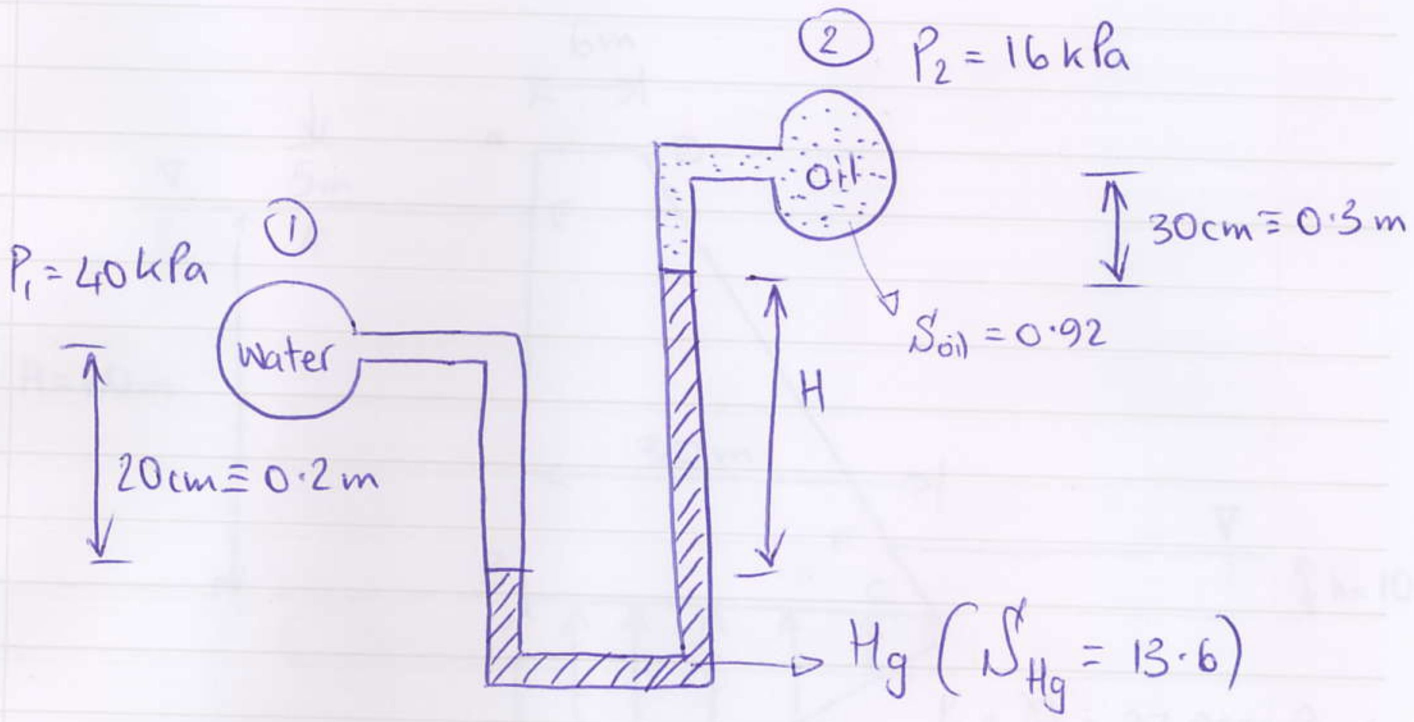
$$r_0 = 8\,cm \sin 45 = 5.66\,cm = 0.0566\,m$$

Eq$\underline{n}$ 15

$$T = \frac{9.66 \times 10^{-2} * 209.44 * \pi * 0.0566^4}{2 * 2 \times 10^{-4} * \sin 45}$$

$$= 2.31 \; [Nm]$$

PROBLEM 3 (10 MARKS)



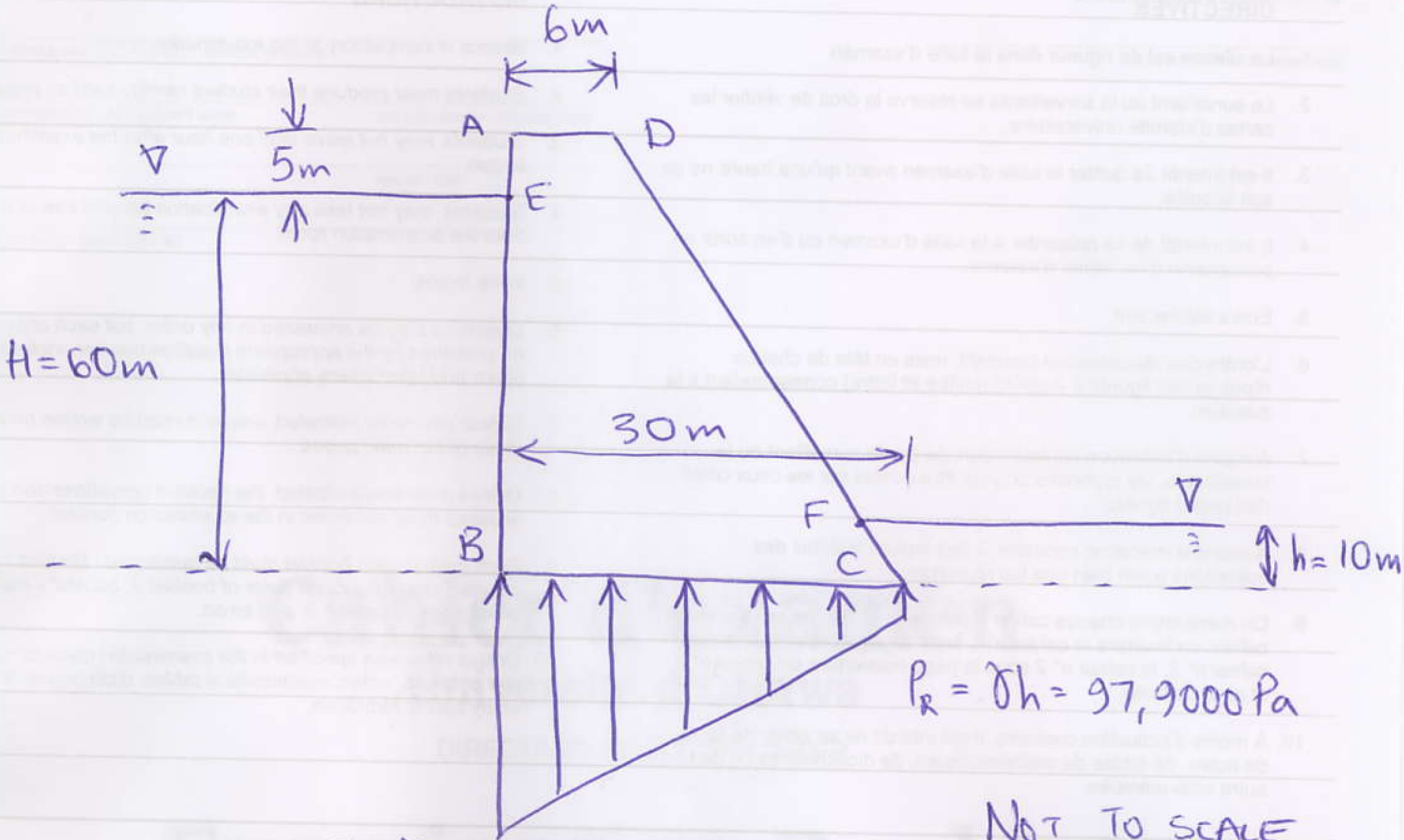
$$P_1 = P_2 + \gamma_{oil}(0.3) + \gamma_{Hg}(H) - \gamma_{water}(0.2)$$

$$= P_2 + \gamma_w S_{oil}(0.3) + \gamma_w S_{Hg}(H) - \gamma_w(0.2)$$

$$40000 = 16{,}000 + 9{,}790\left[0.92 * 0.3 + 13.6H - 0.2\right]$$

$$H = 0.1747\,m \quad = 17.5\,cm$$

PROBLEM 4 (15 MARKS)



$P_R = \gamma h = 97,9000\ Pa$

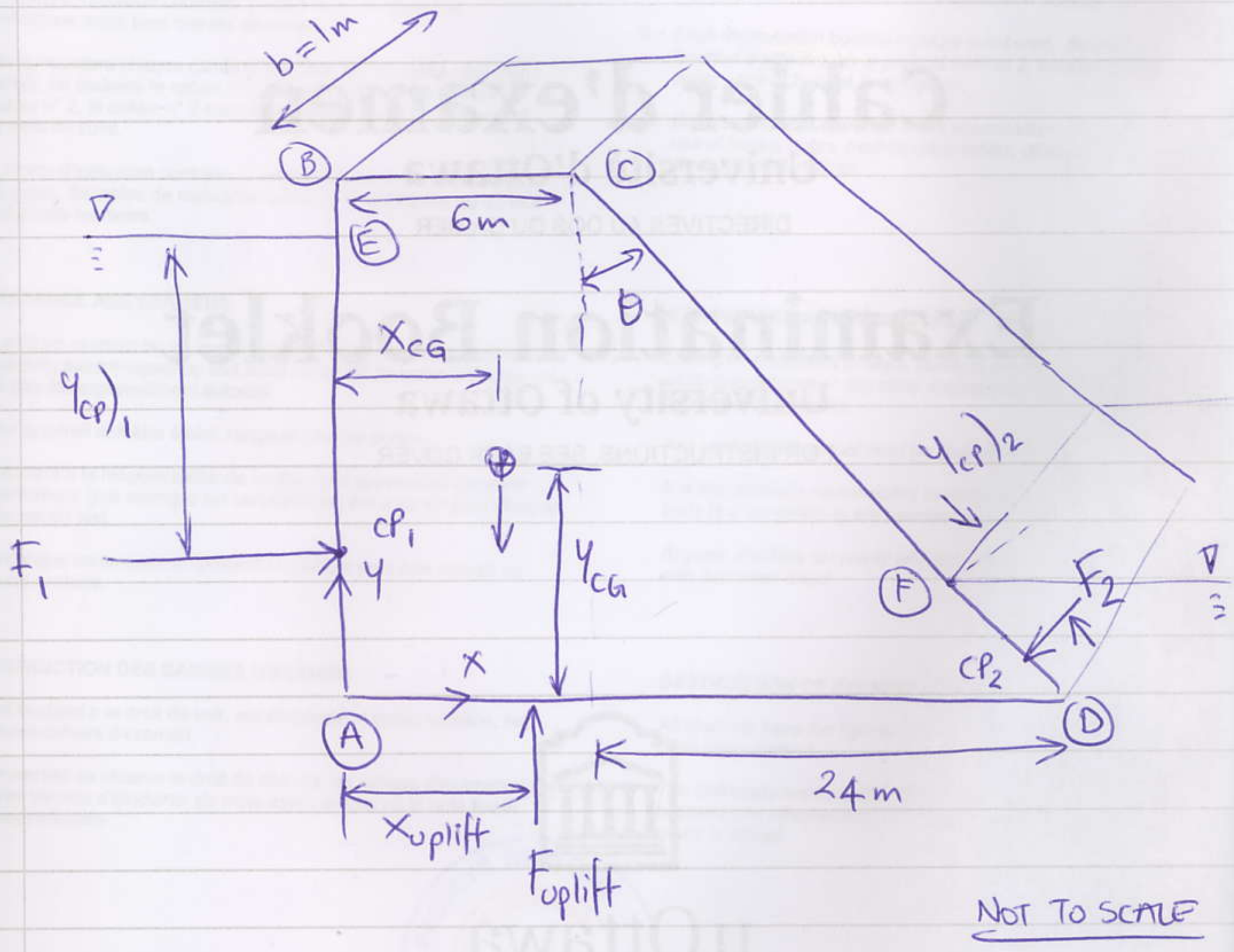
NOT TO SCALE

$P_L = \gamma H = 9,790 * 60$
$= 587,400\ Pa$

Four forces acting on the structure

- 2 Hydrostatic forces (1 on the upstream (i.e. left) side & 1 on the downstream (i.e. right side)
- 1 Uplift force (which is reall a hydrostatic force present in the soil/ground)
- 1 weight of the structure

All the forces mentioned above must be analyzed as to how will they influence stability (overturning) of the structure. It seems that the uplift & left hydrostatic force will cause the structure to turn clockwise while the right hydrostatic force & weight would cause the structure to turn counter-clockwise about the right-hand corner. However, this overturning moment must be quantified.



① F_1

$$F_1 = \bar{P}_1 A_1$$
$$= \gamma \bar{h}_1 A_1$$
$$= \gamma \frac{H}{2} b H$$
$$= \frac{\gamma H^2 b}{2}$$
$$= \frac{9{,}790\,(60)^2 b}{2} = 17{\cdot}62\, b \;[\text{MN}]$$

$\bar{h}_1$ = height to centroid
$= H_1/2$

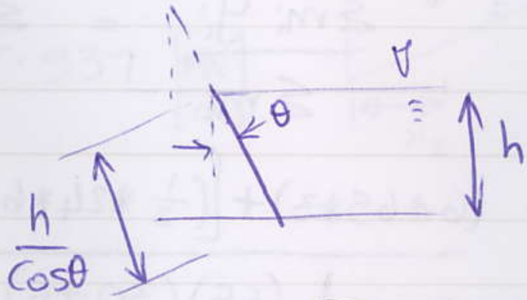
$A_1 = bH_1$

$I_1 = bH_1^3/12$

$\bar{y}_1 = H_1/2$

$$y_{cp})_1 = \bar{y}_1 + \frac{\bar{I}_1}{\bar{y}_1 A_1}$$
$$= \frac{H_1}{2} + \frac{bH_1^3}{12(H_1/2)\,bH_1} = \frac{2H}{3} = 40\,[\text{m}]$$

② F_2



$$F_2 = \bar{P}_2 A_2$$
$$= \gamma \bar{h}_2 A_2$$
$$= \gamma \frac{h}{2} \frac{bh}{\cos\theta}$$
$$= \frac{\gamma b h^2}{2\cos\theta} = 0{\cdot}522\, b \;[\text{MN}]$$

$\bar{h}_2$ = height to centroid
$= h/2$

$A_2 = bh/\cos\theta$

$\theta = \tan^{-1}\frac{24}{65} = 20{\cdot}27^\circ$

$$y_{cp})_2 = \bar{y}_2 + \frac{\bar{I}_2}{\bar{y}_2 A_2}$$

$\cos\theta = 0.938$

$\bar{y}_2 = h/2\cos\theta$

$\bar{I}_2 = b(h/\cos\theta)^3/12$

$$= \frac{h}{2\cos\theta} + \frac{b(h/\cos\theta)^{\cancel{3}}}{12(\cancel{h/\cos\theta})\, b(h/\cancel{\cos\theta}) * \frac{1}{2}} = \frac{2h}{3\cos\theta}$$

$$= 7.11\,m$$

(3) W

$$W = \gamma_{concrete} \forall_{concrete} = \gamma_w S_{concrete} \forall_{concrete}$$

$$\forall_{concrete} = \frac{1}{2}\, 65(6+30) * b = 1{,}170b$$

$$W = 9790 * 2.4 * 1{,}170b = 27.49b\ [MN]$$

$$x_{CG} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{\Sigma W_i x_i}{\Sigma W_i} = \frac{\Sigma A_i x_i}{\Sigma A_i}$$

$$y_{CG} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{\Sigma W_i y_i}{\Sigma W_i} = \frac{\Sigma A_i y_i}{\Sigma A_i}$$

$$x_{CG} = \frac{(6*65*3) + \left[\left(\frac{1}{2}*24*65\right)\left(6+24/3\right)\right]}{\frac{1}{2}(65)(30+6)} = \frac{1{,}170 + 10{,}920}{1{,}170}$$

$$= 10.33\ m$$

$$y_{CG} = \frac{\left(6*65*\frac{65}{2}\right) + \left[\left(\frac{1}{2}*24 \times 65\right)*(65/3)\right]}{\frac{1}{2}(65)(30+6)} = \frac{12{,}675 + 16{,}900}{1{,}170}$$

$$= 25.28\,m$$

$$\cancel{y_{cp}\big|_2 = y_2 + \frac{I_2}{y_2 A_2}}$$

(4) F_{uplift}

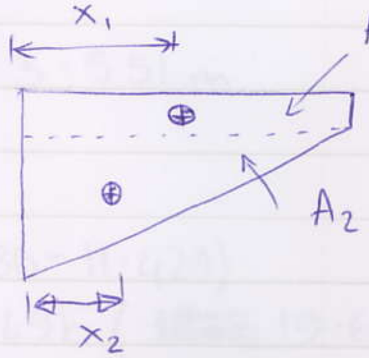
$$F_{uplift} = P_{uplift-resultant} * A_{bottom}$$

$$A_{bottom} = 30 * b = 30b \ [m]$$

$$P_{uplift-resultant} = \frac{P_L + P_R}{2} = \frac{587{,}400 + 97{,}900}{2} = 342{,}650 \ Pa$$

$$F_{uplift} = 342{,}650 * 30b = 10.28 \ [MN]$$

$$X_{uplift} = \frac{A_1 x_1 + A_2 x_2}{\Sigma A}$$



$$A_1 = 97{,}900 * 30 = 2.937 \left[\frac{MN}{m}\right]$$

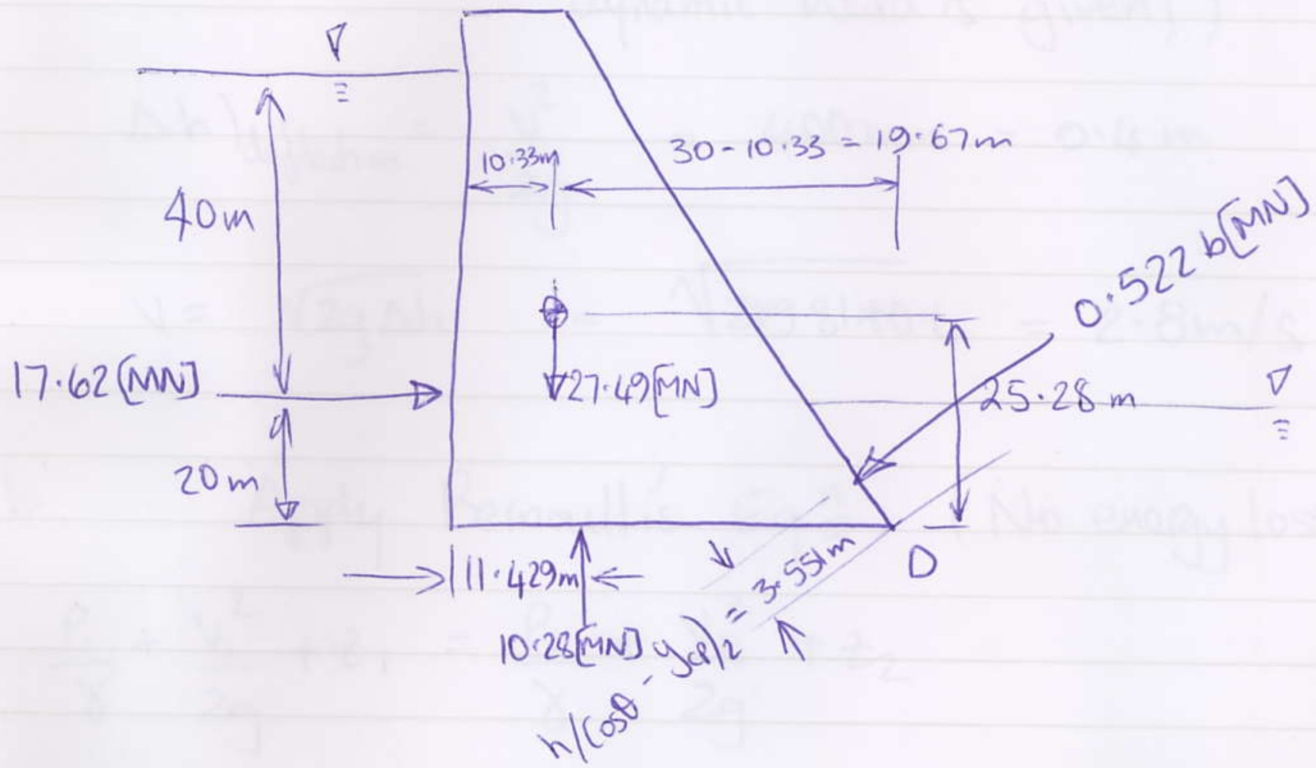
$$x_1 = 15m$$

$$A_2 = (587{,}400 - 97{,}900) * \frac{1}{2} * 30 = 7.343 \left[\frac{MN}{m}\right]$$

$$X_2 = 30/3 = 10m$$

$$X_{uplift} = \frac{2.937 * 15 + 7.343 * 10}{2.937 + 7.343} = 11.429 [m]$$

→ Summation of Moments



$$h/(\cos\theta - y_{cp})_2 = \frac{10}{0.938} - 7.11\,m = 3.551\,m$$

$$\circlearrowright^{+} \sum M_D = +17.62b\,(20) + 10.28b\,(30 - 11.429) - 0.522b\,(3.551) - 27.49b\,(19.667)$$

$$= 0.81\,[N\,m] \rightarrow +ve \;\therefore\; \text{will overturn about } D$$

PROBLEM 5 (10 MARKS)

a. $\Delta h)_{deflection}$ $\longrightarrow$ dynamic head (i.e. no need to consider static head, since the dynamic head is given!)

$$\Delta h)_{deflection} = \frac{V^2}{2g} = 400\,mm = 0.4\,m$$

$$V = \sqrt{2g\Delta h} = \sqrt{2*9.81*0.4} = 2.8\,m/s$$

b. Apply Bernoulli's Eq$^{\underline{n}}$ (No energy loss)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$z_1 = z_2$ (same datum)

$P_1/\gamma = 0.25\,m$ $\qquad\qquad$ $P_2/\gamma = H$

$$0.25 + \frac{0.5^2}{2(9.81)} = H + \frac{1.125^2}{2(9.81)}$$

$$H = 0.198\,m$$

PROBLEM 6 (10 MARKS)

CONTINUITY EQ$^{\underline{N}}$

$$\frac{d}{dt}\int_{CV} \rho \, d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A} = 0$$

$$\frac{dM_{cv}}{dt} = \Sigma \dot{m}_{in} - \Sigma \dot{m}_{out}$$

$$M_{cv} = \rho \forall_{cv}$$

$$\frac{dM_{cv}}{dt} = \frac{d}{dt}(\rho \forall_{cv}) = \frac{d}{dt} \rho A_{cv} h_{cv} = \rho A_{cv} \frac{dh_{cv}}{dt}$$

$$\dot{m}_{in})_1 = \rho A_1 \vartheta_1 = 998 * \frac{\pi}{4}(0.04)^2 * 10 = 12.54 \text{ kg/s}$$

$$\dot{m}_{in})_2 = \rho A_2 \vartheta_2 = \rho Q_2 = \dot{m}_2 = 10 \text{kg/s}$$

$$\Sigma \dot{m}_{in} = 12.54 + 10 = 22.54 \text{ kg/s} \qquad \left[\frac{1 \text{min}}{60 \text{sec}}\right] \cdot \left[\frac{m^3}{1000 L}\right]$$

$$\Sigma \dot{m}_{out} = \dot{m}_3 = \rho Q_3 = 998\left[\frac{Kg}{m^3}\right] * 600\left[\frac{L}{min}\right] * \text{Conversion}$$

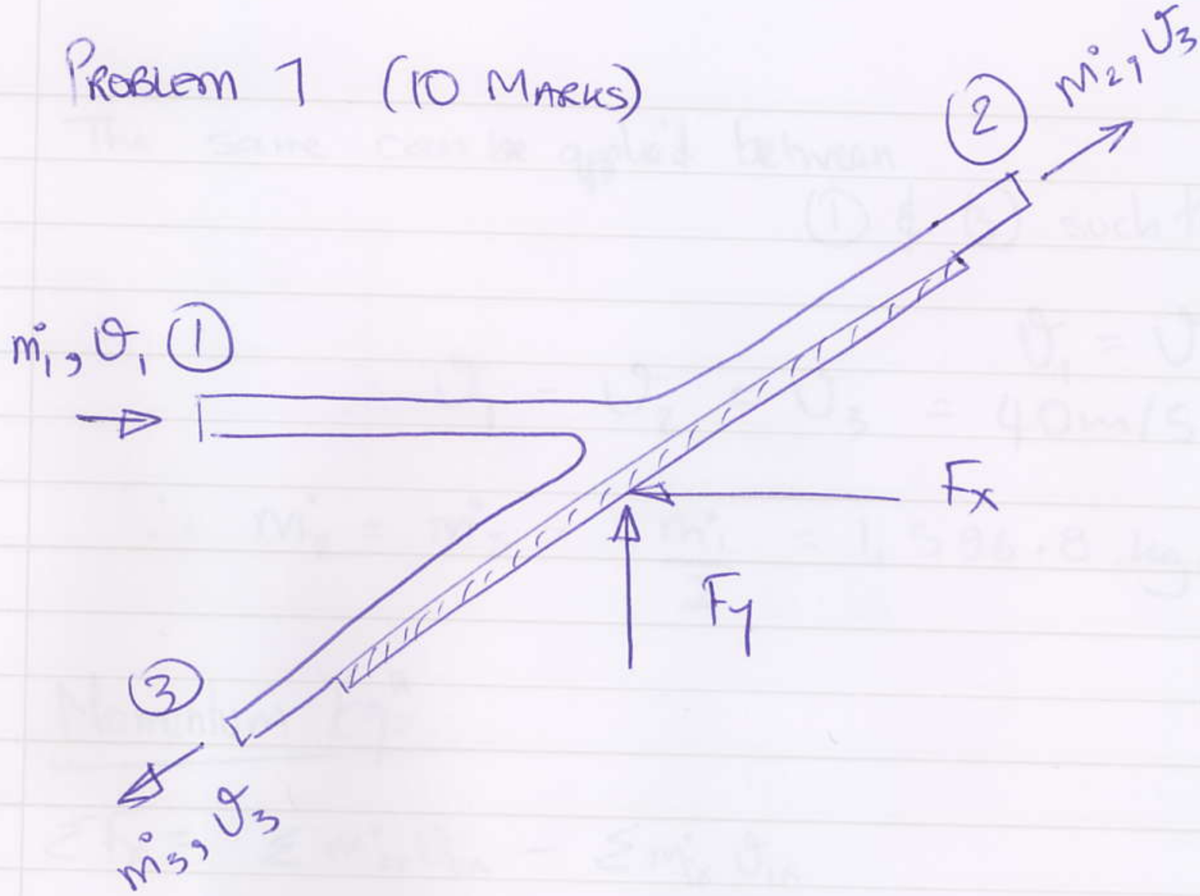
$$= 9.98 \text{ kg/s}$$

$$\frac{dh_{cv}}{dt} = \frac{\Sigma \dot{m}_{in} - \Sigma \dot{m}_{out}}{\rho A_{cv}} = \frac{22.54 - 9.98}{998 * \pi (1.2^2)/4}$$

$$= 0.0111 [m/s]$$

$$= 1.11 [cm/s]$$

Problem 7 (10 Marks)



Continuity

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$
$$\dot{m}_1 = \rho \vartheta_1 A_1 = \rho \vartheta_1 (h * b)$$
$$= 998 * 40 \, m/s * 0.2 * 0.4$$
$$= 3193.6 \, kg/s$$

Bernoulli's Eqn:

$$\underbrace{\cancel{P_1/\gamma}}_{=0} + \frac{V_1^2}{2g} + \cancel{z_1} = \underbrace{\cancel{P_2/\gamma}}_{=0} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$z_1 = z_2$
$P_1 = P_2 = 0$

$$\therefore \vartheta_1 = \vartheta_2$$

The same can be applied between ① & ③ such that

$$\vartheta_1 = \vartheta_3$$

$$\therefore \vartheta_1 = \vartheta_2 = \vartheta_3 = 40 m/s$$

$$\therefore \dot{m}_2 = \dot{m}_3 = \frac{\dot{m}_1}{2} = 1,596.8 \ kg/s$$

Momentum Eq$^{\underline{n}}$

$$\Sigma F_x = \Sigma \dot{m}_{ox} \vartheta_{ox} - \Sigma \dot{m}_{ix} \vartheta_{ix}$$

$$-F_x = [-\dot{m}_3 * 40 \cos 45 + \dot{m}_2 * 40 \cos 45] - \dot{m}_1 * 40$$

$$-F_x = -\dot{m}_1 * 40 = -127,744 \ [N]$$

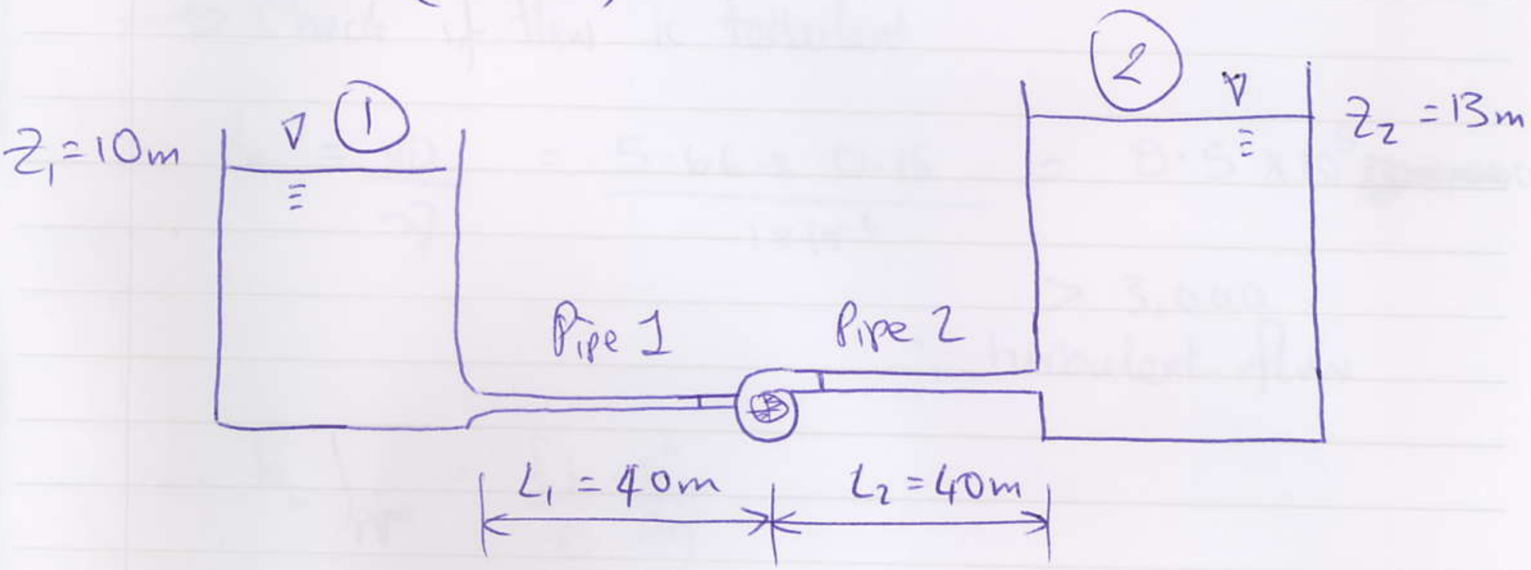
$$\therefore F_x = 127.74 \ [kN]$$

$$\Sigma F_y = \Sigma \dot{m}_{oy} \vartheta_{oy} - \Sigma \dot{m}_{iy} \vartheta_{iy}$$

$$F_y = [-\dot{m}_3 * 40 \sin 45 + \dot{m}_2 * 40 \sin 45] - [0]$$

$$F_y = 0 \ [N]$$

PROBLEM 8 (15 MARKS)



$Q = 0.10\ m^3/s$ $\qquad$ $\vartheta_{p_1} = \vartheta_{p_2} = \frac{Q}{A}$

$D_1 = D_2 = 0.15m$

(where "p" stands for "pipe"!)

$\vartheta_{pipe\ 1} = \frac{0.10}{\pi (0.15)^2/4} = 5.66\ m/s$

$= \vartheta_{pipe\ 2}$

Energy Equation

$$z_1 + \cancel{\frac{p_1}{\gamma}}^{\,0} + \alpha_1 \cancel{\frac{V_1^2}{2g}}^{\,0} + h_p = z_2 + \cancel{\frac{p_2}{\gamma}}^{\,0} + \alpha_2 \cancel{\frac{V_2^2}{2g}}^{\,0} + \cancel{h_t}^{\,0} + \Sigma h_L$$

(atm.) (negl.)* $\qquad$ (atm.) (negl.)* (no turbine)

* NOTE: $\vartheta_1 \neq \vartheta_{pipe\ 1}$ & $\vartheta_2 \neq \vartheta_{pipe\ 2}$

ϑ_1 = vel. of water surface at ①

ϑ_2 = vel. of water surface at ②

$\therefore\ h_p = [z_2 - z_1] + \Sigma h_L$

→ Check if flow is turbulent

$$Re = \frac{VD}{\nu} = \frac{5.66 * 0.15}{1 \times 10^{-6}} = 8.5 \times 10^5 \cancel{3{,}000}$$

$$> 3{,}000$$

$\therefore$ turbulent flow

$$\therefore h_L \Big)_{pipe} = \frac{fL}{D} \frac{V_{pipe}^2}{2g}$$

$$\Sigma h_L = \frac{f_1 L_1}{D_1} \frac{V_{pipe\,1}^2}{2g} + \frac{f_2 L_2}{D_2} \frac{V_{pipe\,2}^2}{2g}$$

$$= \frac{f}{D}(L_1 + L_2) \frac{V_p^2}{2g}$$

$f = 0.015$; $L = 40 + 40 = 80 m$; $D = 0.15 m$

$$\Sigma h_L = \frac{0.015(80)}{0.15} * \frac{5.66^2}{2(9.81)} = 13.192 m$$

$$h_p = (z_2 - z_1) + \Sigma h_L = 13 - 10 + 13.192 m$$

$$= 16.192 [m]$$

$$P_{pump} = \gamma Q h_p = 9{,}790 * 0.1 * 16.192$$

$$= 15{,}851.95 W$$

$$= 15.85 kW$$

BONUS QUESTION (5 MARKS)

$K_s = 0.046\ mm = 0.046 \times 10^{-3}\ m$

$$\frac{K_s}{D} = \frac{0.046 \times 10^{-3}}{0.15} = 0.0003$$

$Re = 8.5 \times 10^5$ (previously calculated)

Using $K_s/D = 0.0003$ & $Re = 8.5 \times 10^5$ in Moody's Diagram

$\rightarrow f \cong 0.016$ (hence, it is not correct to assume $f = 0.015$)

If Q is unknown, f is unknown & K_s is known, and h_p is known, then K_s/D can be calculated, but without, Q, v_p cannot be calculated & Reynold's number, Re, cannot be obtained either. However, the following steps can be made to obtain a value for v_p:

1- Assume fully turbulent conditions (straight line region in Moody's diagram)
  $\hookrightarrow$ get "f" from K_s/D
2- From energy eq^n get v_p (h_p, v_p & Δz & Σh_L known)
3- Calculate Re from v_p above.
4- Obtain new value for "f" using Moody diagram
5- Repeat steps 1-4 until "f" converges (does not change)

PROBLEM 9

a. No. PIPE AB is the smallest

b. A

c. C

d. Jet

e. Sudden Expansion