MAT 1330, Fall 2011 Assignment 2

Due WED October 5, at the beginning of class.

Late assignments will **not** be accepted; **nor** will unstapled assignments.

Instructor (circle one): Robert Smith? Jason Levy Olga Vassilieva Catalin Rada

DGD (circle one): 1 2 3 4

Student Name ______________________________ Student Number ______________________________

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature ______________________________

QUESTION 1. Find $\lim_{x \to 0} \left(x^4 + \frac{\sin 3x}{2011} \right)$. Answer: 0

Give one sequence to support your claim. Four terms are enough.

x_n	$f(x_n)$
0.5	0.062996019
0.1	0.000246951
0.05	0.00008056
0.01	0.000014925

QUESTION 2. Does the limit $\lim_{t \to 0} \left(\frac{2011t}{3 - \sqrt{t+9}} \right)$ exist? Answer: Yes

Justify your answer in at most two lines without using sequences of numerical values for t.

Answer:

$$\lim_{t \to 0} \left(\frac{2011t}{3 - \sqrt{t+9}} \right) = \lim_{t \to 0} \frac{2011t(3 + \sqrt{t+9})}{(3 - \sqrt{t+9})(3 + \sqrt{t+9})} = \lim_{t \to 0} \frac{2011t(3 + \sqrt{t+9})}{9 - t - 9}$$

$$= \lim_{t \to 0} \frac{2011(3 + \sqrt{t+9})}{-1} = -12066.$$

QUESTION 3. Let $F(x) = \dfrac{4-x^2}{|2-x|} + |x|$.

a) Find $\lim_{x\to 2^+} F(x)$.

$$\lim_{x\to 2^+} \frac{4-x^2}{|2-x|} + |x| = \lim_{x\to 2^+} \frac{4-x^2}{-(2-x)} + x = \lim_{x\to 2^+} \frac{(2-x)(2+x)}{-(2-x)} + x = \lim_{x\to 2^+} \frac{2+x}{-1} + x = -2$$

b) Find $\lim_{x\to 2^-} F(x)$.

$$\lim_{x\to 2^-} \frac{4-x^2}{|2-x|} + |x| = \lim_{x\to 2^-} \frac{4-x^2}{2-x} + x = \lim_{x\to 2^-} \frac{(2-x)(2+x)}{2-x} + x = \lim_{x\to 2^-} 2+x+x = 6$$

c) Does $\lim_{x\to 2} F(x)$ exist? Answer: No

Justify your answer:

$\lim_{x\to 2^+} F(x) \neq \lim_{x\to 2^-} F(x) \Longrightarrow$ The limit at 2 does not exist.

QUESTION 4. Consider the function $f(x) = \dfrac{2011}{x^2}$. Use the definition of the derivative to compute $f'(2)$.

Answer:

$$f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{\frac{2011}{(2+h)^2} - \frac{2011}{2^2}}{h} = \lim_{h\to 0} \frac{2011\{2^2-(2+h)^2\}}{h2^2(2+h)^2}$$
$$= \lim_{h\to 0} \frac{2011(2-2-h)(2+2+h)}{h2^2(2+h)^2} = \lim_{h\to 0} \frac{2011(-1)(4+h)}{2^2(2+h)^2} = -\frac{2011}{4}.$$