

**GNG 1105  
ENGINEERING MECHANICS**

Final Examination  
December 9<sup>th</sup>, 2011

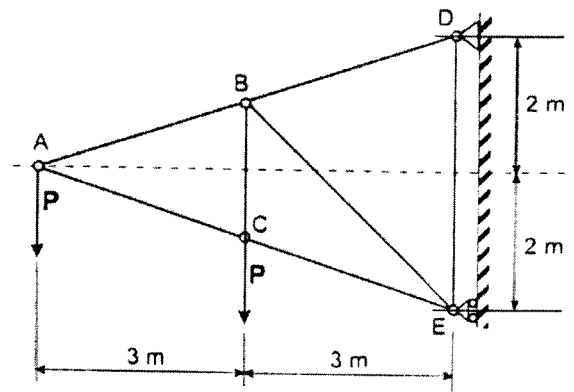
Duration: 3 hours  
Page: 1 of 2

Prof. A. Skaff, Ph. Girault, Y. Haddad and A. Ahmed

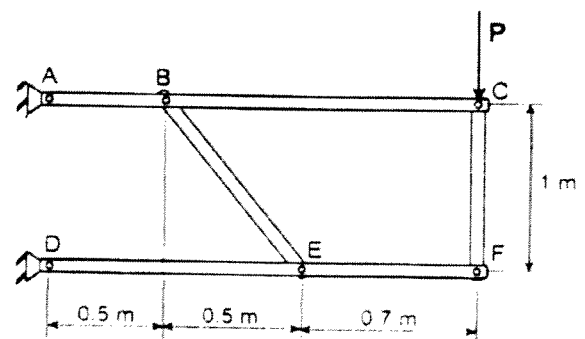
**Closed book examination.** Programmable calculators are not allowed.  
Free-body diagrams must be drawn where applicable.

**Problem 1.** (10/60) The truss shown is loaded by two forces  $P = 8 \text{ kN}$ . All joints are pinned.

- Determine the reactions at the supports D and E.
- Using the method of sections, determine the forces in members BD, BE and CE in the truss, specifying for each whether it is in tension or compression.

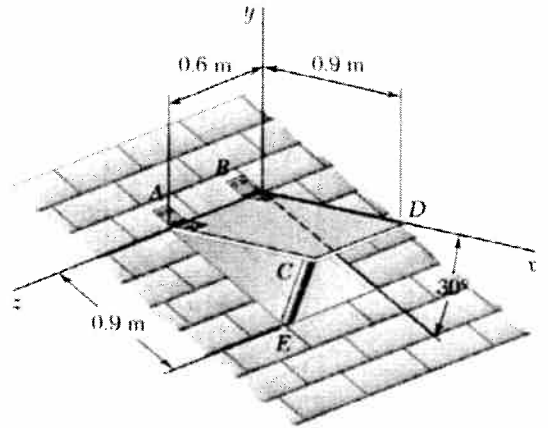


**Problem 2.** (12/60) The diagram shows a frame which is loaded by a force  $P = 20 \text{ kN}$ . All joints are frictionless pins. Determine the reactions at A and D and all forces acting on member DEF. Note that members ABC and DEF are continuous solid members.

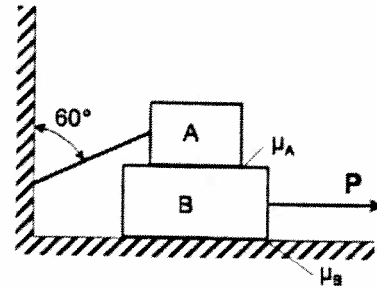


**Problem 3.** (14/60) A 30 kg cover for a roof-opening is hinged at corners A and B. The roof forms an angle of  $30^\circ$  with the horizontal, and the cover is maintained in a horizontal position by the brace CE. Determine:

- (a) The magnitude of the force exerted by the brace CE,
- (b) The reactions at the hinges A & B. Assume that the hinge at A does not exert any axial thrust.

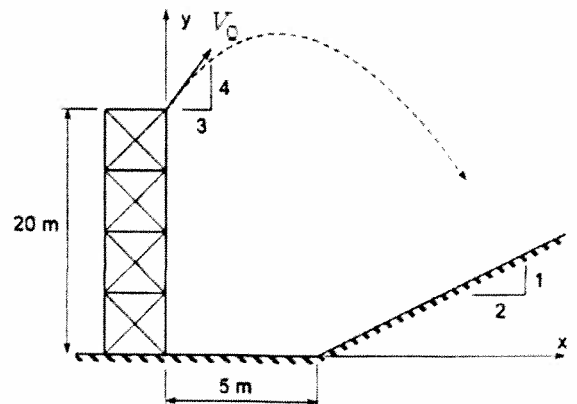


**Problem 4.** (14/60) Block A in the sketch has a mass of 20 kg and is attached to the wall by means of a cord at an angle of  $60^\circ$  with the vertical, while block B has a mass of 40 kg. The coefficient of static friction between A and B is 0.2, while the coefficient of static friction between B and the floor is 0.3. Determine the minimum force P required to cause Block B to slide (i.e. Motion impending).



**Problem 5.** (10/60) A ball is thrown from the top of a 20m high tower with an initial velocity of  $V_0 = 10$  m/s in the direction shown.

- (a) Determine the x and y coordinates of the point at which the ball strikes the slope.
- (b) Determine the velocity of the ball at the point at which it strikes the slope.



GNG 1105 A & F  
FINAL EXAMINATION  
SOLUTIONS

Dec 9, 2011

1. FBD - Entire truss

a)  $\uparrow \Sigma F_y = 0$

$D_y - 8\text{KN} - 8\text{KN} = 0$

$\therefore D_y = \underline{\underline{16\text{KN}\uparrow}}$

b)  $\Sigma M_D = 0$

$8\text{KN} \times 6\text{m} + 8\text{KN} \times 3\text{m} - E_x \times 4\text{m} = 0$

$48 + 24 = 4E_x$

$\therefore E_x = \frac{72}{4} = \underline{\underline{18\text{KN}\leftarrow}}$

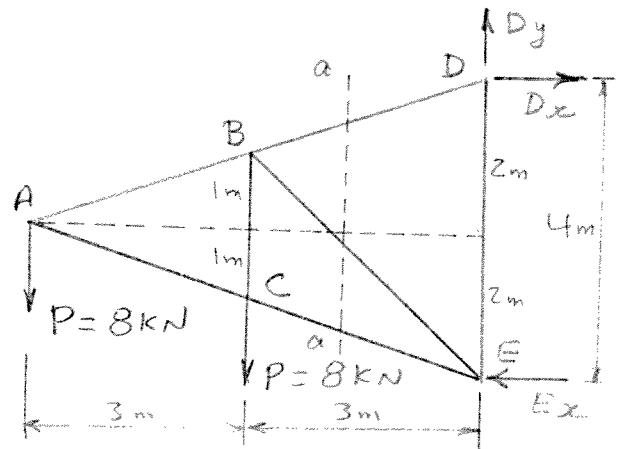
$\rightarrow \Sigma F_x = 0$

$D_x - E_x = 0 ; \therefore D_x = E_x = \underline{\underline{18\text{KN}\rightarrow}}$

$\therefore D = \sqrt{(16)^2 + (18)^2} = 24.08\text{KN}; \angle A = \tan^{-1} \frac{18}{12} = \underline{\underline{48.366^\circ}}$

$\therefore D = \underline{\underline{24.08\text{KN}}}$

ANS.



ANS.

ANS.

ANS.

b) FBD - Left side of section a-a.

a)  $\Sigma M_A = 0$

$- 8\text{KN} \times 3\text{m} - F_{BE} \times \frac{1}{1.414} \times 3\text{m}$

$- F_{BE} \times \frac{1}{1.414} \times 1\text{m} = 0$

$-24 - 2.122 F_{BE} - 0.707 F_{BE} = 0$

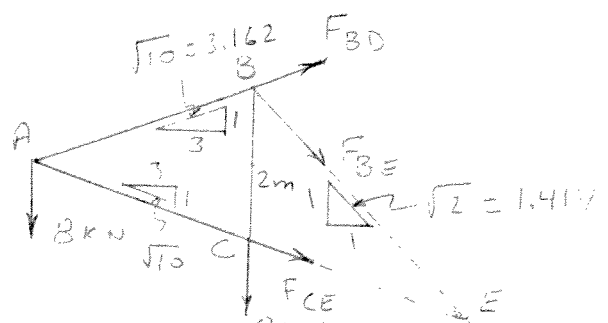
$2.829 F_{BE} = -24 ; \therefore F_{BE} = \frac{-24}{2.829} = \underline{\underline{8.484\text{KN (C)}}}$

b)  $\Sigma M_E = 0$

$8\text{KN} \times 3\text{m} + 8\text{KN} \times 6\text{m} - F_{BD} \times \frac{1}{2.162} \times 3\text{m} - F_{BD} \times \frac{1}{3.162} \times 3\text{m} = 0$

$24 + 48 - 0.463 F_{BD} - 0.949 F_{BD} = 0$

$3.795 F_{BD} = 72 ; \therefore F_{BD} = \underline{\underline{18.972\text{KN (T)}}}$  ANS.



ANS.

1. Cont'd.

$$\uparrow \sum M_B = 0$$

$$8 \text{ kN} \times 3 \text{ m} + F_{CE} \times \frac{3}{3.162} \times 2 \text{ m} = 0$$

$$1.898 F_{CE} = -24 \quad \therefore F_{CE} = -\frac{24}{1.898} = \underline{\underline{-12.645 \text{ kN (C)}}} \text{ ANS.}$$

2. FBD - Entire Frame

$$\uparrow \sum M_A = 0$$

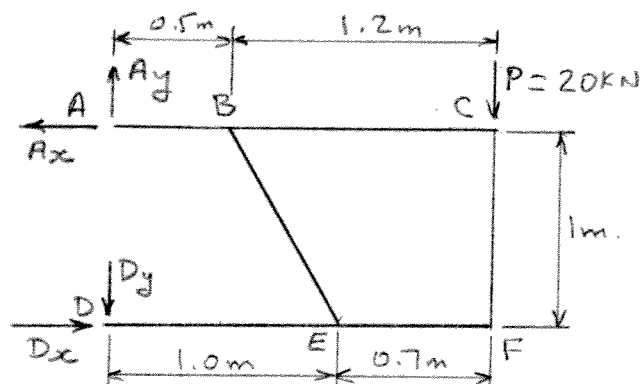
$$D_x \times 1 \text{ m} - 20 \text{ kN} \times 1.7 \text{ m} = 0$$

$$\therefore D_x = \underline{\underline{34.0 \text{ kN}}} \text{ ANS.}$$

$$\rightarrow \sum F_x = 0$$

$$D_x - A_x = 0$$

$$D_x = A_x = \underline{\underline{34.0 \text{ kN}}} \text{ ANS.}$$



FBD - Members ABC and DEF

Member DEF:

$$\rightarrow \sum F_x = 0$$

$$D_x - \frac{1}{2.236} F_{EB} = 0$$

$$F_{EB} = 2.236 \times 34.0 = \underline{\underline{76.06 \text{ kN}}}$$

$$\uparrow \sum M_F = 0$$

$$D_y \times 1.7 \text{ m} - F_{EB} \times \frac{2}{2.236} \times 0.7 \text{ m} = 0$$

$$1.7 D_y = 76.06 \times \frac{2}{2.236} \times 0.7 \text{ m}; \quad D_y = \underline{\underline{47.62 \text{ kN}}} \text{ ANS.}$$

$$\uparrow \sum F_y = 0$$

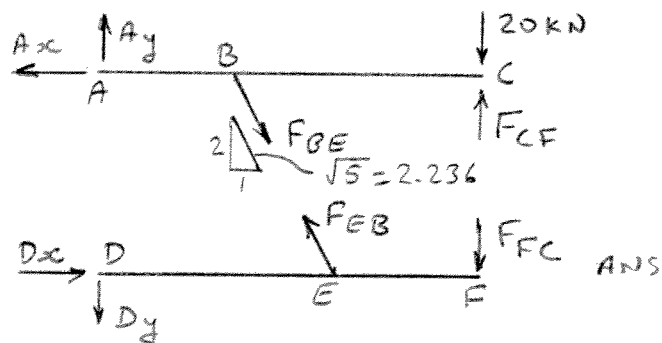
$$-47.62 - F_{FC} + 76.06 \times \frac{2}{2.236} = 0;$$

$$F_{FC} = 47.62 - 68.03; \quad \therefore F_{FC} = \underline{\underline{-20.41 \text{ kN}}} \text{ ANS.}$$

Entire truss:

$$\uparrow \sum F_y = 0;$$

$$A_y - D_y - 20 = 0; \quad A_y = 47.62 + 20 = \underline{\underline{67.62 \text{ kN}}} \text{ ANS.}$$



3. a) FBD - Cover

Force exerted by CE:

$$AC = AE. \text{ Assume } \vec{F}_{CE} = \vec{F}$$

$$\vec{F} = F(\sin 75^\circ)\vec{i} + F(\cos 75^\circ)\vec{j}$$

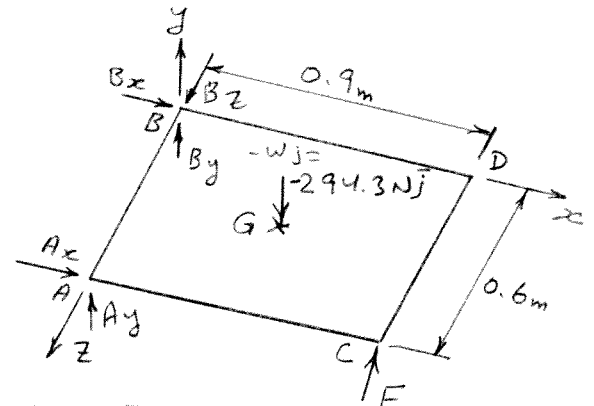
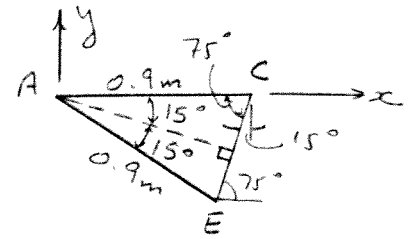
$$\vec{F} = F(0.2588\vec{i} + 0.9659\vec{j})$$

$$W = mg = 30 \text{ kg} \times 9.81 \text{ m/s}^2 = 294.3 \text{ N}$$

$$\sum \vec{M}_B = 0$$

$$\vec{r}_{A/B} = 0.6\vec{k}; \quad \vec{r}_{C/B} = 0.9\vec{i} + 0.6\vec{k}$$

$$\vec{r}_{G/B} = 0.45\vec{i} + 0.3\vec{k}$$



b)

$$\therefore \sum M_B = 0 = \vec{r}_{G/B} \times (-294.3\vec{j}) + \vec{r}_{C/B} \times \vec{F} + \vec{r}_{A/B} \times \vec{A}$$

$$\text{i.e. } (0.45\vec{i} + 0.3\vec{k}) \times (-294.3\vec{j}) + (0.9\vec{i} + 0.6\vec{k}) \times (0.2588\vec{i} + 0.9659\vec{j})F + 0.6\vec{k} (A_x\vec{i} + A_y\vec{j}) = 0$$

$$-132.435\vec{k} + 88.29\vec{i} + 0.869F\vec{k} + 0.153F\vec{j} - 0.579F\vec{i}$$

$$+ 0.6A_x\vec{j} - 0.6A_y\vec{i} = 0$$

$$\text{or } \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.45 & 0 & 0.3 \\ 0 & -294.3 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.9 & 0 & 0.6 \\ 0.2588 & 0.9659 & 0 \end{vmatrix} F + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 0.6 \\ A_x & A_y & 0 \end{vmatrix} = 0$$

$$\text{Coeff. of } \vec{i}: 88.29 - 0.579F - 0.6A_y = 0 \quad \text{--- (1)}$$

$$\text{Coeff. of } \vec{j}: 0.153F + 0.6A_x = 0 \quad \text{--- (2)}$$

$$\text{Coeff. of } \vec{k}: -132.435 + 0.869F = 0; \quad \therefore F = \underline{\underline{152.4 \text{ N}}} \quad \text{ANS.}$$

$$\text{Insert in (1): } 88.29 - 0.579 \times 152.4 - 0.6A_y = 0$$

$$0.6A_y = 88.29 - 88.23$$

$$0.6A_y = 0.06, \quad \therefore A_y = \underline{\underline{0.1 \text{ N}}} = 0 \quad \text{ANS.}$$

$$\text{Insert in (2): } 0.153 \times 152.4 + 0.6A_x = 0$$

$$0.6A_x = -23.317, \quad \therefore A_x = \underline{\underline{-38.86 \text{ N}}} \quad \text{ANS.}$$

3. Cont'd.

$$\Sigma \vec{F} = \vec{A} + \vec{B} + \vec{F} - W\vec{j} = 0$$

$$\text{Coeff. of } \vec{i}: \quad A_x + B_x + 152.4 \times 0.2588 = 0$$

$$-38.8 + B_x + 39.44 = 0,$$

$$\therefore B_x = \underline{\underline{-0.64 \text{ N}}} = 0$$

ANS.

$$\text{Coeff. of } \vec{j}: \quad A_y + B_y + 0.9659 \times 152.4 - 294.3 = 0$$

$$0 + B_y + 147.20 - 294.3 = 0$$

$$\therefore B_y = \underline{\underline{147.2 \text{ N}}}$$

ANS.

Coeff. of  $\vec{k}$ :

$$B_z = \underline{\underline{0}}$$

ANS.

4. FBD - Block A

$$20 \text{ kg} \times 9.81 = 196.2 \text{ N}$$

$$+\uparrow \Sigma F_y = 0$$

$$N_1 - T \cos 60^\circ - 196.2 = 0$$

$$N_1 - 0.5T - 196.2 = 0 \quad \text{--- (1)}$$

$$\pm \Sigma F_x = 0$$

$$\mu_A N_1 - T \sin 60^\circ = 0$$

$$0.2 N_1 - 0.866 T = 0 \quad \text{--- (2)}$$

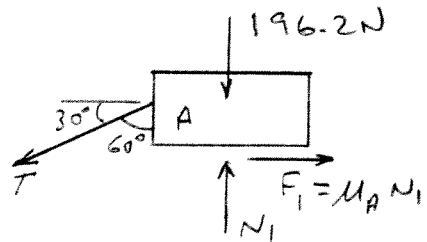
Multiply (1) by 0.866 and (2) by 0.5 and subtract:

$$0.866 N_1 - 0.433 T - 169.91 - 0.1 N_1 + 0.433 T = 0$$

$$0.866 N_1 - 0.1 N_1 = 169.91$$

$$0.766 N_1 = 169.91;$$

$$\therefore N_1 = \underline{221.81 \text{ N}}$$

FBD - Block B

$$40 \text{ kg} \times 9.81 = 392.4 \text{ N}$$

$$+\uparrow \Sigma F_y = 0$$

$$N_2 - N_1 - 392.4 = 0$$

$$N_2 - 221.81 - 392.4 = 0$$

$$\therefore N_2 = 221.81 + 392.4 = \underline{614.21 \text{ N}}$$

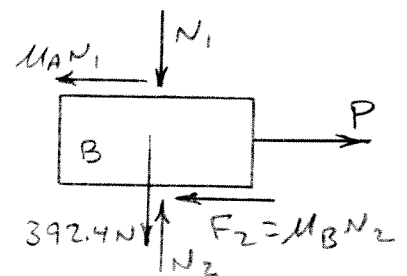
$$\pm \Sigma F_x = 0$$

$$P - \mu_A N_1 - \mu_B N_2 = 0$$

$$P - 0.2 \times 221.81 - 0.3 \times 614.21 = 0$$

$$P = 44.36 + 184.26$$

$$\therefore \underline{P = 228.62 \text{ N}}$$



ANS.

5. a)

Assume, the ball hits the slope at B whose coordinates are  $x$  &  $y$ .

$$(v_0)_x = 10 \text{ m/s} \times \frac{3}{5} = 6 \text{ m/s}$$

$$(v_0)_y = 10 \text{ m/s} \times \frac{4}{5} = 8 \text{ m/s}$$

\* - Horizontal Motion

$$x = x_0 + (v_0)_x t = 0 + 6 \text{ m/s} t$$

$$\therefore x = 6t \text{ m} \quad \text{--- (1)}$$

\* - Vertical Motion

$$y = y_0 + (v_0)_y t - \frac{1}{2} \times 9.81 t^2$$

$$y = 20 \text{ m} + 8t - \frac{1}{2} \times 9.81 t^2 \quad \text{--- (2)}$$

From the diagram:  $\frac{y}{x-5} = \frac{1}{2}$ ;  $\therefore y = \frac{x}{2} - \frac{5}{2}$  --- (3). Insert in (2):

$$\therefore \frac{x}{2} - \frac{5}{2} = 20 + 8t - \frac{1}{2} \times 9.81 t^2$$

$$\text{i.e. } x - 5 = 40 + 16t - 9.81 t^2$$

Insert  $x = 6t$  from (1):

$$\therefore 6t - 5 = 40 + 16t - 9.81 t^2$$

$$9.81 t^2 - 10t - 45 = 0$$

$$\therefore t = 2.71 \text{ sec.} \quad \left( t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$$

Another value of  $t = -1.68 \text{ s}$  is rejected since it is -ve.

$$\text{Insert in (1): } x = 6 \times 2.71 = \underline{\underline{16.26 \text{ m}}}$$

ANS.

$$\text{Insert in (3): } y = \frac{16.26}{2} - \frac{5}{2} = \underline{\underline{5.63 \text{ m}}}$$

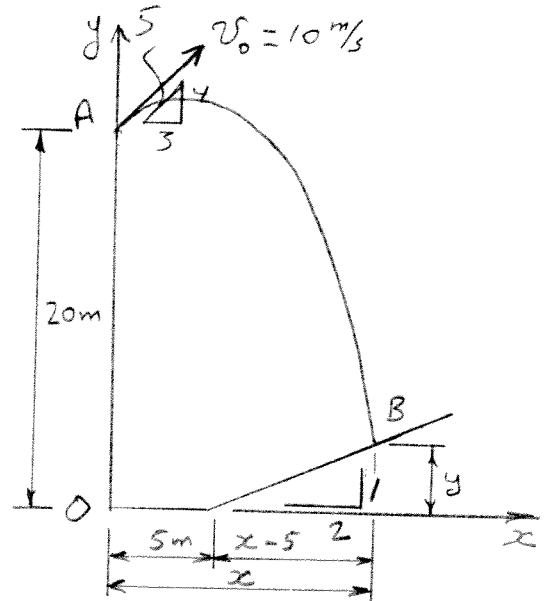
ANS.

$$\text{b) At point B: } v_x = (v_0)_x = 6 \text{ m/s}$$

$$v_y = (v_0)_y - gt = 8 \text{ m/s} - 9.81 \times 2.71 = -18.6 \text{ m/s}$$

$$\therefore v_B = \sqrt{(6)^2 + (-18.6)^2} = \underline{\underline{19.6 \text{ m/s}}}$$

ANS.



END

5. a)

$$(v_0)_x = 10 \text{ m/s} \times \frac{3}{5} = 6 \text{ m/s} ; (v_0)_y = 10 \text{ m/s} \times \frac{4}{5} = 8 \text{ m/s}$$

\* - Horizontal Direction:

$$a_x = 0, (v_0)_x = 6 \text{ m/s} ; x = (v_0)_x t = 6t \text{ m} \quad \text{--- (1)}$$

\* - Vertical Direction:

$$a_y = -9.81 \text{ m/s}^2 ; v_y = (v_0)_y + a_y t \\ = 8 - 9.81t$$

$$y = y_0 + (v_0)_y t + \frac{1}{2} a_y t^2 \\ = 20 \text{ m} + 8t - \frac{9.81}{2} t^2 \quad \text{--- (2)}$$

Equation of slope is:

$$y = -\frac{5}{2} + \frac{1}{2} x ; \therefore x = 2y + 5$$

Substitute into (1):

$$2y + 5 = 6t ; \therefore y = 3t - \frac{5}{2} \quad \text{--- (3)}$$

Equate (2) and (3):

$$20 \text{ m} + 8t - 4.905t^2 = 3t - \frac{5}{2}$$

$$4.905t^2 - 5t - 22.5 = 0$$

$$t = 2.71 \text{ sec.}$$

$$\text{Insert in (1): } x = 6t = 6 \times 2.71 = 16.26 \text{ m}$$

ANS.

$$\text{Insert in (3): } y = 3t - \frac{5}{2} = 3 \times 2.71 - 2.5 = 5.63 \text{ m.}$$

ANS.

b)  $v_x = 6 \text{ m/s} ;$

$$v_y = 8 - 9.81t = 8 - 9.81 \times 2.71 = 8 - 26.6 = -18.6 \text{ m/s}$$

$$\therefore v = \sqrt{(6)^2 + (-18.6)^2} = 19.6 \text{ m/s}$$

ANS.

END