Lecture

March 19, 2014

11.8 Turbine and Pump Efficiency

Turbine: $\left[\eta_T = \frac{\text{shaft output power}}{\text{ideal power}} = \frac{P_{OUT}}{P_i} \right]$

P_{OUT}: real power output of a turbine shaft
P_i : ideal power if there were no losses.

PUMP: $\left[\eta_P = \frac{\text{ideal power}}{\text{shaft input power}} = \frac{P_i}{P_{IN}} \right]$

P_i : ideally, only this much power would be required if there were no losses in:

- the pump
- surrounding pump system

P_{IN} : real power input to the pump shaft (must be larger than P_i b/c of losses).

Note: efficiency (η_T or η_P) is always less than 1.0 or 100%

Example: (PUMP) A pump delivers $0.3 \frac{m^3}{s}$ of water. The suction (inlet) pipe has a diameter of 0.25 m and the static pressure at the inlet is -25 KPa (g). The discharge (outlet) pipe has a diameter of 0.2 m and a pressure of 700 KPa (g). It is located 10 m above the suction pipe.

- what is the ideal pumping power, P_i?
- what is the efficiency of the pump if 310 kW of power is required to drive it?

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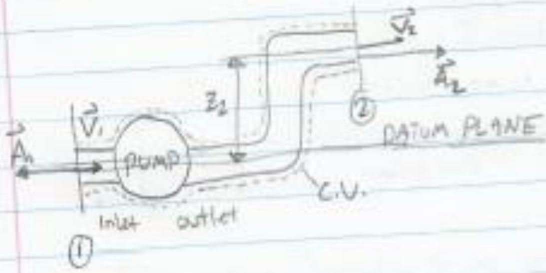


Given: $Q = 0.3\ m^3/s$
$d_1 = 0.25\ m$
$d_2 = 0.2\ m$
$P_1 = -25\ kPa(g) = -25 \cdot 10^3\ Pa(g)$
$P_2 = 700\ kPa(g) = 700 \cdot 10^3\ Pa(g)$
$Z_2 = 10m$
$Z_1 = 0m$

Find: $P_i = ?$
$\eta_p = ?$ for $P_{IN} = 310kW$

Solution:

a. $P_i = ?$

First, calculate V_1 & V_2:

$$V_1 = \frac{Q}{A_1} = \frac{0.3}{\frac{\pi d_1^2}{4}} = \frac{0.3}{\frac{(\pi \cdot 0.25^2)}{4}} = 6.11\ m/s$$

$$V_2 = \frac{Q}{A_2} = \frac{0.3}{\frac{\pi d_2^2}{4}} = \frac{0.3}{\frac{(\pi\ 0.2^2)}{4}} = 9.55\ m/s$$

Now apply SFEE btwn ① & ②

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + Z_2 + \frac{\dot{W}_s}{\dot{m} g} + H_L$$

$\rho_1 = \rho_2 = \rho$ (assume incompressible)

For P_i, w/ no losses, $H_L = \emptyset$

$$P_i = \dot{W}_s = \frac{1}{g}\left(\frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + \emptyset - Z_2 g\right) \cdot g \dot{m}$$

$$\dot{W}_s = \underbrace{\dot{m}}_{\rho \cdot Q}\left(\frac{P_1 - P_2}{\rho} - \frac{V_1^2 - V_2^2}{2} - Z_2 \cdot g\right)$$

$$\dot{W}_s = 1000 \cdot 0.3\left(\frac{-25 \cdot 10^3 - 700 \cdot 10^3}{10^3} + \frac{6.11^2 - 9.55^2}{2} - 10 \cdot 9.81\right) =$$

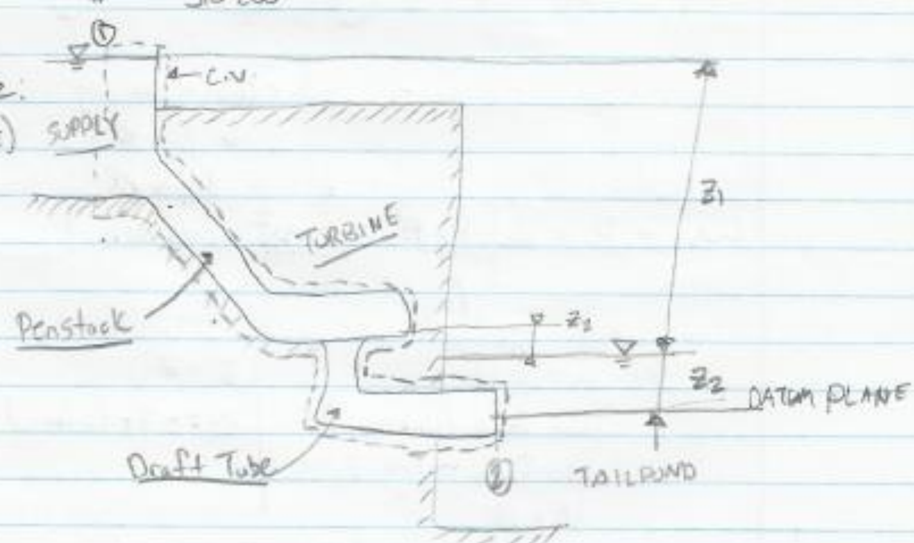
$$\boxed{P_i = \dot{W}_s = -255{,}010\,W = -255\,kW}$$

Note: ⊖ sign means that this is indeed a pump and not a turbine

b, $\eta_p = ?$ for $P_{IN} = 310\,kW$

$$\eta_P = \frac{P_i}{P_{IN}} = \frac{255\,kW}{310\,kW} = 0.823 = 82.3\%$$

Example:
(TURBINE)



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$210\ m^3/s$ of water pass through a hydroelectric turbine whose efficiency is 95%. The water level in the supply is $z_1 = 100m$ above the tailwater pond. The draft tube inlet and exit areas are $47\ m^2$ ($= A_e$) and $94\ m^2$ ($= A_2$).

a, neglect losses in the penstock, what shaft power is delivered by the turbine?

b, what shaft power is delivered by the turbine if head loss in the penstock is $H_L = 2.5m$?

c) for the conditions of a) what is the static pressure at the turbine exit/draft tube inlet plane, if this lies 2m above the tailpond level?

a, SFEE btwn ① & ②

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho_2 g} + \frac{V_1^2}{2g} + z_2 + \frac{\dot{W}_s}{\dot{m} g} + H_L \quad \Big| \cdot (g \cdot \dot{m})$$

and assuming
↳ $\rho_1 = \rho_2 = \rho$
$H_L = \emptyset$ in penstock

$$\dot{W}_s = P_i = \dot{m}\left[\frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + (z_1 - z_2)g\right]$$

to $\dot{m} = \rho \cdot Q$
$p_1 = p_{atm}$ (unknown)
$P_2 = P_{atm} - \rho \cdot g \cdot z_2$
$V_1 = \emptyset$
$V_2 = \frac{Q}{A_2} = \frac{210}{94} = 2.23\ m/s$

$z_1 = \emptyset$
$z_2 = -z_2$ (unknown)

$$\dot{W}_S = P_i = 1{,}000 \cdot 210 \left[\frac{P_{atm} - P_{atm} - \rho g \cdot z_2}{\rho} + \frac{0.25^2}{2} + g \cdot (100 - z_2) \right]$$

$$= \ldots 205.9 \cdot 10^9 = 206 \text{ MW}$$

This is the ideal power, the real power will be less by

$$\eta_T = \frac{P_{OUT}}{P_I} \Rightarrow P_{OUT} = \eta_T \cdot P_i = 0.95 \cdot 206 = 195.7 \text{ MW}$$

b) Same as a) but w/ $H_L = 2.5\text{m}$

$$P_i = P_{i,H_L = \emptyset} - H_L \dot{m} g = \quad = 200.8 \text{ MW}$$

c) Bernoulli btwn ① and ②: $P_{real} = 0.95 \cdot 200.8 = 190.8 \text{ MW}$

$$P_{1(g)} = -27.12 \text{ kPa (g)}$$

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