

This test paper has two parts. Total of 40 marks. Part I has 6 multiple choice questions. Part II has 2 long answer questions. This test paper cannot be taken from the examination room. Duration: 50 Minutes. Only nonprogrammable calculators are allowed.

NAME :

STUDENT NO :

PART I: Multiple Choice Questions. Circle the correct answer. Three marks each. No part marks.

1) Consider the following augmented matrix of a system of linear equations:

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \end{array} \right]$$

The system has

- a) infinitely many solutions with one free variable
- b) infinitely many solutions with two free variables
- c) unique solution
- d) no solutions

2) Let $A = \left[\begin{array}{ccc|c} 1 & 2 & 0 & 10 \\ 0 & 3 & -1 & -2 \\ 0 & 0 & k^2 - 16 & k + 4 \end{array} \right]$ be the augmented matrix of a given system.

For which value(s) of k will the system has infinitely many solutions?

- a) $k = 4$
- b) $k = -4$
- c) $k \neq \pm 4$
- d) $k \neq -4$

3) Let $A = \left[\begin{array}{cccc} 1 & -3 & -4 & -3 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & -3 & -4 \end{array} \right]$.

Reduced row echolon form (RREF) of A is

- a) $\left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$
- b) $\left[\begin{array}{cccc} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$,
- c) $\left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array} \right]$,
- d) $\left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right]$,

4) Which of the following statements is TRUE?

a) Row echelon form of a matrix is unique.

b) The number of vectors in $\text{Span}\{u, v\}$ is 2.

c) The matrix equation $AX = 0$ has always infinitely many solutions.

d) If the matrix equation $AX = b$ is consistent, then b is in the span of the columns of A .

5) Consider the matrix equation $AX = b$, where $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 1 \\ -1 & 1 & -1 \end{bmatrix}$. For which of the following vectors b , the above matrix equation is inconsistent?

a) $b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, b) $b = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, c) $b = \begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}$, d) Always consistent.

6) What is the general solution of the following system of linear equations?

$$x + y - 3z = 0$$

$$-x + 3y - z = 0$$

$$x - y - z = 0.$$

a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ t \\ 2t \end{bmatrix}$, b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ 3t \\ t \end{bmatrix}$, c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2t \\ t \\ t \end{bmatrix}$, d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2t \\ -t \\ t \end{bmatrix}$,

where t is a parameter

PART II: Long answer questions. Show all your work.

[10] 1) Let $A = \begin{bmatrix} 1 & -3 & 1 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 2 & 6 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 2 \\ 8 \end{bmatrix}$.

- a) Find the reduced echelon form of the augmented matrix $[A|b]$.
- b) Express b , if possible, as a linear combination of the columns of A . If not possible, give your reason.
- c) Express b , if possible, as a linear combination of the first three columns of A . If not possible, give your reason.
- d) Express b , if possible, as a linear combination of the last three columns of A . If not possible, give your reason.

[12] 2) Solve the following system of linear equations.

Write the solution in vector form.

$$x_1 - 2x_2 + x_3 - x_4 = 4$$

$$-3x_1 + 5x_2 - 3x_3 + 4x_4 = -3$$

$$-2x_1 + 3x_2 - 2x_3 + 3x_4 = 1$$

$$x_1 - x_2 + x_3 - 2x_4 = -5$$