MATH 3705B
Test 1 Solutions
January 23, 2012

[Marks]

[4] 1. $\mathcal{L}\left\{e^{2t}\cos(3t)\right\} =$

(a) $\dfrac{s-2}{(s-2)^2+9}$ (b) $\dfrac{s}{(s-2)^2+9}$ (c) $\dfrac{s+2}{(s+2)^2+9}$ (d) $\dfrac{s-2}{s^2+9}$ (e) None of these

<u>*Answer:*</u> (a)

[4] 2. $\mathcal{L}^{-1}\left\{\dfrac{3s+1}{s^2-1}\right\} =$

(a) $3\cos(t)+\sin(t)$ (b) $e^t - 2e^{-t}$ (c) $2e^{-t}+e^t$ (d) $2e^t + e^{-t}$ (e) None of these

<u>*Answer:*</u> (d)

[4] 3. $\mathcal{L}\left\{u(t-3)e^{-2t}\right\} =$

(a) $\dfrac{e^6 e^{-3s}}{s+2}$ (b) $\dfrac{e^{-3s}}{s+2}$ (c) $\dfrac{e^{3s}}{s+2}$ (d) $\dfrac{e^{-6}e^{-3s}}{s+2}$ (e) None of these

<u>*Answer:*</u> (d)

[4] 4. $\mathcal{L}^{-1}\left\{\dfrac{se^{-2s}}{s^2-2s+4}\right\} =$

(a) $e^t\cos(\sqrt{3}t) + \dfrac{1}{3}e^{3t}\sin(\sqrt{3}t)$

(b) $u(t-2)e^{t-2}\left\{\cos[\sqrt{3}(t-2)] + \sin[\sqrt{3}(t-2)]\right\}$

(c) $u(t-2)e^{t-2}\left\{\cos[\sqrt{3}(t-2)] + \dfrac{1}{3}\sin[\sqrt{3}(t-2)]\right\}$

(d) $u(t-2)e^{t-2}\left\{\cos[\sqrt{3}(t-2)] + \dfrac{1}{\sqrt{3}}\sin[\sqrt{3}(t-2)]\right\}$

(e) None of the above

<u>*Answer:*</u> (d)

[7] 5. Solve the initial-value problem $y'' + y' - 6y = 0,\ y(0) = 1,\ y'(0) = 12.$

<u>*Solution:*</u>

$[s^2Y(s) - sy(0) - y'(0)] + [sY(s) - y(0)] - 6Y(s) = 0 \ \Rightarrow$

$(s^2+s-6)Y(s) - s - 13 = 0 \ \Rightarrow\ Y(s) = \dfrac{s+13}{s^2+s-6} = \dfrac{s+13}{(s-2)(s+3)} = \left[\dfrac{3}{s-2} - \dfrac{2}{s+3}\right]$ by partial fractions. Hence, $y(t) = 3e^{2t} - 2e^{-3t}$.

[7] 6. Solve the initial-value problem $y'' + 6y' + 13y = 0,\ y(0) = 1,\ y'(0) = 0.$

<u>*Solution:*</u>

$[s^2Y(s) - sy(0) - y'(0)] + 6[sY(s) - y(0)] + 13Y(s) = 0 \ \Rightarrow$

$$(s^2+6s+13)Y(s)-s-6=0 \;\Rightarrow\; Y(s)=\frac{s+6}{s^2+6s+13}=\frac{(s+3)+3}{(s+3)^2+4}=$$
$$\frac{s+3}{(s+3)^2+4}+\frac{3}{2}\frac{2}{(s+3)^2+4} \;\Rightarrow\; y(t)=e^{-3t}\cos(2t)+\frac{3}{2}e^{-3t}\sin(2t).$$