Carleton University
COMP1805, Fall 2013
Assignment 1

Due Monday, September 23 at 10:00 am

Submit your solution in the dropbox for COMP1805 located in HP3115.

- Write down your name and student number on **every** page.
- Write the course number and tutorial section on the first page of the assignment.
- The questions should be answered in order and your assignment sheets must be stapled, if there are multiple pages, otherwise the assignment will not be marked.
- Read Section E. Student Conduct (Parts 14 & 15), of the Academic Regulations of the University. (Link to document here.)
- Every part of every question is worth 2 marks. This means question 1 is worth 8 marks, question 2 is worth 6 marks, etc.

There are 82 possible marks. The assignment is out of 80.

1. Let a be the proposition "It is Tuesday", b be the proposition "I can submit my assignment online". Translate the following expressions into English.

   (a) $a \rightarrow \neg b$

   If it is Tuesday, then I cannot submit my assignment online.

   (b) $\neg a \wedge b$

   It is not Tuesday and I can submit my assignment online.

   (c) $\neg b \rightarrow \neg a$

   If I cannot submit my assignment online, then it is not Tuesday.

   (d) $b \oplus a$

   Either I can submit my assignment online and it is not Tuesday or I cannot submit my assignment online and it is Tusday, but not both.

2. Translate the following logical propositions into English expressions. Let a be the proposition "I have a job", b be "I make money", c be "The economy is improving", and d be "I finish my degree at Carleton".

   (a) $\neg b \to \neg c$

   If I do not make money then the economy is not improving.

   (b) $(c \wedge d) \to (a \wedge b)$

   If the economy is improving and I finish my degree at Carleton, then I have a job and I make money.

   (c) $(b \vee c) \to \neg a$

   If I make money or the economy is improving, then I do not have a job.

3. Translate the following English expressions into logical statements. You must explicitly state what are the atomic propositions and then show their logical relation.

   (a) If it is sunny outside and I am running fast then I fall on the ground.

   Let S be the proposition "It is sunny outside", R be the proposition "I am running fast", and F be the proposition "I fall on the ground".
   The English expression can then be written as $(S \wedge R) \to F$

   (b) If I study hard then I have a better result in the test and if I have a better result in the test then I study hard.

   Let S be the proposition "I study hard" and B be the proposition "I have a better result in the test". Then the English expression can be written as $(S \to B) \wedge (B \to S)$

   (c) It is raining or it is snowing or I am sleeping.

   Let r be the proposition "it is raining", s be the proposition "it is snowing", and z be the proposition "I am sleeping". Then the English expression can be written as $r \vee s \vee z$

   (d) If I can skate then I can ski, and, if I can ski and I can skate, then I do not like winter.

   Let s be the proposition "I can skate", t be the proposition "I can ski", and w be the propositoin "I like winter". The given English expression can then be written as $(s \to t) \wedge \big((t \wedge s) \to \neg w\big)$

4. Consider the following proposition: "If hackers penetrated the system or client data was not secured then the company will get bad publicity and will go out of business".

   (a) Translate the following statement into logic. You must explicitly state what are the atomic propositions.

   Let h be the proposition "hackers penetrated the system", s be the proposition "the client data was secured", p be the proposition "the company will get good publicity", and b be the proposition "the company will stay in business". The English expression then becomes $(h \vee \neg s) \rightarrow (\neg p \wedge \neg b)$

   (b) **Negate** the logical statement from part (a). Translate the negated logical statement back into English.

   First, we have
   $\neg\big((h \vee \neg s) \rightarrow (\neg p \wedge \neg b)\big)$
   $\equiv \neg\big(\neg(h \vee \neg s) \vee (\neg p \wedge \neg b)\big)$ (Relation of Implication)
   $\equiv \neg\neg(h \vee \neg s) \wedge \neg(\neg p \wedge \neg b)$ (DeMorgan's Law)
   $\equiv (h \vee \neg s) \wedge \neg(\neg p \wedge \neg b)$ (Double Negation)
   $\equiv (h \vee \neg s) \wedge (\neg\neg p \vee \neg\neg b)$ (DeMorgan's Law)
   $\equiv (h \vee \neg s) \wedge (p \vee b)$ (Double Negation)

   The negated logical statement then becomes: Hackers penetrated the system or the data was not secured, and the company will get good publicity or will stay in business.

   Alternatively,

   First, we have
   $\neg\big((h \vee \neg s) \rightarrow (\neg p \wedge \neg b)\big)$
   $\equiv \neg\big(\neg(h \vee \neg s) \vee (\neg p \wedge \neg b)\big)$ (Relation of Implication)
   $\equiv \neg\neg(h \vee \neg s) \wedge \neg(\neg p \wedge \neg b)$ (DeMorgan's Law)
   $\equiv \neg(\neg h \wedge \neg\neg s) \wedge \neg(\neg p \wedge \neg b)$ (DeMorgan's Law)
   $\equiv \neg(\neg h \wedge s) \wedge \neg(\neg p \wedge \neg b)$ (Double Negatation)
   $\equiv (\neg h \wedge s) \rightarrow \neg(\neg p \wedge \neg b)$ (Relation of Implication)
   $\equiv (\neg h \wedge s) \rightarrow (\neg\neg p \vee \neg\neg b)$ (DeMorgan's Law)
   $\equiv (\neg h \wedge s) \rightarrow (p \vee b)$ (Double Negtation, twice)

   The negated logical statement can also be read as: If hackers did not penetrate the system and the data is secured, then the company will get good publicity or will stay in business.

5. Determine which of the following statements are True and explain why.

   (a) If $3 + 5 + 3 < 8$ then the moon will explode on Tuesday.

   This statement is True. The antecedent in the implication is False (it is not the case that $3+5+3 < 8$) and so regardless of the truth value of the consequent, the statement is True. (See the footnote at the bottom of the page if you are unfamiliar with these words.)

   If you draw the truth table for this problem, we know that $3 + 5 + 3 < 8$ is False and but we do not know the truth value of the proposition "the moon will explode Tuesday", you will see that the implication is True for all valid cases.

   (b) If the earth has one moon then the earth is flat.

   This is False. Let X be the proposition "the earth has one moon", and let Y be the proposition "the earth is flat". The English statement corresponds to $X \to Y$ which is equivalent to $\neg X \vee Y$. But, X is True and Y is False, and this is the only situation in which an implication is False.

   (c) $-2 > 0$ or $9 \times 3 > 27$ or $\sqrt{2}$ is irrational.

   This is True. The proposition "$\sqrt{2}$ is irrational" is a True, and so the conjunction is also True (by the Domination Rule).

6. Find a logical proposition that is equivalent to $(\neg p \leftrightarrow q)$ but that only uses the connectives $\vee, \wedge, \neg$. Prove your answer is correct.

   We know from class that $(\neg p \leftrightarrow q) \equiv (\neg p \to q) \wedge (q \to \neg p)$, and applying the Relation of Implication twice we will have $(p \vee q) \wedge (\neg q \vee \neg p)$. Unfortunately, we don't have a rule on the logic sheet to remove the $\leftrightarrow$ for us and we need to prove that this is correct. Using truth tables, we see

p	q	$\neg p$	$\neg p \leftrightarrow q$	$(p \vee q) \wedge (\neg q \vee \neg p)$
T	T	F	F	F
T	F	F	T	T
F	T	T	T	T
F	F	T	F	F

In an implication $A \to B$, A is called the antecedent and B is called the consequent.

7. Evaluate the following bitwise logical expressions:

   (a) $(10110101 \wedge (01010101 \vee 10100101))$

   First, compute the disjunction.

   $$\begin{array}{r} 01010101 \\ \underline{\vee 10100101} \\ 11110101 \end{array}$$

   Now, compute the conjunction (and final answer)

   $$\begin{array}{r} 10110101 \\ \underline{\wedge 11110101} \\ 10110101 \end{array}$$

   Therefore, $(10110101 \wedge (01010101 \vee 10100101)) = 10110101$

   (b) $11001011 \oplus 10111001 \oplus 10010010$

   $= 1110000$

   (c) $(10111101 \oplus 11001101) \wedge (10110001 \vee 11010011)$

   $=01110000$

8. Determine, for each statement, whether it is a proposition or not. Justify your answer in either case.

   (a) "There are signs that the global financial crisis is over."

   This statement **is** a proposition since it is a declarative sentence that is either true or false (and not both).

   (b) "Get us out of the global financial crisis!"

   This statement is **not** a proposition since it is a command (imperative sentence) that does not have a truth value.

9. Determine if the following are tautologies, contradictions or contingencies and justify your answers. You may use truth tables.

   (a) $\neg(\neg x \wedge \neg y) \leftrightarrow (\neg(y \wedge x) \wedge \neg(\neg x \vee y))$

   This compound proposition is a contingency. The truth table is given by

p	q	$\neg(\neg x \wedge \neg y) \leftrightarrow (\neg(y \wedge x) \wedge \neg(\neg x \vee y))$
T	T	F
T	F	F
F	T	T
F	F	T

   Since the last column (corresponding to the compound proposition) is True for some truth values of the p and q and False for others, we know that this is a contingency.

(b) $(a \to b) \to (\neg b \to \neg a)$

This proposition is a tautology. We see this from the truth table

a	b	$\overbrace{a \to b}^{A}$	$\overbrace{\neg b \to \neg a}^{B}$	$\overbrace{a \to b}^{A} \to \overbrace{\neg b \to \neg a}^{B}$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

Since the last column in the truth table (corresponding to the compound proposition) is True for all truth values of a and b, we know this is a tautology.

(c) $((q \vee \neg p) \wedge (\neg q \vee p)) \wedge ((\neg q \wedge p) \vee (\neg p \wedge q))$

This is a contingency. When the truth values of p and q are the same the proposition is True and when the truth values of p and q are different, the proposition is False. That is, the proposition is equivalent to $\neg(p \oplus q)$.

10. Determine if the following are tautologies, contradictions or contingencies and justify your answers. You cannot use truth tables to justify your answers. Use either logical equivalences or some other means that does not use truth tables.

(a) $((a \vee c) \wedge \neg(\neg c \wedge \neg b)) \vee ((c \to a) \wedge (c \to \neg b))$

$\big((a \vee c) \wedge \neg(\neg c \wedge \neg b)\big) \vee \big((c \to a) \wedge (c \to \neg b)\big)$
$\equiv \big((a \vee c) \wedge (c \vee b)\big) \vee \big((c \to a) \wedge (c \to \neg b)\big)$ (DeMorgan, Double Negation $\times 2$)
$\equiv \big((a \vee c) \wedge (c \vee b)\big) \vee \big((\neg c \vee a) \wedge (\neg c \vee \neg b)\big)$ (Relation of Implication $\times 2$)
$\equiv \big((c \vee a) \wedge (c \vee b)\big) \vee \big((\neg c \vee a) \wedge (\neg c \vee \neg b)\big)$ (Communtivity)
$\equiv \big(c \vee (a \wedge b)\big) \vee \big(\neg c \vee (a \wedge \neg b)\big)$ (Distributivity $\times 2$
$\equiv c \vee \neg c \vee (a \wedge b) \vee (a \wedge \neg b)$ (Associativity, Commutivity (multiple times))
$\equiv T \vee (a \wedge b) \vee (a \wedge \neg b)$ (Excluded Middle)
$\equiv T$ (Domination)

The proposition is logically equivalent to True. Therefore, the proposition is a tautology.

(b) $\neg(\neg(p \to \neg(\neg x \vee \neg y)) \to \neg(x \wedge y))$

$$\neg(\neg(p \to \neg(\neg x \vee \neg y)) \to \neg(x \wedge y))$$

$\equiv \neg((p \to \neg(\neg x \vee \neg y)) \vee \neg(x \wedge y))$ } Relation of Implication, Double Negation

$\equiv \neg((p \to (x \wedge y)) \vee \neg(x \wedge y))$ } DeMorgan, Double Negation

let $A = x \wedge y$

$\equiv \neg((p \to A) \vee \neg A)$

$\equiv \neg(p \to A) \wedge A$ } DeMorgan, Double Negation

$\equiv \neg(\neg p \vee A) \wedge A$ } Relation of Implication

$\equiv (p \wedge \neg A) \wedge A$ } DeMorgan, Double Negation

$\equiv p \wedge (\neg A \wedge A)$ } Associativity

$\equiv p \wedge F$ (contradiction) $\equiv F$ (Domination)

$\therefore$ It is a contradiction.

(c) $((\neg p \vee z) \wedge (p \vee q)) \to (z \vee q)$

$$((\neg p \vee z) \wedge (p \vee q)) \to (z \vee q)$$

(Relation of Implication)

$\equiv \neg((\neg p \vee z) \wedge (p \vee q)) \vee (z \vee q)$

$\equiv \neg(\neg p \vee z) \vee \neg(p \vee q) \vee (z \vee q)$ (DeMorgan)

$\equiv (p \wedge \neg z) \vee (\neg p \wedge \neg q) \vee (z \vee q)$ (DeMorgan ×2, Double Negation)

$\equiv (p \wedge \neg z) \vee z \vee (\neg p \wedge \neg q) \vee q$ (Associativity, Commutivity)

$\equiv ((p \vee z) \wedge \underbrace{(\neg z \vee z)}_{T}) \vee ((\neg p \vee q) \wedge \underbrace{(\neg q \vee q)}_{T})$ (Distributivity)

$\equiv (p \vee z) \vee (\neg p \vee q)$ (Excluded middle ×2, Identity ×2)

$\equiv (\underbrace{p \vee \neg p}) \vee (z \vee q)$ (associativity, commutivity)

$\equiv T \vee (z \vee q)$ (Excluded middle)

$\equiv T$ (Domination)

$\therefore$ the proposition is a tautology.

11. Translate the following into English, where $D(x)$ is "x knows how to dance", $M(x)$ is "x likes to listen to music" and $C(x)$ is "x has a ticket to the Melanie C concert". The universe of discourse is all humans.

   (a) $\exists x(C(x) \wedge \neg(D(x) \vee \neg M(x))$

   There is someone who has a ticket to the Melanie C concert, does not know how to dance and likes to listen to music.

   (b) $\forall x((D(x) \wedge M(x)) \rightarrow C(x))$

   Every person that knows how to dance and likes to listen to music has a ticket to the Melanie C concert.

   (c) $\forall x(\neg M(x) \rightarrow \neg D(x))$

   Everyone that does not like to listen to music does not know how to dance.

   Or,

   All people like to listen to good music or do not know how to dance.

   Or,

   Everyone that knows how to dance likes to listen to good music.

   (d) $\neg\forall x(\neg M(x) \rightarrow \neg(D(x) \wedge C(x)))$

   First, we can simplify this logical expression as follows:
   $\neg\forall x(\neg M(x) \rightarrow \neg(D(x) \wedge C(x)))$
   $\equiv \neg\forall x(M(x) \vee \neg(D(x) \wedge C(x)))$ (Relation of Implication, Double Negation)
   $\equiv \neg\forall x(M(x) \vee \neg D(x) \vee \neg C(x))$ (DeMorgan)
   $\equiv \exists x\neg(M(x) \vee \neg D(x) \vee \neg C(x))$ (DeMorgan)
   $\equiv \exists x\neg M(x) \wedge D(x) \wedge C(x))$ (DeMorgan, Double Negation $\times$ 2)

   From this, we get the expression: There exists a person that knows how to dance, has a ticket to the Melanie C concert and does not like to listen to music.

12. Let $D(x)$ be "x can dance", $S(x)$ be "x can sing", $R(x)$ be "x loves listening to the radio" and $P(x)$ be "x plays soccer". The universe of discourse is all humans. State the following logically.

   (a) No one can dance and sing.

   $\neg\exists x\ (D(x) \wedge S(x))$

   $\forall x\ \neg(D(x) \wedge S(x))$

   $\forall x\ (\neg D(x) \vee \neg S(x))$

   (b) At least one person loves listening to the radio and plays soccer.

   $\exists x\ \big(R(x) \wedge P(x)\big)$

(c) Not everyone who can sing can also dance.

$\neg\forall x\ \big(S(x) \to D(x)\big)$

$\exists x\ \neg\big(S(x) \to D(x)\big)$

$\exists x\ \neg\big(\neg S(x) \lor D(x)\big)$

$\exists x\ \big(S(x) \land \neg D(x)\big)$

(d) Everyone loves listening to the radio and can sing, and no one loves listening to the radio and plays soccer.

$\big(\ \forall x\ (R(x) \land S(x))\big) \land \big(\neg\exists x\ (R(x) \land P(x))\big)$

(e) There is exactly one person who can sing but cannot dance.

$\big(\exists x\ (S(x) \land \neg D(x))\big) \land \forall x\ \neg\exists y\ (S(x) \land \neg D(x)) \land \big((S(y) \land \neg D(y)) \land (x \neq y)\big)$

Which is easier to see as

$\big(\exists x\ (S(x) \land \neg D(x))\big) \land \forall x \forall y\ \neg(S(x) \land \neg D(x)) \lor \neg(S(y) \land \neg D(y)) \lor (x = y)$

13. Express the negation of the following statements:

(a) All bunnies are not vampires.

There is a vampire bunny.

(b) There is one bunny that does not want to be a zombie.

All bunnies want to be a zombie.

14. Let the universe of discourse for x and y be $\{a, b, c, d\}$.

(a) Write out the statement $\exists x.P(x) \land \neg\exists y.\neg Q(y)$.

$\exists x\ P(x) \equiv P(a) \lor P(b) \lor P(c) \lor P(d)$
$\neg\exists y\ \neg Q(y) \equiv \forall y\ \neg\neg Q(y) \equiv \forall y\ Q(y) \equiv Q(a) \land Q(b) \land Q(c) \land Q(d)$
Conjoin these two together.

(b) Is $\forall x.A(x) \to \exists y.B(y)$ equivalent to $\forall x.(A(x) \to B(x))$? Prove your answer.

No. These two logical statements are not equivalent.

The first statement is
$\big(A(a) \land A(b) \land A(c) \land A(c)\big) \to B(a) \lor B(b) \lor B(c) \lor B(c)$
The second is
$(A(a) \to B(a)) \land (A(b) \to B(b)) \land (A(c) \to B(c)) \land (A(d) \to B(d))$

Notice that if $A(a) = A(b) = A(c) = A(d) = True$, and $B(a) = True, B(b) = B(c) = B(d) = False$, the first statement is True and the second is False.

Therefore, we have a **counter example**, which proves that they are not equivalent.