MATH 3705* A Test 1 January 2012

LAST NAME: ______________________________ **ID#:**______________

Questions 1-6 are multiple choice. Circle the correct answer. Only the answer will be marked.

1. **[2]** $\mathcal{L}\{e^{-2t}\cos(3t)\} =$

(a) $\dfrac{s+2}{(s+2)^2+9}$ (b) $\dfrac{s}{s^2+9}$ (c) $\dfrac{s-2}{(s-2)^2+9}$ (d) None of the above

2. **[2]** $\mathcal{L}\{t\sin(3t)\} =$

(a) $\dfrac{6s}{(s^2+9)^2}$ (b) $\dfrac{s^2-9}{(s^2+9)^2}$ (c) $\dfrac{9-s^2}{(s^2+9)^2}$ (d) None of the above

3. **[2]** $\mathcal{L}\{u(t-1)\,e^{2t}\} =$

(a) $\dfrac{e^{-s}}{s-2}$ (b) $\dfrac{e^{1-s}}{s+2}$ (c) $\dfrac{e^{2-s}}{s-2}$ (d) None of the above

4. **[3]** $\mathcal{L}^{-1}\left\{\dfrac{2s}{s^2-25}\right\} =$

(a) $e^{5t}-e^{-5t}$ (b) $e^{5t}+e^{-5t}$ (c) $-\cos(5t)$ (d) None of the above

5. **[3]** $\mathcal{L}^{-1}\left\{\dfrac{se^{-3s}}{s^2+4}\right\} =$

(a) $u(t-3)\,\cos(2t)$ (b) $u(t+3)\,\cos[2(t+3)]$

(c) $u(t-3)\,\cos[2(t-3)]$ (d) None of the above

6. **[3]** $\mathcal{L}^{-1}\left\{\dfrac{3s+6}{s^2-4s+13}\right\} =$

(a) $3e^{2t}\cos(3t)+4e^{2t}\sin(3t)$ (b) $3e^{-2t}\cos(3t)+\dfrac{9}{4}e^{-2t}\sin(3t)$

(c) $3e^{2t}\cos(3t)+\dfrac{9}{4}e^{2t}\sin(3t)$ (d) None of the above

Answers: 1.(a), 2.(a), 3.(c), 4.(b), 5.(c), 6.(a).

7. [**5 marks**] Let $f(t) = e^t$ for $0 < t < 1$ and $f(t+1) = f(t)$ for all $t \geq 0$. Find $\mathcal{L}\{f(t)\}$.

Solution:

Since f is periodic with the period $\omega = 1$, then

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-s}} \int_0^1 e^t\, e^{-st}\, dt = \frac{1}{1-e^{-s}} \int_0^1 e^{t(1-s)}\, dt = \frac{1}{1-e^{-s}} \left\{ \frac{1}{1-s}\, e^t \right\}_0^1 =$$

$$= \frac{1}{1-e^{-s}} \left(\frac{1}{1-s} \right) (e-1) = \frac{e-1}{(1-e^{-s})(1-s)}.$$

8. [**10 marks**] Employ the Laplace transform to solve the initial-value problem

$$y'' + 4y' + 8y = 0, \quad y(0) = 2, \; y'(0) = 4.$$

Solution:

$$[s^2 Y(s) - sy(0) - y'(0)] + 4[sY(s) - y(0)] + 8Y(s) = 0$$

$$\Rightarrow (s^2 + 4s + 8)Y(s) - 2s - 12 = 0 \Rightarrow Y(s) = 2\frac{s+6}{s^2+4s+8} = 2\frac{(s+2)+4}{(s+2)^2+4} =$$

$$2\frac{s+2}{(s+2)^2+4} + 4\frac{2}{(s+2)^2+4} \quad \Rightarrow \quad y(t) = 2e^{-2t}\cos(2t) + 4e^{-2t}\sin(2t).$$

Marking Guidelines: Give 5 marks for the correct direct transformation and 5 marks for the correct inverse. Take 1 mark off for a minor arithmetic mistake.