ECON 345

Notes

Present Value and Bonds.

345 NOTES: PRESENT VALUE / BONDS

Reference: [Axel Leijonhufvud: 'Corridor']

Asset Valuation

	t	t+1	t+2	t+3
Compounding				
	X	$X(1+r)$	$X(1+r)^2$	
	100	110	121	
Discounting				
	91	100		
	$\frac{100}{1+r}$			
	$\frac{X}{1+r}$			
	$\frac{X}{(1+r)^2}$	91	100	
	83			100
	$\frac{X}{(1+r)^3}$			
	75			

$$P_A = PDV = \frac{X_1}{(1+r)} + \frac{X_2}{(1+r)^2} \cdots\cdots \frac{X_n}{(1+r)^n}$$

$$P_A = PDV = \frac{\overset{200}{\overset{100}{X_1}}}{(1+i)} + \frac{\overset{200}{\overset{100}{X_2}}}{(1+i)^2} + \dots + \frac{\overset{200}{\overset{100}{X_n}}}{(1+i)^n}$$

① P_A is positive in X's

(a) permanent (biggest effect on P_A)

(b) temporary

② P_A is negative in i

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x: $P_x > PDV_x$ ES_x $P_x \downarrow$

y: $P_y < PDV_y$ ED_y $P_y \uparrow$

$$P_A = PDV = \frac{X_1}{1+r} + \frac{X_2}{(1+r)^2} \cdots \cdots \frac{X_n}{(1+r)^n}$$

x) $P_x > PDV$ sell $P_x \downarrow$

y) $P_y < PDV$ buy $P_y \uparrow$

arbitrage process

Way 1) P_A is given $r \Rightarrow$ yield (unique)

" 2) explain P_A $r \Rightarrow$ interest rates (different due time)

Basic Asset Valuation Eqn

$$P_A = \frac{X_1}{(1+i)} + \frac{X_2}{(1+i)^2} + \frac{X_3}{(1+i)^3} + \cdots + \frac{X_n}{(1+i)^n}$$

Application 1 Bonds

Coupon Bonds

Excel IRR

face value : $1000 (FP)

coupon interest rate $= \frac{C}{FP} = 0.1$

$$P_B = PDV = \frac{\overset{100}{C_1}}{(1+i)} + \frac{\overset{100}{C_2}}{(1+i)^2} + \cdots + \frac{\overset{100}{C_{10}}}{(1+i)^{10}} + \frac{\overset{1000}{FP_{10}}}{(1+i)^{10}}$$

1000 — $i = 10\%$

900 — $i > 10\%$

1100 — $i < 10\%$

$$\overset{900}{P_B} = (PDV) = \frac{\overset{100}{C_1}}{(1+i)} + \frac{\overset{100}{C_2}}{(1+i)^2} + \cdots \frac{\overset{100}{C_{10}}}{(1+i)^{10}} + \frac{\overset{1000}{FP_{10}}}{(1+i)^{10}}$$

Discount Bond: no coupon $\Rightarrow P_B < FP$

$$P_B = \frac{FP_{10}}{(1+i)^{10}}$$

Perpetuity:

$$P_B = \frac{C}{1+i} + \frac{C}{(1+i)^2} - - - + \frac{C}{(1+i)^n}$$

$$P_B = \frac{C}{i}$$

$\Rightarrow i = \frac{C}{P_B}$ current yield i_c

$\Rightarrow \uparrow i \cdot P_B \downarrow = \bar{C}$

Tradeoff $P_B = \frac{C \uparrow 10\%}{i \uparrow 10\%}$

$$P_B = \frac{C}{i}$$

i_c = current yield

$$i_c = \frac{C}{P_B}$$

Selling Before Maturity Problems

$$RET(1): \frac{\overset{100}{C_1} + (\overset{1100}{P_{t+1}} - \overset{1000}{P_t})}{\underset{1000}{P_t}}$$

20%

$$\overset{i_c \; + \; g}{}$$

$$RET(2): \overset{100}{C_1} + \overset{100}{C_2} + \frac{(\overset{1100}{P_{t+2}} - \overset{1000}{P_t})}{\underset{1000}{P_t}}$$

30%/2 = 15%

RET(3): 400/1000 = 13.3%

RET(4): = 12.5%

(9): = 11.1%

(10): = 10%

$RET_0 = i_c + g$

$$\frac{C + [P_{t+1} - P_t]}{P_t}$$

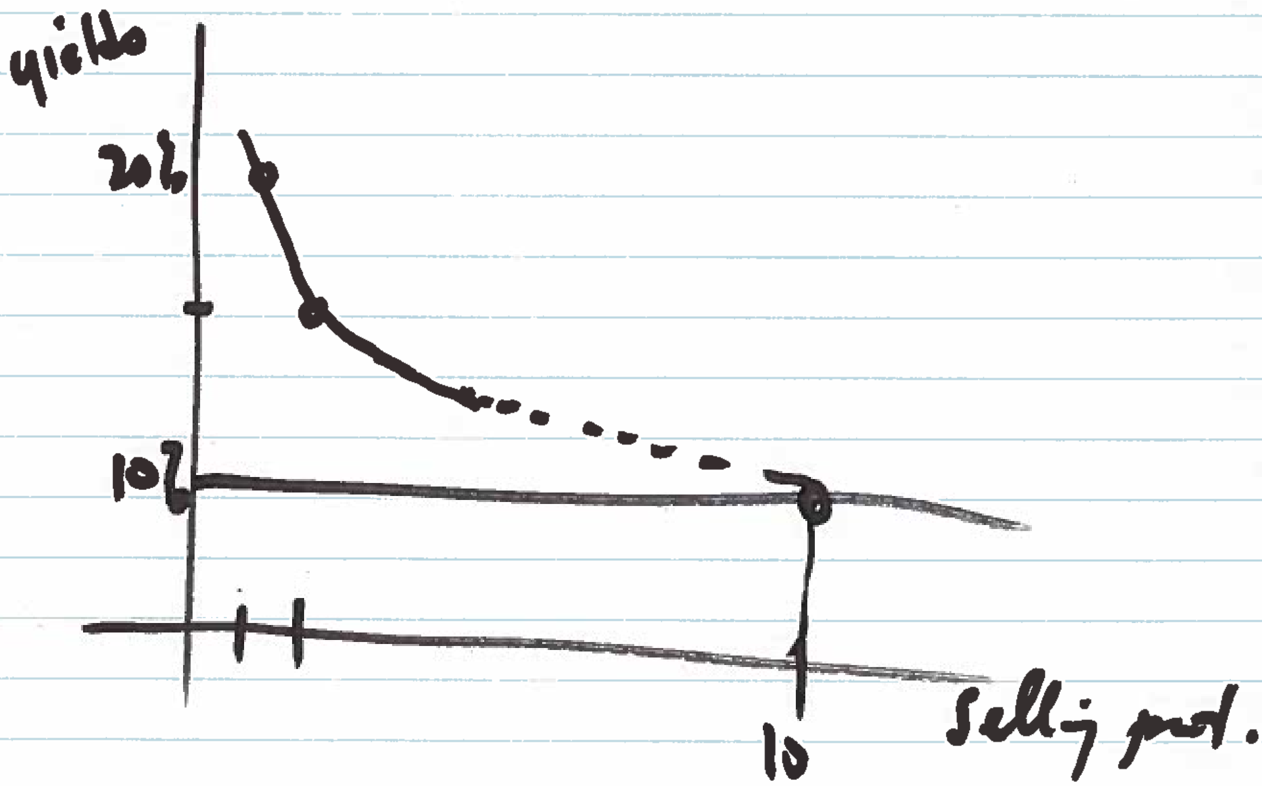
20% $RET_1 = \dfrac{\overset{C}{100} + (\overset{1100}{P_{t+1}} - \overset{1000}{P_t})}{1000}$ $\quad \dfrac{200}{1000}$

15% $RET_2 = \dfrac{100 + 100 + (P_{t+2} - \overset{100}{P_t})}{1000}$ $\quad \dfrac{300}{1000}$

13.3% $RET_3 = \dfrac{100 + 100 + 100 + 100}{1000}$ $\quad \dfrac{400}{1000}$

12.5% RET_4

12% RET_5

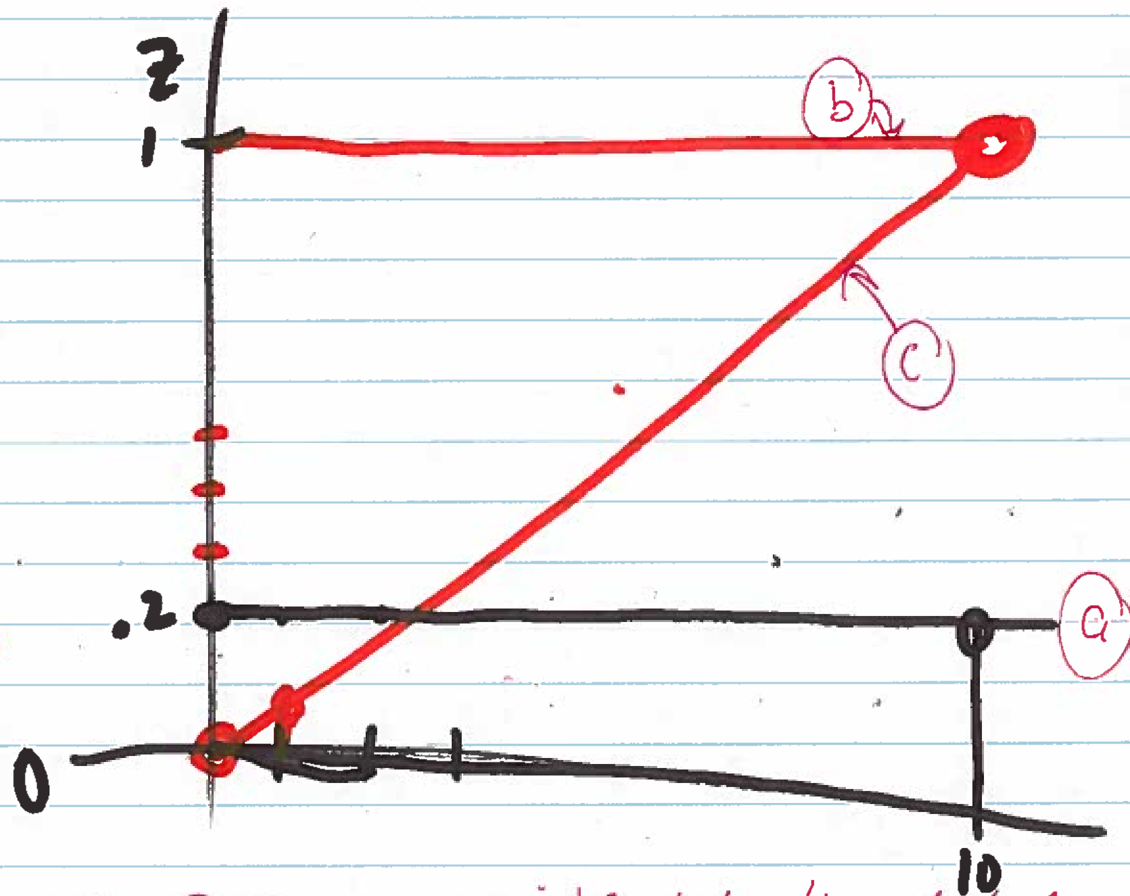


Default Risk:

$$\frac{C_i(1-z_i)}{(1+i)^i} \qquad z: \text{prob. of default}$$

$$\overset{1000}{P_B} = \frac{\overset{100(1-.2)}{C_1(1-z_1)}}{(1+i)} + \frac{\overset{80}{C_2(1-z_2)}}{(1+i)^2} + \cdots \frac{\overset{80}{C_{10}(1-z_{10})}}{(1+i)^{10}} + \frac{\overset{800}{FP_{10}(1-z_{10})}}{(1+i)^{10}}$$

80



ⓐ ⓑ ⓒ are possible default schedules

RISK: Two Effects:

1. $Z\uparrow$ for some terms
   for given P_B $\Rightarrow$ lower i

2. Market: lower i $\Rightarrow$ selling of bond
   Excess Supply $\Rightarrow$ $P_B\downarrow$, $i\uparrow$

   RISK PREMIUM

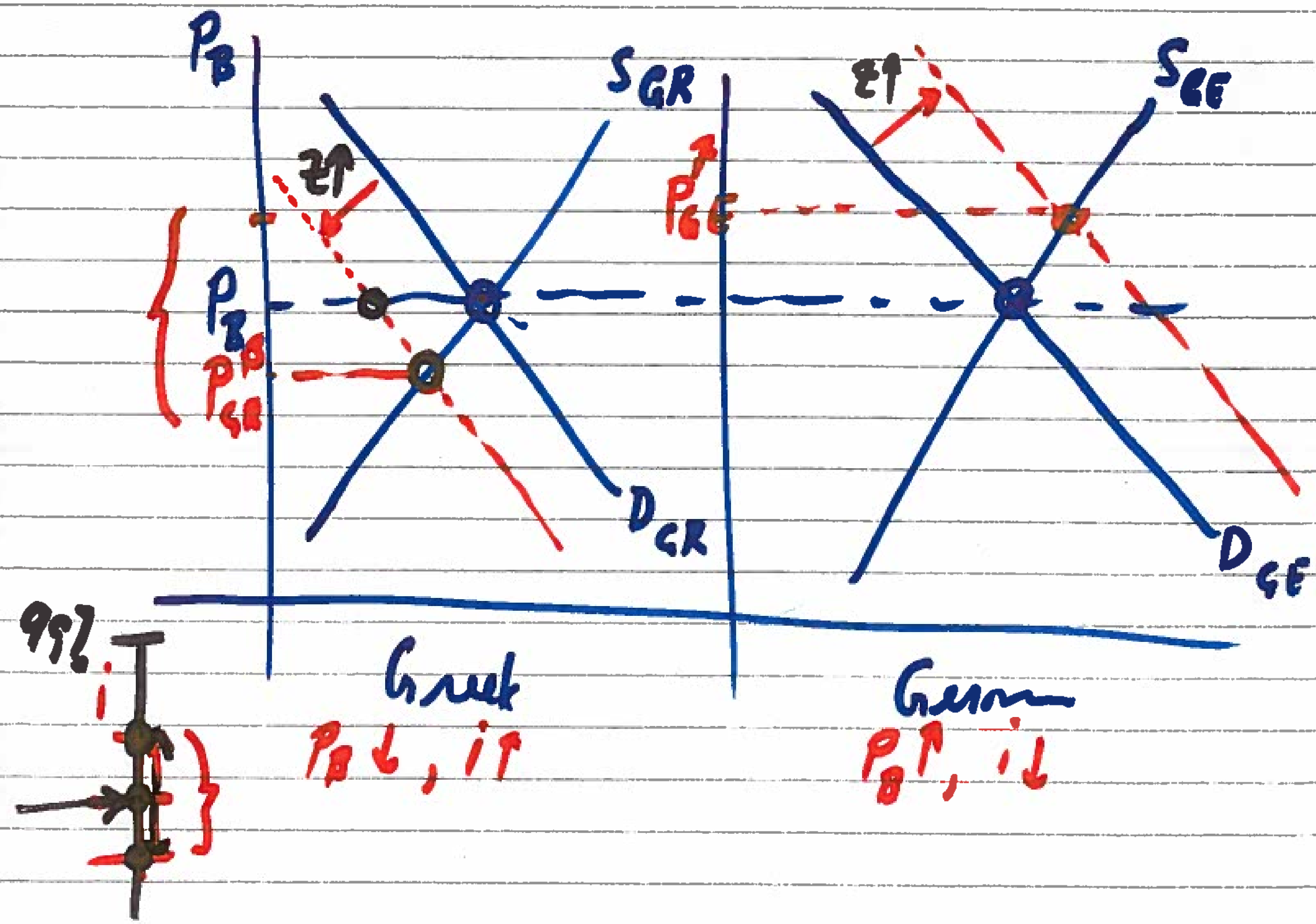
Risk Premium (first part of ch. 6, text)

① $z > 0$, $(\bar{P}_B) i \downarrow$, sell the bond

② Market Effect, $ES_{bond} \longrightarrow P_B \downarrow$

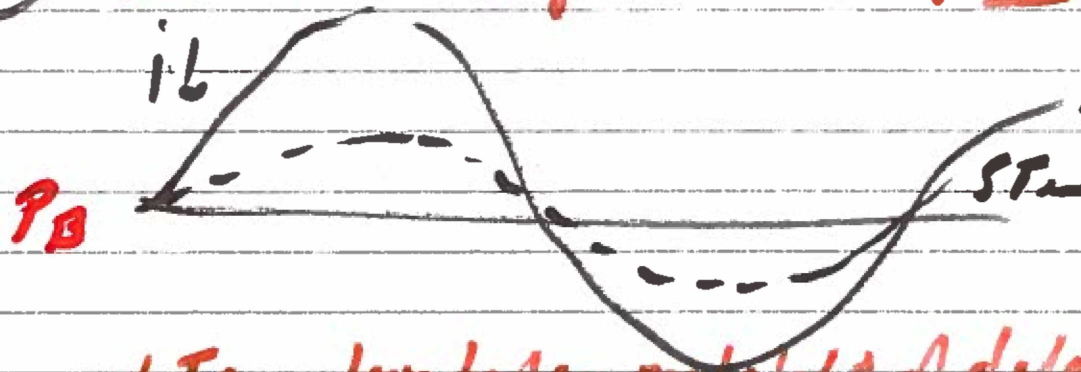
③ New holder, $P_B \downarrow \Rightarrow i \uparrow$ (gets risk premium)

$z \uparrow$ Greek bond risk



Summary: LT Bonds Riskier than Short-Term

① L Term have higher 'variance'; more PDV terms to be ↑ when i change



L Term have higher probability of default

② 5 yr. NO RISK

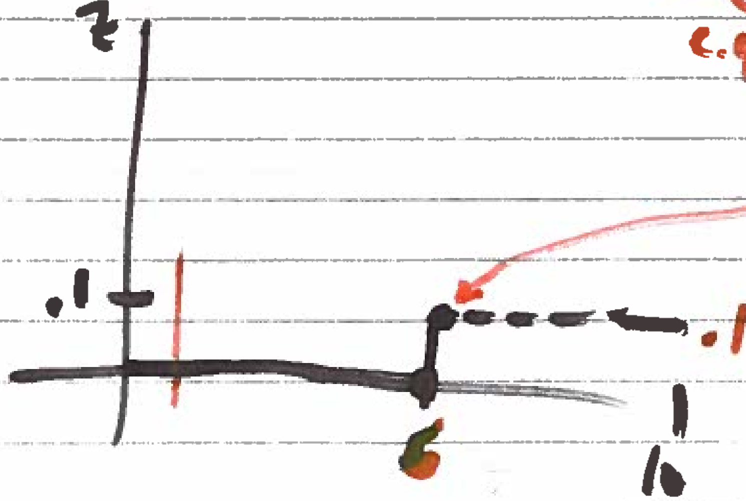
$$P_B = \frac{\overset{100}{C_1}}{(1+i)} + \dots + \frac{C_5}{(1+i)^5} + \frac{\overset{1000}{FP_5}}{(1+i)^5} \qquad (P_B = 1000)$$

$z = 0;\ i = 10\%$

Default for later periods

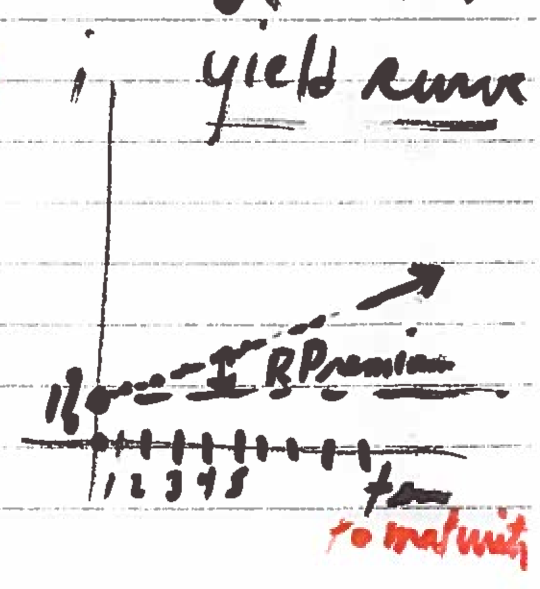
10 yr.

$$\overset{1000}{P_B} = \overset{100}{C_1} + \dots + \overset{100}{C_5} + \overbrace{\underbrace{C_6 + \dots + \frac{C_{10}}{(1+i)^{10}}}_{\text{e.g. } C_6(1-z) = 90}}^{z = .1} + \frac{\overset{1000}{FP_{10}}}{(1+i)^{10}}$$



$C_6(1-z) = 90$

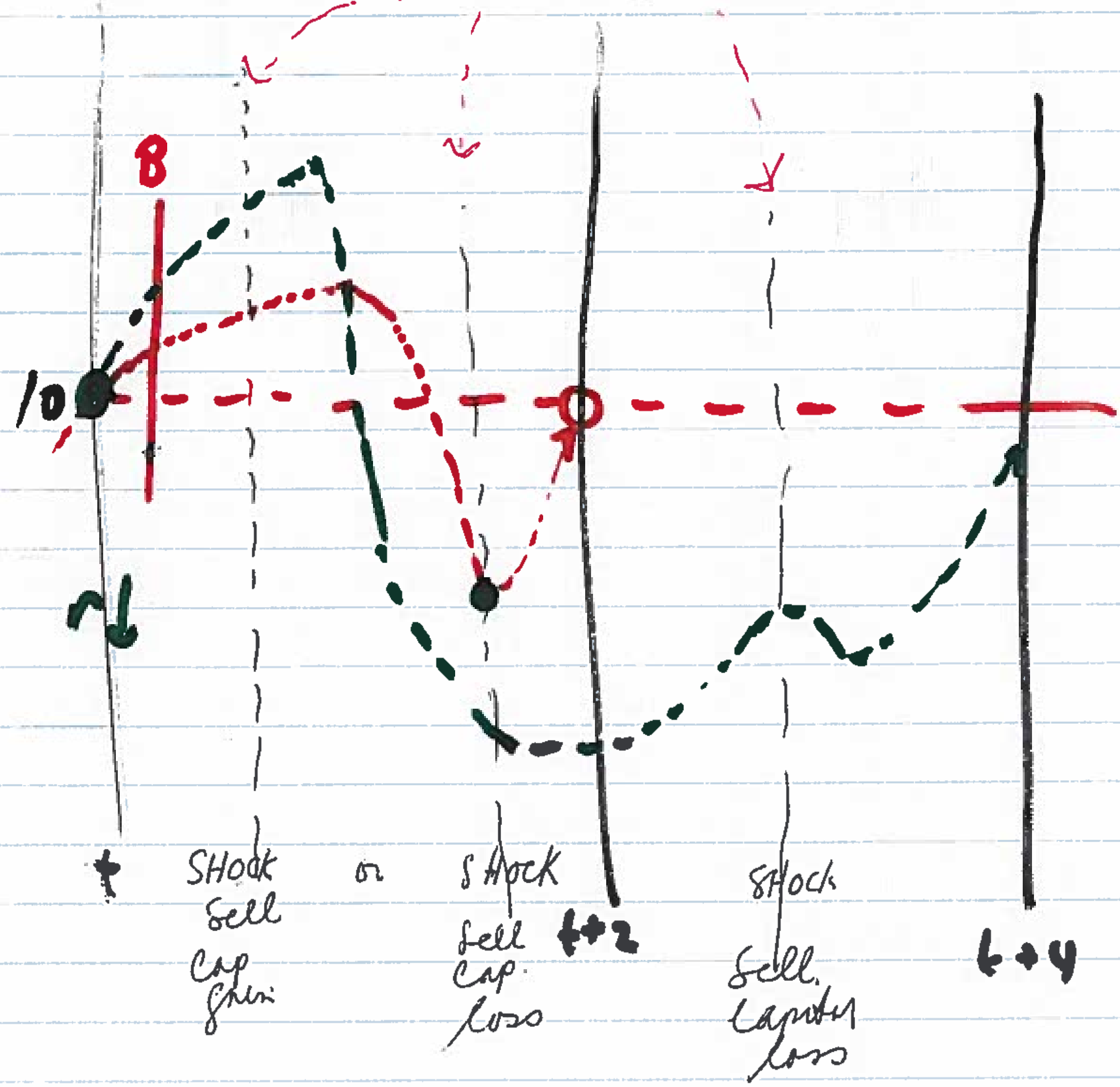
yield curve



RESULT: Solve for $i < 10\%$; ∴ $ES_b \Rightarrow$ risk premium on 10 yr. bond.

Long Term Bonds Riskier than Short Term
2 yr bond / 4 year bond.

Possible timings of liquidity shocks



Possible Liquidity Shocks (forced sale)

liquidity shocks are ⊕ function of time

① L Term have greater price volatility
② L Term " " time to maturity
③ L Term " " probability of liquidity shock (which forces sale)