Questions 1–4 are multiple choice. Enter the **letter** corresponding to each correct answer in the appropriate box below.

Question	1	2	3	4
Answer	E	D	A	A

[2pts] 1. Which of the following statements are **false**?

(i) The compound proposition $(a \to b) \vee a$ is a tautology. ✓
(ii) The compound propositions $a \to \neg b$ and $b \to \neg a$ are logically equivalent. ✓
(iii) If a is false, b is false, and c is true, then $(a \wedge b) \vee c$ is true. ✓
(iv) The compound propositions $\neg(a \to \neg b)$ and $a \wedge b$ are logically equivalent. ✓
(v) If the set of premises of an argument is inconsistent, then the argument is valid. ✓

A. only (iii) **B.** only (iv) **C.** only (i) **D.** (ii) and (v)

E. none **F.** only (ii)

(iv) $\neg(a \to \neg b) \equiv \neg(\neg a \vee \neg b) \equiv a \wedge b$

(v) $P_1 \wedge P_2 \wedge \dots \wedge P_k \to C$

$\{P_1, \dots, P_k\}$ inconsistent means $P_1 \wedge \dots \wedge P_k$ is a contradiction. Then $P_1 \wedge \dots \wedge P_k \to C$ is a tautology

[2pts] 4. Below, let A and B be finite sets, and $f : A \to B$ a function. Furthermore, let $g : \mathbb{R}^+ \to \mathbb{R}^+$ be a function defined by $g(x) = x^2 + 3$.

Which of the following statements are **true**?

(i) If $|A| < |B|$, then f can not be one-to-one. X
(ii) If $|A| = |B|$, then f is a bijection. X
(iii) If f is onto, then $|A| \geq |B|$. ✓
(iv) g is one-to one. ✓
(v) g is onto. X

A. (iii) and (iv) **B.** (i) and (iii) **C.** (ii) and (iii) **D.** (ii) and (iv)

E. (iv) and (v) **F.** (ii) and (v)

In each of the following five questions, write your final answer in the answer box.
Show your work below the answer box to receive partial marks.

[2pts] 5. Let A and B be finite sets with $|A| = 3$. If the **cardinality of the power set of** $A \times B$ is 4096, what is the cardinality of B? Hint: $2^{10} = 1024$

$|B| = 4$

$|\mathcal{P}(A \times B)| = 2^{|A \times B|} = 2^{|A||B|} = 4096 = 2^{12}$

$|A||B| = 12$, $|A| = 3$

$|B| = 4$

[2pts] 8. Define the following atomic propositions:

H: "The tiger hides."
F: "The hunt is finished soon."
E: "The hunter is eaten by the tiger."
N: "The hunt is happening at night."

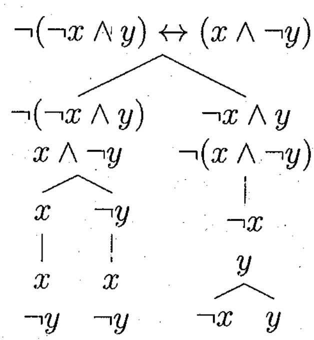
Translate the following sentence into a compound proposition (parentheses are there to help you):

(The tiger does not hide or the hunter is eaten by the tiger) only if (the hunt is happening at night and is not finished soon).

Compound proposition: $(\neg H \vee E) \rightarrow (N \wedge \neg F)$

9. Let P be the compound proposition $\neg(\neg x \wedge y) \leftrightarrow (x \wedge \neg y)$. Below is a complete truth tree for P. Answer the questions about P in the answer box below.

[3pts]



P is a contradiction *(circle)*: YES NO

If NO, give a counterexample: $x = T, y = F$ and $x = F, y = T$

P is a tautology *(circle)*: YES NO

If NO, give a counterexample: $x = T, y = T$. and $x = F, y = F$

Give a DNF for P: $(x \wedge \neg y) \vee (\neg x \wedge y)$

11. Let n be an integer. Give an **indirect proof** of the following theorem.

[5pts] *If $n^2 + 4n - 1$ is odd, then n is even.*

Let p : "n^2+4n-1 is odd."
q : "n is even"

We must prove $p \to q$ using an indirect proof.
That is, we prove $\neg q \to \neg p$ using a direct proof.
Now, $\neg q$: "n is odd."
$\neg p$: "n^2+4n-1 is even."

So assume n is odd. That is, $n = 2k+1$ for some integer k. Then:

$$
\begin{aligned}
n^2+4n-1 &= (2k+1)^2 + 4(2k+1) - 1 \\
&= 4k^2 + 12k + 4 \\
&= 2(2k^2 + 6k + 2) \\
&= 2m \quad \text{for } m = 2k^2+6k+2
\end{aligned}
$$

Since $m = 2k^2+6k+2$ is an integer and $n^2+4n-1 = 2m$, we conclude that n^2+4n-1 is even.
We showed that $\neg q \to \neg p$ is true.
Therefore, $p \to q$ is true.

12

Additional work space. Do not detach this page.

Alternative solution

A	B	C	$\neg(A \Leftrightarrow B)$	$(A \to C) \to \neg B$	$\neg(B \vee C)$	$\neg A \wedge \neg B$
T	T	T	F	F	F	F
T	T	F	F	T	F	F
T	F	T	T	T	F	F
T	F	F	T	T	T	F
F	T	T	T	F	F	F
F	T	F	T	F	F	F
F	F	T	F	T	F	T
F	F	F	F	T	T	T

The only truth assignment resulting in all premises true is $A=T$, $B=F$, $C=F$. In this case, the conclusion is false. Hence the argument is invalid, and the only counterexample is $\boxed{A=T,\ B=F,\ C=F}$.

Questions 1–4 are multiple choice. Enter the **letter** corresponding to each correct answer in the appropriate box below.

Question	1	2	3	4
Answer	F	B	A	C

[2pts] 1. Let $S = \{a, b, \{a, \emptyset\}, \{\emptyset\}\}$. Which of the following statements are **false**?

(i) $\{a, \{\emptyset\}\} \subseteq S$ ✓

(ii) $\{a, b\} \in S$ ✗

(iii) $\{a, \emptyset\} \in S$ ✓

(iv) $\{a, \emptyset\} \subseteq S$ ✗

(v) $\{\emptyset\} \in S$ ✓

A. only (iv) **B.** (i) and (iv) **C.** only (v) **D.** only (iii)

E. (ii) and (v) **F.** (ii) and (iv)

[2pts] 4. Which of the following arguments (rules of inference) are **invalid**?

(i) $\begin{array}{l} a \to b \\ \neg b \\ \hline \therefore \neg a \end{array}$ ✓ (ii) $\begin{array}{l} a \to b \\ \neg b \\ \hline \therefore a \end{array}$ X (iii) $\begin{array}{l} a \vee b \\ \neg a \vee c \\ \hline \therefore b \vee c \end{array}$ ✓

(iv) $\begin{array}{l} a \vee b \\ \neg b \\ \hline \therefore a \end{array}$ ✓ (v) $\begin{array}{l} a \vee b \\ \neg a \vee c \\ \hline \therefore b \wedge c \end{array}$ X (vi) $\begin{array}{l} a \to b \\ \neg a \to c \\ \hline \therefore \neg b \to c \end{array}$ ✓

A. (ii) and (iii) **B.** (i) and (iv) **C.** (ii) and (v) **D.** (iii), (v), and (vi)

E. only (v) **F.** (i) and (v)

In each of the following five questions, write your final answer in the answer box.
Show your work below the answer box to receive partial marks.

5. The truth table of a compound proposition P with atomic propositions x, y, and z is shown below. Give a **disjunctive normal form** of P. *You do not need to simplify your answer.*

[2pts]

x	y	z	P
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

DNF of P: $(x \wedge y \wedge z) \vee (x \wedge y \wedge \neg z) \vee (x \wedge \neg y \wedge \neg z)$

[2pts] 8. On the Island of Knights and Knaves we meet two inhabitants A and B. Person A says: "B is a knave if and only if I am a knave." What is person B?

Answer: B is a knight

p: "A is a knight."
q: "B is a knight."
A says: $\neg q \Leftrightarrow \neg p$

p	q	$\neg q \Leftrightarrow \neg p$
T	T	T
T	F	F
F	T	F
F	F	T

must match

[2pts] 9. Define the following atomic propositions:

H: "The tiger hides."
F: "The hunt is finished soon."
E: "The hunter is eaten by the tiger."
N: "The hunt is happening at night."

Translate the following sentence into a compound proposition (parentheses are there to help you):

(The hunt is finished soon and the hunter is not eaten by the tiger) only if (the tiger hides or the hunt is happening at night).

Compound proposition: $(F \wedge \neg E) \rightarrow (H \vee N)$

11. Let n be an integer. Give an **indirect proof** of the following theorem.

[5pts] *If $n^2 - 2n + 3$ is odd, then n is even.*

Let p : " n^2-2n+3 is odd "

q : " n is even "

We must prove $p \to q$ using an indirect proof.

That is, we prove $\neg q \Rightarrow \neg p$ using a direct proof.

Observe that $\neg p$: " n^2-2n+3 is even "

$\neg q$: " n is odd. "

Hence assume n is odd. Then $n = 2k+1$ for some integer k. Now:

$$
\begin{aligned}
n^2-2n+3 &= (2k+1)^2 - 2(2k+1)+3 \\
&= 4k^2+2 \\
&= 2(2k^2+1) \\
&= 2m \quad \text{for } m = 2k^2+1
\end{aligned}
$$

Since $m = 2k^2+1$ is an integer and $n^2-2n+3 = 2m$, we conclude that n^2-2n+3 is even.

We showed that $\neg q \Rightarrow \neg p$ is true.

Hence $p \Rightarrow q$ is true.

(12)

Additional work space. Do not detach this page.

Alternative solution

A	B	C	$\neg(B \Leftrightarrow \neg A)$	$(B \to C) \to A$	$\neg(\neg A \vee C)$	$\neg B \wedge A$
T	T	T	T	T	F	F
T	T	F	T	T	T	F
T	F	T	F	T	F	T
T	F	F	F	T	T	T
F	T	T	F	F	F	F
F	T	F	F	T	F	F
F	F	T	T	F	F	F
F	F	F	T	F	F	F

Note: $\neg(B \Leftrightarrow \neg A) \equiv A \Leftrightarrow B$

$\neg(\neg A \vee C) \equiv A \wedge \neg C$

All premises are T only when $A=T$, $B=T$, $C=F$. In this case we have the conclusion F. Thus, the argument is invalid and $\boxed{A=T, B=T, C=F}$ is the only counterexample.