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Digital Logic Design: Principles and Practices ELG5195 (EACJ5705), Carleton CRN: 18371

Assignment #1

Question 1:

a) Using iterated consensus find all the prime implicants of the following function:

$$F(a,b,c,d) = \bar{a}\cdot\bar{b}\cdot c\cdot\bar{d} + \bar{a}\cdot b\cdot d + a\cdot b\cdot\bar{c} + a\cdot\bar{b}\cdot\bar{c}\cdot\bar{d} + a\cdot\bar{b}\cdot c\cdot\bar{d} + a\cdot b\cdot c\cdot d$$

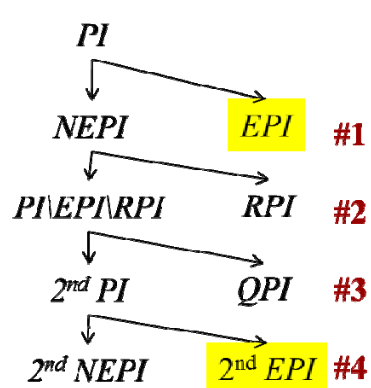
b) Find the minimum set of prime implicants (PI) that covers F by evaluating how each PI implies subsets of the other PI's. What is the expression(s) of the minimized function F?

Consensus	Term	Implicants	abcd	Covered by
	~~T1~~	~~a'b'cd'~~	~~0010~~	<T9
	~~T2~~	~~a'bd~~	~~01-1~~	<T12
	T3	abc'	110-	
	~~T4~~	~~ab'c'd'~~	~~1000~~	<T8
	~~T5~~	~~ab'cd'~~	~~1010~~	<T9
	~~T6~~	~~abcd~~	~~1111~~	<T10
T2¢T1		-		
T3¢T2	T7	bc'd	-101	<T12
T3¢T1		-		
T4¢T3	T8	ac'd'	1-00	
T5¢T3		-		
T5¢T2		-		
T5¢T1	T9	b'cd'	-010	
T6¢T3	~~T10~~	~~abd~~	~~11-1~~	<T12
T7¢T3		-		
T7¢T2		-		
T8¢T7		~~abc'~~		=T3
T8¢T3		-		
T8¢T2		-		
T9¢T8	T11	ab'd'	10-0	
T9¢T7		-		
T9¢T3		-		
T9¢T2		-		
T10¢T9		-		
T10¢T8		~~abc'~~		=T3
T10¢T7		-		
T10¢T3		-		
T10¢T2	T12	bd	-1-1	
T11¢T9		-		
T11¢T8		-		
T11¢T3		~~ac'd'~~		=T8
T12¢T11		-		
T12¢T9		-		
T12¢T8		~~abc'~~		=T3
T12¢T3		-		

PI =
{ a.b.c', **ac'd'**, **b'cd'**, **ab'd'**, **bd**}

Verification with K-maps:

ab \ cd	00	01	11	10
00	0	0	0	1*
01	0	1*	1*	
11	**1**	1	1*	
10	**1**	0	0	1



#1 Find EPI's $E_j = E_{PI \setminus P_j}(P_j) = \sum_{P_k \in PI \setminus P_j} P_k \; ; \; j = 1..N$ **PI = {abc', ac'd', b'cd', ab'd', bd}**

$E_{PI\setminus abc'}$(abc') = ac'd' + b'cd' + ab'd' + bd = $E_{PI\setminus abc'}|_{(a=1,\, b=1,\, c=0)}$ = d' + 0 + 0 + d = 1 **abc'**=> NEPI

$E_{PI\setminus ac'd'}$ (ac'd') = abc' + b'cd' + ab'd' + bd = $E_{PI\setminus ac'd'}|_{(a=1,\, c=0,\, d=0)}$ = b + 0 + b' + 0 = 1 **ac'd'**=> NEPI

$E_{PI\setminus b'cd'}$ (b'cd') = abc' + ac'd' + ab'd' + bd = $E_{PI\setminus b'cd'}|_{(b=0,\, c=1,\, d=0)}$ = 0 + 0 + a + 0 $\neq$ 1 **b'cd' => EPI**

$E_{PI\setminus ab'd'}$ (ab'd') = abc' + ac'd' + b'cd' + bd = $E_{PI\setminus ab'd'}|_{(a=1,\, b=0,\, d=0)}$ = 0 + c' + c + 0 = 1 **ab'd'**=>NEPI

$E_{PI\setminus bd}$ (bd) = abc' + ac'd' + b'cd' + ab'd' = $E_{PI\setminus bd}|_{(b=1,\, d=1)}$ = ac' + 0 + c + 0 $\neq$ 1 **bd => EPI**

EPI: {b'cd', bd}

NEPI: {abc', ac'd', ab'd'}

#2 Find Redundant PI's from NEPI: {abc', ac'd', al $E_{EPI}(P_r) = \sum_{P_e \in EPI} P_e$ and $P_r \in NEPI$

E_{EPI} (abc') = b'cd' + bd = $E_{EPI}|_{a=1,\, b=1,\, c=0}$ = 0+d $\neq$ 1 $ab\bar{c} \notin RPI$ abc'=>PI/EPI/RPI

E_{EPI} (ac'd') = b'cd' + bd = $E_{EPI}|_{a=1,\, c=0,\, d=0}$ = 0+0 $\neq$ 1 $a \cdot \bar{c} \cdot \bar{d} \notin RPI$ ac'd'=>PI/EPI/RPI

E_{EPI} (**ab'd'**) = b'cd' + bd = $E_{EPI}|_{a=1,\, b=0,\, d=0}$ = c+0 $\neq$ 1 $a \cdot \bar{b} \cdot \bar{d} \notin RPI$ ab'd'=>PI/EPI/RPI

So, there are no RPI's (**RPI = Φ**), and all 3 NEPI's = PI\EPI\RPI: {abc', ac'd', ab'd}

#3 Find **Eclipsed PI's** (QPI) $P_q \neq P_s$ of $PI \setminus EPI \setminus RPI$ that are covered by EPI U P_s

$$E_s(P_q) = \sum_{P_e \in EPI} P_e + P_s, \text{ with } P_s, P_q \in PI \setminus EPI \setminus RPI$$

Add the NEPIs (P_s) to the set of EPIs, one at a time, to check if they eclipse the other NEPI's (P_q)

Add abc' to *EPI*:

$E_{EPI\, U\, abc'}$(a.c'.d') = [(b'.c.d'+b.d) + a.b.c']$|_{a=1,c=0,d=0}$ = 0+0+b≠1 => NQPI

$E_{EPI\, U\, abc'}$(**ab'd'**) = [(b'.c.d'+b.d)+ a.b.c']$|_{a=1,b=0,d=0}$ = c+0+0 ≠1 => NQPI

Add a.c'.d' to *EPI*:

$E_{EPI\, U\, a.c'.d'}$ (a.b.c') = [(b'.c.d'+b.d)+a.c'.d']$|_{a=1,b=1,c=0}$ = 0+d+d' = 1 => a.b.c' eclipsed by *EPI* U a.c'.d'

$E_{EPI\, U\, a.c'.d'}$ (**ab'd'**) = [(b'.c.d'+b.d)+a.c'.d']$|_{a=1,b=0,d=0}$ = c+0+c' = 1 => **ab'd'** eclipsed by *EPI* U a.c'.d'

⇨ keep a.c'.d' 2nd PI and eliminate eclipsed PIs: QPI = { a.b.c', **ab'd'**}

Add **ab'd'** to *EPI*:

$E_{EPI\, U\, a.b'.d'}$ (a.b.c') = [(b'.c.d'+b.d)+ **ab'd'**] $|_{a=1,b=1,c=0}$ = 0+d+0 ≠1 => NQPI

$E_{EPI\, U\, a.b'.d'}$ (a.c'.d') = [(b'.c.d'+b.d)+ **ab'd'**] $|_{a=1,c=0,d=0}$ = 0+0+b' ≠1 =>NQPI

Adding a.c'.d' to the *EPI* *2nd PI =**SPI** =***{**ac'd'**}; they eclipse the other 2 NEPIs ➔**QPI** ={ a.b.c' , **ab'd'**}

#4. Find 2nd EPI from 2nd PI (SPI = PI\EPI\RPI\QPI= a.c'.d')

$E_{SEPI} = E_{SPI\, U\, EPI \setminus Pj}(P_j) = \Sigma P_k + \Sigma P_{EPI}, P_k \in SPI \setminus P_j$;

SPI ∪ EPI \ Pj = [a.c'.d' + (b'.c.d'+b.d)] \ a.c'.d' = (b'.c.d'+b.d)

E_{SEPI} (ac'd') = $E_{SEPI}|_{a=1,c=0,d=0}$ = (b'.c.d'+b.d)$|_{ac'd'=1}$ = 0+0≠1 ➔ ac'd'=*SEPI*

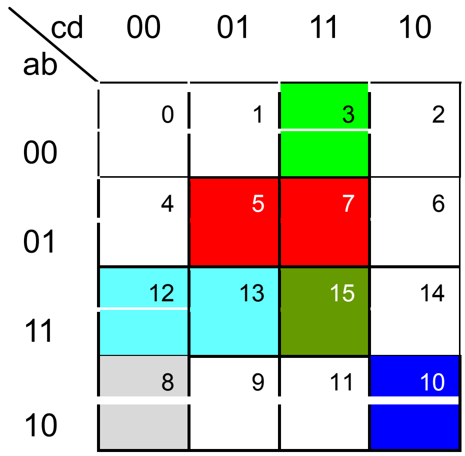
➔ **F_{min} = (b'cd'+bd)+ac'd'**

Question 2:

a) Find PI's of the following function using Iterated Consensus :

$F(a,b,c,d) = a'b'cd + a'bd + abc' + ab'c'd' + ab'cd' + abcd$

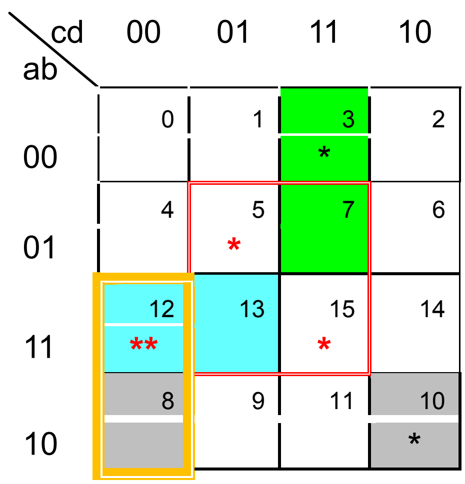
Consensus	Term	Implicants	Covered by	Covers & Removes
	~~T1~~	~~a'b'cd~~	<T7	
	~~T2~~	~~a'bd~~	<T13	
	T3	**abc'**		
	~~T4~~	~~ab'c'd'~~	<T9	
	~~T5~~	~~ab'cd'~~	<T11	
	~~T6~~	~~abcd~~	<T10	
T2¢T1	**T7**	**a'cd**		>T1
T3¢T2	~~T8~~	~~bc'd~~	<T13	
T4¢T3	**T9**	**ac'd'**		>T4
T5¢T3		-		
T5¢T2		-		
T6¢T5		-		
T6¢T3	~~T10~~	~~abd~~	<T13	>T6
T7¢T5		-		
T7¢T3		-		
T7¢T2		-		
T8¢T7		~~a'bd = T2~~		
T8¢T5		-		
T8¢T3		-		
T8¢T2		-		
T9¢T8		~~abc' = T3~~		
T9¢T7		-		
T9¢T5	**T11**	**ab'd'**		>T5
T9¢T3		-		
T9¢T2		-		
T10¢T9		~~abc' = T3~~		
T10¢T8		-		
T10¢T7	~~T12~~	~~bcd~~	<T13	
T10¢T3		-		
T10¢T2	**T13**	**bd**		>T2,T10
T11¢T9		-		
T11¢T7		-		
T11¢T3		~~ac'd'=T9~~		
T13¢T11		-		
T13¢T9		~~abc' = T3~~		
T13¢T3		-		



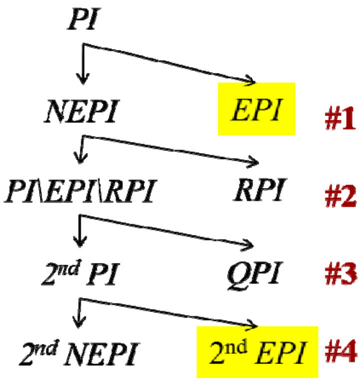
K-maps are for verification only:

PI : { T3,T7,T9,T11, T13}

*PI : {abc', **ab'd'**, **a'cd**, ac'd', **bd**}*



b) Find a minimum set of Prime Implicants that covers F:



#1. Find Essential Prime Implicants (EPI's) by evaluating $E_{PI \setminus Pj}(P_j) = \Sigma P_k, P_k \in PI \setminus P_j$

If $E_j = 1 \rightarrow P_j$ implies $PI \setminus P_j \rightarrow P_j \in NEPI$ = set of Non-Essential PI's
If $E_j \neq 1 \rightarrow P_j$ does not imply $PI \setminus P_j \rightarrow P_j \in EPI$= set of Essential PI's

PI : {*abc'*, *ab'd'*, *a'cd*, *ac'd'*, ***bd***}
PI : { T3, T11, T7, T9, T13}

T3 = abc': **a=1 b=1 c=0**

$E_{PI \setminus T3}$ (**T3**) = (ab'd'+a'cd+ac'd'+bd)$|_{abc'=1}$ = 0+0+d'+d = 1 ➔ **T3** ∈ **NEPI**

T11=ab'd' : **a=1 b=0 d=0**

$E_{PI \setminus T5}$ (**T11**) = (abc'+a'cd+ac'd'+bd)$|_{ab'd'=1}$ = 0+0+c+0 = c ≠ 1 ➔ **T11 ∈ EPI**

T7=a'cd: **a=0 c=1 d=1**

$E_{PI \setminus T7}$ (**T7**) = abc'+ab'd'+ ac'd'+bd = 0+0+0+b ≠ 1 ➔ **T7 ∈ EPI**

T9=ac'd': **a=1 c=0 d=0**

$E_{PI \setminus T9}$ **(T9)**= abc'+ab'd'+a'cd+bd = b+b'+0+0 = 1 ➔ **T9** ∈ **NEPI**

T13=bd: **b=1 d=1**

$E_{PI \setminus T13}$ (**T13**) = abc'+ab'd'+a'cd+ac'd' = ac'+0+a'c+0= ac'+a'c≠ 1 ➔ **T13 ∈ EPI**

#1 EPI* : { T11, T7, T13 } = { ab'd', a'cd, bd }	NEPI: { T3, T9 } = { abc', ac'd' }

#2 Find Redundant Prime Implicants (RPI's) by evaluating each NEPI

T3: **a=1, b=1, c=0 =>** E_{EPI}(T3) = (**ab'd' + a'cd + bd**)$|_{abc'=1}$ = 0+0+d = d ≠ 1 abc' ∉ RPI

T9: **a=1, c=0, d=0 =>** E_{EPI}(T9) = (**ab'd' + a'cd + bd**)$|_{ac'd'=1}$ = b'+0+0 = b' ≠ 1 => ac'd' ∉ RPI

Hence | **#2 RPI = Φ** |

#3 Find Eclipsed Prime Implicants (QPI's) $P_q \in PI \setminus EPI \setminus RPI$ which are covered by $EPI \cup P_s$, with $P_s \in PI \setminus EPI \setminus RPI$ and $P_q \neq P_s$

$E_{EPI \cup T3}$ (P_q)=(**ab'd'**+**a'cd**+**bd**)+**abc'**, P_q={ **ac'd'** }

$E_{EPI \cup T3}$ (*T9*)$|_{a=1,c=0,d=0}$ =b'+0+0+b=1

➔ *EPI* ∪ *T3* > *T9* i.e. *T9* eclipsed by *EPI* ∪ *T3*

➔ ***T9* ∈ QPI**; | 2^{nd} *PI* = ***SPI*** = {**abc'**} = { **T3**} |

If only 1 minimized expression is required ➔ #4,

else, find all other 2^{nd} PI's lists as follows:

$E_{EPI \cup T9}$ (P_q)=(**ab'd'** + **a'cd** + **bd**)+**ac'd'**, P_q = {**abc'**}

$E_{EPI \cup T9}$ (**T3**)$|_{a=1, b=1, c=0}$ = 0+0+d +d' = 1

➔ *EPI* ∪ **T9** > **T3** i.e. **T3** eclipsed by *EPI* ∪ **T9**

➔ ***T3* ∈ QPI**$_1$; | 2^{nd} *PI'* = ***SPI***$_1$ = { **ac'd'**} = { **T9**} |

#4. Find Secondary EPI's by evaluating $E_{SPI \cup EPI \setminus Pj}(P_j) = \Sigma P_k + \Sigma P_{EPI}, P_k \in SPI \setminus P_j$

SPI ∪ ***EPI*** \ ***Pj*** = **T3** + (**ab'd'** + **a'cd** + **bd**) \ **T3** =

= **ab'd'**+ **a'cd**+ **bd** = ***EPI***➔ | **T3=abc'=SEPI**** |

SPI$_1$ ∪ ***EPI*** \ ***Pj*** = **T9** + (**ab'd'** + **a'cd** + **bd**)**T9** =

= **ab'd'**+**a'cd**+**bd** = ***EPI***➔ | **T9=ac'd'=SEPI$_1$**** |

2^{nd} NEPI = Φ ➔ **STOP**

***F*(*a,b,c,d*) = (ab'd' + a'cd + bd) + abc'**	***F*$_1$(*a,b,c,d*) = (ab'd' + a'cd + bd) + ac'd'**

Question 3:

a) Find the prime implicants of the following function by applying the Quine-McCluskey method:
$G(v,w,x,y,z) = \Sigma m(0, 2, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 21, 23, 26, 28, 29, 30, 31)$

Class 0

Group	Min term	vwxyz	
0	0	00000	√
1	2	00010	√
	4	00100	√
	8	01000	√
2	5	00101	√
	6	00110	√
	9	01001	√
	10	01010	√
3	7	00111	√
	11	01011	√
	13	01101	√
	21	10101	√
	26	11010	√
	28	11100	√
4	15	01111	√
	23	10111	√
	29	11101	√
	30	11110	√
5	31	11111	√

Class 1

Group	Dec	vwxyz	
0	(0,2)	000-0	√
	(0,4)	00-00	√
	(0,8)	0-000	√
1	(2,6)	00-10	√
	(2,10)	0-010	√
	(4,5)	0010-	√
	(4,6)	001-0	√
	(8,9)	0100-	√
	(8,10)	010-0	√
2	(5,7)	001-1	√
	(5,13)	0-101	√
	(5,21)	-0101	√
	(6,7)	0011-	√
	(9,11)	010-1	√
	(9,13)	01-01	√
	(10,11)	0101-	√
	(10,26)	-1010	A
3	(7,15)	0-111	√
	(7,23)	-0111	√
	(11,15)	01-11	√
	(13,15)	011-1	√
	(13,29)	-1101	√
	(21,23)	101-1	√
	(21,29)	1-101	√
	(26,30)	11-10	B
	(28,29)	1110-	√
	(28,30)	111-0	√
4	(15,31)	-1111	√
	(23,31)	1-111	√
	(29,31)	111-1	√
	(30,31)	1111-	√

Class 2

Group	Dec	vwxyz	
0	(0,2,4,6)	00--0	C
	(0,2,8,10)	0-0-0	D
	~~(0,4,2,6)~~	~~00--0~~	
	~~(0,8,2,10)~~	~~0-0-0~~	
1	(4,5,6,7)	001--	E
	~~(4,6,5,7)~~	~~001--~~	
	(8,9,10,11)	010--	F
	~~(8,10,9,11)~~	~~010--~~	
2	(5,7,13,15)	0-1-1	√
	(5,7,21,23)	-01-1	√
	~~(5,13,7,23)~~	~~0-1-1~~	
	(5,13,21,29)	--101	√
	~~(5,21,7,23)~~	~~-01-1~~	
	~~(5,21,13,29)~~	~~--101~~	
	(9,11,13,15)	01--1	G
	~~(9,13,11,15)~~	~~01--1~~	
3	(7,15,23,31)	--111	√
	~~(7,23,15,31)~~	~~--111~~	
	(13,15,29,31)	-11-1	√
	~~(13,29,15,31)~~	~~-11-1~~	
	(21,23,29,31)	1-1-1	√
	~~(21,29,23,31)~~	~~1-1-1~~	
	(28,29,30,31)	111--	H
	~~(28,30,29,31)~~	~~111--~~	

Class 3

Group	Dec	vwxyz	
2	(5,7,13,15,21,23,29,31)	--1-1	I
	~~(5,7,21,23,13,15,29,31)~~	~~--1-1~~	
	~~(5,13,21,29,7,15,23,31)~~	~~--1-1~~	

Since no other implicants can be formed, this is the end of the procedure.

PI = {A, B, C, D, E, F, G, H, I}

PI= {wx'yz', vwyz', v'w'z', v'x'z', v'w'x, v'wx', v'wz, vwx, xz}

b) Find the essential prime implicant(s) from the corresponding *Prime Implicant Table.*

PI		CU	0	2	4	5	6	7	8	9	10	11	13	15	21	23	26	28	29	30	31
						√		√					√	√	√	√		√	√	√	√
(10,26)	A	4									x						x				
(26,30)	B	4															x			x	
(0,2,4,6)	C	3	x	x	x		x														
(0,2,8,10)	D	3	x	x					x		x										
(4,5,6,7)	E	3			x	x	x	x													
(8,9,10,11)	F	3							x	x	x	x									
(9,11,13,15)	G	3								x		x	x	x							
(28,29,30,31)	H*	3																X	x	x	x
(5,7,13,15,21, 23,29,31)	I*	1				x		x					x	x	X	X			x		x

EPI: H*, I* => **G= H*+ I*+ …**

Eliminate the EPI's rows and the columns of their corresponding minterms:

PI		CU	0	2	4	6	8	9	10	11	26
(10,26)	A	4							x		x
(26,30)	B	4									x
(0,2,4,6)	C	3	X	X	X	X					
(0,2,8,10)	D	3	X	X			X		X		
(4,5,6,7)	E	3			X	X					
(8,9,10,11)	F	3					X	X	X	X	
(9,11,13,15)	G	3						X		X	

Eliminate double columns:

0 = 2, 4 = 6, 8 = 10, 9 = 11

PI		CU	0	4	8	9	26
(10,26)	A	4					x
(26,30)	B	4					x
(0,2,4,6)	C	3	X	X			
(0,2,8,10)	D	3	X		X		
(4,5,6,7)	E	3		X			
(8,9,10,11)	F	3			X	X	
(9,11,13,15)	G	3				X	

Remove dominated rows implemented at the same cost: A=B, C>E, F>G

PI		CU	0	4	8	9	
			√	√	√	√	√
(10,26)	A**	4					X*
(0,2,4,6)	C**	3	X	X*			
(0,2,8,10)	D	3	X		X		
(8,9,10,11)	F**	3			X	X*	

Secondary EPI: A**, C**, F** cover all the minterms remained uncovered by EPI's =>

⇨ **A minimum function is**

⇨ **f = H*+ I*+ A**+ C**+ F** = (**vwx + xz) + (wx'yz' + v'w'z' + v'wx')

c) Apply the Petrick method to determine ALL the minimal covers of the function G.

After finding the EPI's, the Petrick method is used to find all the function's covers:

⇨ PI		CU	0	2	4	6	8	9	10	11	26
(10,26)	A	4							x		x
(26,30)	B	4									x
(0,2,4,6)	C	3	X	X	X	X					
(0,2,8,10)	D	3	X	X			X		X		
(4,5,6,7)	E	3			X	X					
(8,9,10,11)	F	3					X	X	X	X	
(9,11,13,15)	G	3						X		X	

P = (C+D)(C+E)(D+F)(A+D+F)(F+G) (A+B)

P = (C+CD+CE+DE)(D+F)(F+G) (A+B)

P = (C+DE)(F+DG)(A+B) = **ACF**+**BCF**+ACDG + BCDG+AFDE + BFDE + ADEG + BDEG

G_1 = (H+ I) + (A+C+ F)=
= (vwx + xz) + (wx'yz' + v'w'z' + v'wx')

G_2 = (H+ I) + (B+C+ F) =
= (vwx + xz) + (vwyz'+ v'w'z' + v'wx')

Question 4:

a) Find the prime implicants of the following multiple-output function by applying the modified Quine-McCluskey method: $Z_1\ (a,b,c,d) = \Sigma(0, 3, 4, 5, 7, 8, 12, 13, 15)$; $Z_2\ (a,b,c,d) = \Sigma(1, 5, 7, 8, 9, 10, 11, 13, 14, 15)$; $Z_3\ (a,b,c,d) = \Sigma(1, 2, 4, 5, 7, 10, 13, 14, 15)$

Class	1	a b c d	$Z_1Z_2Z_3$	
0	0	0000	-00	√
1	1	0001	0--	√
	2	0010	00-	√
	4	0100	-0-	√
	8	1000	--0	I
2	3	0011	-00	√
	5	0101	---	√
	9	1001	0-0	√
	10	1010	0--	√
	12	1100	-00	√
3	7	0111	---	√
	11	1011	0-0	√
	13	1101	---	√
	14	1110	0--	√
4	15	1111	---	√

Class 2		a b c d	$Z_1Z_2Z_3$	
0	~~(0,1)~~	~~000-~~	~~000~~	
	~~(0,2)~~	~~00-0~~	~~000~~	
	(0,4)	0-00	-00	√
	(0,8)	-000	-00	√
1	~~(1,3)~~	~~00-1~~	~~000~~	
	(1,5)	0-01	0--	K
	(1,9)	-001	0-0	√
	~~(2,3)~~	~~001-~~	~~000~~	
	(2,10)	-010	00-	H
	(4,5)	010-	-0-	J
	(4,12)	-100	-00	√
	(8,10)	10-0	0-0	√
	(8,12)	1-00	-00	√
2	(3,7)	0-11	-00	C
	~~(3,11)~~	~~-011~~	~~000~~	
	(5,7)	01-1	---	√
	(5,13)	-101	---	√
	(9,11)	10-1	0-0	√
	(9,13)	1-01	0-0	√
	(10,11)	101-	0-0	√
	(10,14)	1-10	0--	L
	~~(12,14)~~	~~11-0~~	~~000~~	
3	(7,15)	-111		√
	(11,15)	1-11	0-0	√
	(13,15)	11-1	---	√
	(14,15)	111-	0--	M

Class 3		a b c d	$Z_1Z_2Z_3$	
0	(0,4,8,12)	--00	-00	A
	~~(0,1,4,5)~~	~~0-0-~~	~~000~~	
	~~(0,2,8,10)~~	~~-0-0~~	~~000~~	
	~~(0,1,8,9)~~	~~-00-~~	~~000~~	
1	(1,5,9,13)	--01	0-0	D
	(4,5,12,13)	-10-	-00	B
	(8,9,10,11)	10--	0-0	E
	~~(8,9,12,13)~~	~~1-0-~~	~~000~~	
	~~(1,3,5,7)~~	~~0--1~~	~~000~~	
~~2~~	~~(3,7,11,15)~~	~~--11~~	~~000~~	
	(5,7,13,15)	-1-1	---	N
	(9,11,13,15)	1--1	0-0	F
	(10,11,14,15)	1-1-	0-0	G

Class 4

~~(0,1,4,5,8,9,12,13)~~	~~--0-~~	~~000~~	

b) Using reduction methods of the *prime implicant table* find all <u>*sets*</u> of minimum sum of products expressions that covers the given *set* of functions.

Find EPI					0	3	4	5	7	8	12	13	15	1	5	7	8	9	10	11	13	14	15	1	2	4	5	7	10	13	14	15	
					●	●	●	●	●	●	●	●	●		●	●					●		●	●	●	●	●	●	●	●		●	
c'd'	(0,4,8,12)	--00 -00	A	3	1*		1			1	1																						Z_1
bc'	(4,5,12,13)	-10- -00	B	3			1	1			1	1																					
a'cd,	(3,7)	0-11 -00	C	4		1*			1																								
c'd	(1,5,9,13)	--01 0-0	D	3										1	1			1			1												Z_2
ab'	(8,9,10,11)	10-- 0-0	E	3													1	1	1	1													
ad	(9,11,13,15)	1--1 0-0	F	3														1		1	1		1										
ac	(10,11,14,15)	1-1- 0-0	G	3															1	1		1	1										
b'cd'	(2,10)	-010 00-	H	4																					1*				1				Z_3
ab'c'd'	8	1000 --0	I	5						1							1																Z_1Z_2
a'bc'	(4,5)	010- -0-	J	4			1	1																		1*	1						Z_1Z_3
a'c'd	(1,5)	0-01 0--	K	4										1	1									1*			1						Z_2Z_3
acd'	(10,14)	1-10 0--	L	4															1			1							1		1		
abc	(14,15)	111- 0--	M	4																		1	1								1	1	
bd	(5,7,13,15)	-1-1 ---	N	3				1	1			1	1*		1	1*					1		1				1	1*		1*		1	$Z_1Z_3Z_3$

EPI: Z_1 = *bd* + *c'd'* + *a'cd* - done! Z_2 = *bd* + ….. Z_3 = *bd* + *b'cd'* + *a'bc'* + *a'c'd*

Eliminate dominated rows					1	8	9	10	11	14	14	
c'd	(1,5,9,13)	--01 0-0	D	3	1		1					Z_2
ab'	(8,9,10,11)	10-- 0-0	E	3		1	1	1	1			
~~ad~~	~~(9,11,13,15)~~	~~1--1 0-0~~	~~F~~	~~3~~			~~1~~		~~1~~			
ac	(10,11,14,15)	1-1- 0-0	G	3				1	1	1		
~~ab'c'd'~~	~~8~~	~~1000 --0~~	~~I~~	~~5~~		~~1~~						Z_1Z_2
a'c'd	(1,5)	0-01 0--	K	1	1							Z_2Z_3
acd'	(10,14)	1-10 0--	L	4				1		1	1	
~~abc~~	~~(14,15)~~	~~111- 0--~~	~~M~~	4						~~1~~	~~1~~	

Find SEPI					1	8	9	10	11	14	14	
						•	•	•	•		•	
c'd	(1,5,9,13)	--01 0-0	D	3	1		1					Z_2
ab'	(8,9,10,11)	10-- 0-0	E	3		1*	1	1	1			
ac	(10,11,14,15)	1-1- 0-0	G	3				1	1	1		
a'c'd	(1,5)	0-01 0--	K	1	1							Z_2Z_3
acd'	(10,14)	1-10 0--	L	4				1		1	1*	

{SEPI} : Z_2 = bd + {ab'} +
Z_3 = bd + b'cd' + a'bc' + + a'c'd + {acd'} -> *done!!!*

					1	14	
c'd	(1,5,9,13)	--01 0-0	D	3	1		Z_2
ac	(10,11,14,15)	1-1- 0-0	G	3		1	
a'c'd	(1,5)	0-01 0--	K	1	1		Z_2Z_3
acd'	(10,14)	1-10 0--	L	1		1	

Z_2 = bd + ab' + a'c'd + acd' -> *done!!!*

Z_1 = bd + c'd' + a'cd

Z_2 = bd + ab' + a'c'd + acd'

Z_3 = bd + b'cd' + a'bc' + + a'c'd + {acd'}

Verification with K-maps:

Z_1 = bd + c'd' + a'cd

ab \ cd	00	01	11	10
00	1	0	1	0
01	1	1	1	0
11	1	1	1	0
10	1	0	0	0

Z_1 = bd + c'd' + a'cd

Z_2 = bd + ab' + a'c'd + acd'

ab \ cd	00	01	11	10
00	0	1	0	0
01	0	1	1	0
11	0	1	1	1
10	1	1	1	1

Z_3 = bd + b'cd' + a'bc' + + a'c'd + acd'

ab \ cd	00	01	11	10
00	0	1	0	1
01	1	1	1	0
11	0	1	1	1
10	0	0	0	1

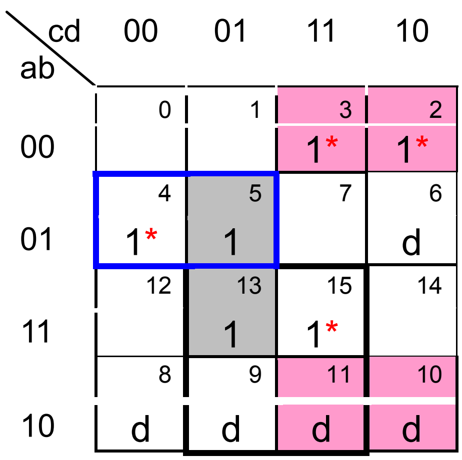
Question 5:

Use Karnaugh maps to find the prime implicants (PI), essential prime implicants (EPI) and redundant prime implicants (RPI) of the following functions.
Provide the Boolean expressions of the minimized functions.

1. f(a,b,c,d) = ∑m(2,3,4,5,13,15) + ∑d(6,8,9,10,11)

PI: ad, a'bc', bc'd, b'c
EPI: ad, a'bc', b'c
RPI: bc'd

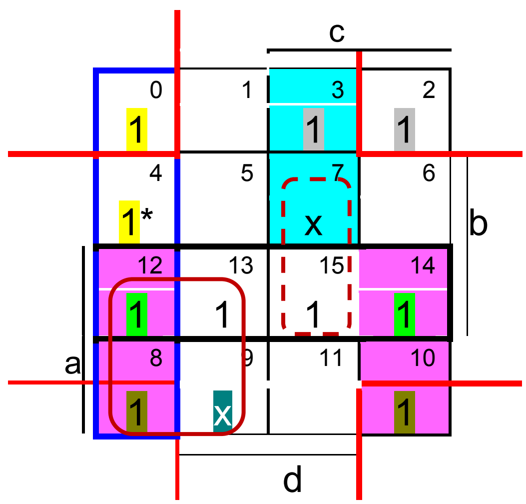
f(a,b,c,d) = ad + a'bc' + b'c



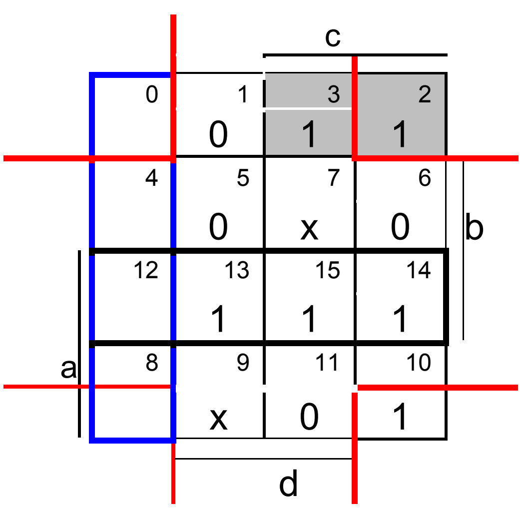
2. f(a,b,c,d) = a'c'd'+a'b'c+abd'+abd+ ab'd' + d(a'bcd + ab'c'd)

PI: ab, c'd', b'd', ad', a'b'c, a'cd, ab'c', ac', bcd
EPI: c'd'
RPI: ac'

f_1(a,b,c,d) = ab + c'd' + b'd' + a'cd



f_2(a,b,c,d) = ab + c'd' + b'd' + a'b'c



f_3(a,b,c,d) = ab + c'd' + ad' + a'b'c