MAT 1320, Winter 2014 Midterm Exam 2
Wednesday March 19
Instructor: Amin Bahmanian
Calculators are NOT allowed!
For rough work, you may use back pages or else, ask me for scrap paper.

Last Name ____________________ First Name ________________ Student Number ________

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature
____________________(should match the signature that you sign on the attendance sheets.)

Questions 1–4 are multiple-choice (5 points each). You must enter the letter corresponding to each correct answer in the table below. No partial marks will be given for other work.

Problem	1 (5 pts.)	2(5 pts.)	3(5 pts.)	4(5 pt.)
Your answer	D	D	A	A
Your marks				

Question 1. *Determine the derivative of* $f(x) = sin^{-1}(2x) = arcsin(2x)$ *with respect to* x.
(a) $\frac{1}{\sqrt{1-x^2}}$ **(b)** $\frac{2}{\sqrt{1-x^2}}$ **(c)** $\frac{1}{\sqrt{1-4x^2}}$ **(d)** $\frac{2}{\sqrt{1-4x^2}}$ **(e)** $\frac{9}{1-9x^2}$ **(f)** $\frac{9}{\sqrt{1-9x^2}}$

Using the chain rule, $f'(x) = \frac{1}{\sqrt{1-(2x)^2}} \times 2$.

Question 2. *Find the slope of the tangent line to the curve* $y = e^{1+x^2}\ln\left(1+x^2\right)$ *at the point* $x = 2$.
(a) $\ln(2)$ **(b)** 1 *(c)* π **(d)** 0 **(e)** $e^2\ln(2)$ **(f)** 1

$y' = 2xe^{1+x^2}\ln(1+x^2) + e^{1+x^2}\frac{2x}{1+x^2}$
$y'(0) = 2(0)e^{1+0^2}\ln(1+0^2) + e^{1+0^2}\frac{2(0)}{1+0^2} = 0 + 0 = 0$

Question 3. *A particle moves with acceleration function* $a(t) = \sin t - 3\cos t$. *Its initial velocity is* $v(0) = 2$ *m/s and its initial displacement is* $s(0) = 5$ *m. Find its position after* π *seconds.*
(a) $3\pi - 1$ **(b)** $3\pi + 1$ **(c)** *0* *(d)* π **(e)** $\pi + 1$ **(f)** $\pi - 1$

$v(t) = -\cos t - 3\sin t + C$
$2 = v(0) = -\cos 0 - 3\sin 0 + C$, so $C = 3$
$s(t) = -\sin t + 3\cos t + 3t + D$
$5 = s(0) = -\sin 0 + 3\cos 0 + 3(0) + D$, so $D = 2$
$s(t) = -\sin t + 3\cos t + 3t + 2$, so $s(\pi) = -\sin\pi + 3\cos\pi + 3\pi + 2 = 3\pi - 1$.

Question 4. *Evaluate the area between the x-axis and* $f(x) = -x^2 + 4x - 3$ *from* $x = 1$ *to* $x = 3$.
(a) 4/3 **(b)** 3/4 *(c)* 0 **(d)** 5/4 **(e)** 4/5 **(f)** 1/2

Area $= \int_1^3 -x^2 + 4x - 3\,dx = -\frac{1}{3}x^3 + 2x^2 - 3x\Big]_1^3 = (-9+18-9) - (-\frac{1}{3} + 2 - 3) = 0 - (-\frac{4}{3}) = \frac{4}{3}$.

Questions 5-12 are long-answer (10 points each). You must clearly show all relevant steps and justify your solution to receive full marks. Clearly indicate the final answer.

Question 5. *Find the derivative of the function*

$$y = (\sin x)^{x^2}.$$

$\ln y = \ln\left((\sin x)^{x^2}\right) = x^2 \ln(\sin x)$
$\frac{y'}{y} = 2x\ln(\sin x) + x^2 \frac{\cos x}{\sin x}$
$y' = y\big(2x\ln(\sin x) + x^2 \cot x\big)$
$y' = (\sin x)^{x^2}\big(2x\ln(\sin x) + x^2 \cot x\big)$

Question 6. **(a)** *Find the linearization of* $f(x) = \sqrt[3]{1+3x}$ *at* $a = 0$*, and state the corresponding linear approximation.*
$f(x) = (1+3x)^{1/3} \Rightarrow f'(x) = (1+3x)^{-2/3}$*, so the linearization of* f *at* $a = 0$ *is* $L(x) = f(0) + f'(0)(x-0) = 1^{1/3} + 1^{-2/3}x = 1 + x$. *Thus,* $\sqrt[3]{1+3x} \approx 1 + x$.
(b) *Use part (a) to give an approximate value for* $\sqrt[3]{1.03}$.
$\sqrt[3]{1.03} = \sqrt[3]{1+3(0.01)} \approx 1 + 0.01 = 1.01.$

Question 7. *Evaluate the following limit by first recognizing the sum as a Riemann sum for a function defined on* $[0,1]$.

$$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt[3]{\frac{1}{n}} + \sqrt[3]{\frac{2}{n}} + \sqrt[3]{\frac{3}{n}} + \cdots + \sqrt[3]{\frac{n-1}{n}} + \sqrt[3]{\frac{n}{n}}\right)$$

$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt[3]{\frac{1}{n}} + \sqrt[3]{\frac{2}{n}} + \sqrt[3]{\frac{3}{n}} + \cdots + \sqrt[3]{\frac{n-1}{n}} + \sqrt[3]{\frac{n}{n}}\right) = \lim_{n\to\infty} \frac{1-0}{n}\sum_{i=1}^{n} \sqrt[3]{\frac{i}{n}} = \int_0^1 \sqrt[3]{x} = [\frac{3x^{4/3}}{4}]_0^1 = 3/4.$

Question 8. *Compute the derivative of the function given by*

$$g(x) = \int_1^{x^3} \sin(1-t^2)\, dt.$$

By the fundamental theorem of calculus we have

$$g'(x) = (x^3)' \sin(1-(x^3)^2) - (1)' \sin(1-(1)^2) = 3x^2 \sin(1-x^6).$$

Question 9. *Find the area of the region bounded by the graphs of* $f(x) = x^2 + 2$, $g(x) = -x$, $x = 0$, *and* $x = 1$.

Area $= \int_0^1 f(x) - g(x)\, dx = \int_0^1 x^2 + 2 - (-x)\, dx = \frac{1}{3}x^3 + 2x + \frac{1}{2}x^2\Big]_0^1 = \frac{1}{3} + 2 + \frac{1}{2} = \frac{2+12+3}{6} = \frac{17}{6}.$

Question 10. *Evaluate*

$$\int \frac{-4x}{(1-2x^2)^2}\, dx.$$

Using substitution, $u = 1 - 2x^2$, we have $du = -4x$ and

$$\int \frac{-4x}{(1-2x^2)^2}\, dx = \int \frac{1}{u^2}\, du = \int u^{-2}\, du = \frac{1}{-2+1}u^{-2+1} + C = -\frac{1}{1-2x^2} + C.$$

Question 11. *Evaluate*

$$\int \frac{1}{x\big(1+(\ln x)^2\big)}\, dx$$

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We use an integration by substitution. Substitute

$$u = \ln(x) \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x du\,.$$

Also, we have

$$\begin{array}{c|c|c} x & 1 & 2 \\ \hline u & \ln(1)=0 & \ln(2) \end{array}.$$

Therefore, we get

$$\int_1^2 \frac{1}{x\big(1+(\ln x)^2\big)}\,dx = \int_0^{\ln(2)} \frac{1}{1+u^2}\,du = [\arctan(u)]_0^{\ln(2)} = \arctan(\ln(2))\,.$$

Question 12. *Evaluate*

$$\int_0^{\pi/2} \sin x\, \cos(\cos x)\, dx$$

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Let $u = \cos x$, so $du = -\sin x\, dx$. When $x = 0$, $u = 1$; when $x = \pi/2$, $u = 0$. Thus, $\int_0^{\pi/2} \sin x\, \cos(\cos x)\, dx = \int_1^0 (-\cos u)\, du = [-\sin u]_1^0 = -\sin 0 - (-\sin 1) = \sin 1$.

Problem	5(10 pts.)	6(10 pts.)	7(10 pts.)	8(10 pts.)	9(10 pts.)	10(10 pts.)	11(10 pts.)	12(10 pts.)
Your marks								

Student Last,First Name:________________________________ Total marks: ___________ out of 100