3.8 Derivative of Log cont

1) $(\ln x)' = \frac{1}{x}$

2) $(\ln f(x))' = \frac{f'(x)}{f(x)}$

$(\ln|x|)' = \frac{1}{x}$

3) $(\log_a x)'$ $\quad \log_a x = \frac{\log_e x}{\log_e a}$ or $\frac{\log_e x}{\log_e a}$

$$(\log_a x)' = \left(\frac{\ln x}{\ln a}\right)' = \frac{1}{\ln a}(\ln x)' = \frac{1}{\ln a}\frac{1}{x}$$

$$\boxed{= \frac{1}{x \ln a}}$$

4) $(\log_a |x|)' = \frac{1}{x \ln a}$

5) $(\log_a f(x))' = \frac{f'(x)}{f(x) \ln a}$

Ex: $[\ln(\sin x)]' = \frac{\sin' x}{\sin x} = \frac{\cos x}{\sin x} = \cot x$

Ex: $[\log_3(x^2+1)]' = \frac{(x^2+1)'}{x^2+1 \ln 3} = \frac{2x}{(x^2+1)\ln 3}$

Logrithmic Differentiation

Ex: $y = \frac{x^2+2x+4}{(x^3+x+1)\sqrt{4x^2+9}}$ find y' by using logrithmic differentiation

1) take log of both sides ($\log_e$) is easiest AKA $\ln x$

$$\ln y = \ln \frac{x^2+2x+4}{(x^3+x+1)\sqrt{4x^2+9}}$$

$$= \ln(x^2+2x+4) - \ln(x^3+x+1) - \ln(\sqrt{4x^2+9})$$

$$\ln(4x^2+9)^{1/2}$$

$$\frac{1}{2}\ln(4x^2+9)$$

left side will always look the same

2) * Differentiate both sides using implicit differentiation

$$\frac{y'}{y} = \frac{2x+2}{x^2+2x+4} - \frac{3x^2+1}{x^3+x+1} - \frac{1}{2}\frac{8x}{4x^2+9}$$

3) Set equation equal to y'

$$y' = y\left(\frac{2x+2}{x^2+2x+4} - \frac{3x^2+1}{x^3+x+1} - \frac{4x}{4x^2+9}\right)$$

Ex: Find y' ① $y = x^x$ * careful, can't use power function *

$$\ln y = \ln x^x \Rightarrow \ln y = x\ln x$$

$$\frac{y'}{y} = \ln x + x\left(\frac{1}{x}\right)$$

$$= \ln x + 1$$

$$y' = y(\ln x + 1) = y' = x^x(\ln x + 1)$$

2) $y = x^{\cos x}$

$$\ln y = \ln x^{\cos x}$$

$$\ln y = (\cos x)(\ln x)$$

$$\frac{y'}{y} = (-\sin x)(\ln x) + \left(\frac{1}{x}\right)(\cos x)$$

$$y' = y\left(-\sin x \ln x + \frac{1}{x}\cos x\right)$$

$$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \lim_{h\to 0}(1+h)^{1/h}$$

3.9 Derivative of Inverse Trig Functions

$$[f^{-1}(x)]' = \frac{1}{f'(f^{-1}(x))}$$

* remember: composition *

① $\boxed{\begin{matrix}(\arcsin x)' \\ (\sin^{-1} x)'\end{matrix} = \frac{1}{\sqrt{1-x^2}}}$

Proof: Let $f(x) = \sin x$, $f^{-1}(x) = \arcsin x$

$$f'(x) = \cos x \Rightarrow f'(f^{-1}(x))$$
$$= \cos(\arcsin x)$$
$$= \sqrt{1-\sin^2(\arcsin x)}$$
$$= \sqrt{1-x^2}$$

② $\boxed{(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}}$

③ $\boxed{(\arctan x)' = \frac{1}{1+x^2}}$

<u>Ex1</u>: $[\arcsin(2x)]' = \frac{(2x)'}{\sqrt{1-(2x)^2}} = \frac{2}{\sqrt{1-4x^2}}$

Ex2: $\left[\arccos\frac{2+x}{3-x}\right]' = -\frac{\left(\frac{2+x}{3-x}\right)'}{\sqrt{1-\left(\frac{2+x}{3-x}\right)^2}} = \frac{(3-x)-(-2x)}{(3-x)^2\sqrt{1-\left(\frac{2+x}{3-x}\right)^2}}$

$$= \frac{-5}{(3-x)^2\sqrt{1-\left(\frac{2+x}{3-x}\right)^2}}$$

Ex 3 $[\sin(\arctan x)]'$

$= \cos(\arctan x)\,[\arctan x]'$

$= \dfrac{\cos(\arctan x)}{1+x^2}$

3.11 Linearization & Differentials

Nov 2/12

$$f'(x) = \frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

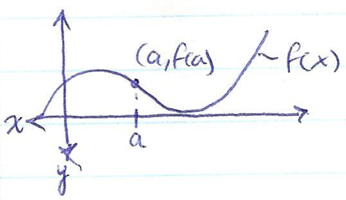
$\approx$ is approximation

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

$$f'(x)h \approx f(x+h) - f(x)$$

$$\boxed{f(x+h) \approx f(x) + f'(x)h}$$

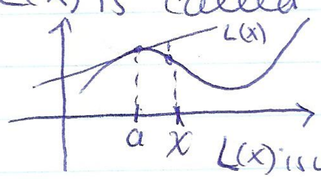
$f(x)$ is a function when $x = a$
the y-value at $f(x+h)$ can be estimated by $f(x) + f'(x)h$.



let $x = a$ $\quad f(a+h) \approx f(a) + f'(a)h$ $\quad \exists$ linear function $\therefore$ called linearization

Definition: Let $L(x) = f(a) + f'(a)(x-a)$
(tangent line approximation)

$L(x)$ is called Linearization of $f(x)$ at $x = a$



$L(x)$ is used to approximate y-value at x.
approximation is not good when x-value is far from a