

University of Ottawa
MAT 1332B Practice Midterm Exam
Duration: 80 minutes.

Family Name: _____

First Name: _____

Do **not** write your student ID number on this front page. Please write your student ID number in the space provided on the second page.

Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 80 minutes to complete this exam.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted**.
- Only the Faculty approved calculators (TI-30X, TI-34X, Casio FX-260X and Casio FX-300X) are allowed.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Where it is possible to check your work, do so.
- Good Luck!

Student number: _____, Total marks: _____ out of 30

Problem	1	2	3	4	5	6
Marks						

Question 1. Consider the following matrix:

$$A = \begin{bmatrix} -4 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & 4 & 2 \end{bmatrix}.$$

- (a) Calculate the determinant and explain why the matrix is invertible. (One short sentence is enough.)
- (b) Find A^{-1} .
- (c) Solve the equation $Ax = [6 \ 6 \ 6]^T$.
- (d) Show that $\lambda_1 = -4$ is an eigenvalues of A and find the other two eigenvalues.
- (e) Find the eigenvectors corresponding to $\lambda_1 = -4$.

Question 2. Consider the system of linear equations

$$\begin{aligned} x + ay &= 1 \\ b + 5y &= 2 \end{aligned}$$

where a and b are parameters.

- (a) Determine the conditions on a and b to get a unique solution.
- (b) Determine the conditions on a and b to get infinitely many solutions.
- (c) Determine the conditions on a and b such that the system has no solutions.

Question 3. Find the complete solution to the system of equations

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 3 \\ x_1 + 2x_2 - 3x_3 &= -2 \\ 3x_1 + 5x_2 + x_3 &= 1 \end{aligned}$$

Be sure to show all your work.

Question 4. Consider the following linear system of differential equations

$$\begin{aligned} \frac{d}{dt}x &= x + 5y, \\ \frac{d}{dt}y &= x - 3y. \end{aligned}$$

- (a) Rewrite the system in matrix form, i.e., $\frac{d}{dt} \begin{bmatrix} x \\ y \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix}$.
- (b) Find the eigenvalues and the eigenvectors of A .
- (c) Find the general solution of the system.
- (d) Find the particular solution with initial conditions $x(0) = 11, y(0) = 1$.
- (e) Will all solutions converge to zero?

Question 5. Let A be the matrix

$$A = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$$

- (a) Show that the eigenvalues of A are $4 + i$ and $4 - i$.
- (b) Find an eigenvector corresponding to the eigenvalue $4 + i$.

Question 6.

- There are three types of trees in the Princeton experimental forest: Red Maple (R), Beech (B), and Grey Birch (G). If a Red Maple dies, it is being replaced by a Red Maple in 70% of the cases. In the other 30%, it is being replaced by Beech or Grey Birch with equal probability. Beech and Grey Birch are both replaced by one of their own in 50% of the cases. In 40%, Red Maple takes over, and in the remaining 10% the other of the three. Write down the transition matrix for this Markov Chain. Use the vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ to denote the probability of a tree being R, B, G, respectively, in this order.
- Now suppose that a disease kills all the Grey Birch trees in the forest. For the remaining Red Maple and Beech, the transition matrix is given by

$$P = \begin{bmatrix} 0.6 & 0.1 \\ 0.4 & 0.9 \end{bmatrix},$$

where 0.6 is the probability that R is replaced by R.

- Assume that one finds a Red Maple at certain site now. What is the probability that there is a Red Maple at the same site in two generations?
- What is the probability that the site is occupied by Red Maple in the long run?