Question 1 [7 marks]

The number of withdrawals at a bank machine was monitored during randomly chosen 10 minute intervals over the past month. Appendix A gives the data distribution and various analyses.

a. Suppose you were to test whether the average number of withdrawals exceeded 3. Identify the most appropriate test, and explain briefly.

[1] The boxplot shows a reasonably symmetric distribution with no outliers. It is reasonable to assume a normal distribution for the pop. Data.

b. Perform the hypothesis test that you selected above.

[3] Ho: mu = 3; Ha: mu > 3;
T = (3.76 – 3) / (1.73877/sqrt(25)) = .76/.34775 = 2.185
Based on 24 d.f. the critical value is 1.71.
Since 2.185 > 1.71, we reject Ho at the .05 level
Conclude the average number of withdrawals exceeds 3.

c. What is the p-value of the test statistic.

[1] Based on 24 d.f. the p-value is between .025 and .01.

d. Suppose you wanted to estimate the average number of withdrawals with a margin of error of ±.5 and a 95% confidence level. What sample size would be required?

n = (1.96*1.73877/ .5)^2 = 47 (rounded up)

[2]

Question 2 [7 marks]

An undercover investigative team decides to determine whether it costs more to have a moderately damaged car repaired at *Jo's Trusty Mechanic shop* or *Uncle Bob's Honest Mechanics*.

a) Design an experiment and an appropriate hypothesis test that would *best* help answer this question. You may assume that the cost for fixing moderately damaged cars is normally distributed. Note that there are at least two types of experiments that would work here but one is clearly better than the other. Justify your choice.

[2] Define moderately damaged more specifically to be a range of repair costs. Collect a random sample of such cars and get an estimate both from Jo's and Uncle Bob's. Compare the mean of the differences using a Matched Pairs test to see if there is a statistically significant difference in costs.

b) The data collected are firm estimates (in thousands of dollars) from the two body shops:

7.1	7.9
9.0	10.1
11.0	12.2
8.9	8.8
9.9	10.4
9.1	9.8
10.3	11.7

The first column represents the estimates from *Jo's Trusty Mechanic shop* and the second column represents the estimates from *Uncle Bob's Honest Mechanics.* We will assume that the repair costs for moderately damaged cars are normally distributed. Perform the hypothesis test you determined was appropriate in part (a). Use a 5% significance level and state any additional assumptions required.

[4] The hypothesis test is
$H_0: \mu_d = 0$; $H_a: \mu_d \neq 0$

The mean of the differences is -0.8 and the standard deviation of the differences is 0.5033 so the test statistic is

$$t = \frac{-0.8-0}{0.5033/\sqrt{7}} = -4.2053$$

Since the absolute value of t is greater than the critical value of $t_{0.025,6} = 2.447$, we reject the null and conclude that there is significant evidence at the 5% level that the two mechanic companies do not charge the same amount to repair moderately damaged cars.

1 for hypotheses, 1 for mean and stdev, 1 for t-statistic and critical value, 1 for decision and conclusion.

A 2-sample t-test would result in a t-statistic of -1.08 and a critical value of either 2.18 based on 12 d.f. assuming equal pop. Var. or 2.20 based on 11 d.f. not assuming equal pop. Var.

c) Explain what impact the normality assumption makes on your analysis and what you would do if you could not make this assumption. You do not need to do any calculations though!

[1] If you could not make the normality assumption, then since the sample size is

small, one would have to do a Wilcoxon signed rank test to determine if the median of the differences was different from zero.

Question 3. [7 marks]

Recent research in health sciences has shown that "Waist to Hip Ratio" (WHR) is an excellent indicator of the general health of an adult. It has been suggested that if the WHR of adult males is 0.9 or higher (0.8 or higher for adult females) the risk of morbidity increases significantly to "high".

In a random sample of 140 adult males, 56 were found to be "high risk" based on their WHR.

a. Test whether the data show that the proportion that are high risk is different from 30%. Use a 5% level of significance.

[3]

Ho: p = .30; Ha: p ≠ .30
Z = (.4 - .3) / sqrt(.3*.7/140) = 2.58
We reject Ho since | z | > 1.96, and conclude that the proportion differs from 30%

b. Find a 95% confidence interval for the proportion of adult males who are at high risk.

[2] .4 ± 1.96 * sqrt (.4*.6/140) = .4 +/- 1.96 * 0.041404

= .4 +/- .08 = (.32, .48)

c. If we wanted to estimate the proportion of high risk adult males with a margin of error of ± 3%, what sample size should be used?

[2] n = (.5*.5) * (1.96 / .03)^2 = 1067
or
N = (.4*.6) * (1.96 / .03)^2 = 1024

d. Ho: p = .3; Ha: p < .3
Prob (X <= 0) = .7^8 = 0.057648
Since the p-value is not < .05, we do not reject Ho and conclude there is insufficient evidence to conclude that the proportion is less than 30%

1 for hypotheses, 1 for p-value, 1 for decision and conclusion.
If they did a z-test, mark out of 1.5 total:
z = (0 - .3) / sqrt(.3*.7/8) = -.3/.162 = -1.85
Since z < -1.645, we reject the null hypothesis.

Question 4. [6 marks]

Two methods for etching semiconductor wafers are under consideration. The question is whether the etch rates of the two solutions are the same or not. Two random samples of size 100 were chosen and each sample was etched with one method. Appendix B shows a number of different Minitab outputs of *possible* tests for determining an answer to this question. Columns C1 and C2 represent method 1 and 2, respectively.

a. Choose the most appropriate test. Make sure to explain why you chose the test you did over the other options.

[2] The appropriate test in this case is a two sample t-test with the alternative being not equal to. This is due to the fact that we have a large sample so the normality assumption is valid and we have two independent samples from independent populations.

I assume students do not have to specify whether to assume equal or unequal variance in this question AND we accept either approach.

b. Perform the test you selected above. Use the .01 level of significance.

[4]

The hypothesis test is

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 \neq 0$$

the test statistic, based on the minitab output, is

$$t = -.17 / (.3961*\sqrt{(2/100)}) = -3.035$$

no change assuming unequal variance.

Using the z-table this yields a p-value of 2(.5 - .4988) = 2*.0012= .0024. Therefore we reject the null at the 1% significance level and conclude that there is a difference between the two etching techniques. Alternatively, the critical value is 2.57 or 2.58 and so reject since the absolute value of the test statistic is greater than critical value.

1 for hypotheses, 1.5 for t-statistic, .5 for critical value or p-value, 1 for decision and conclusion