Student number:____________________, Total marks: __________ out of 25

Problem	1	2	3	4	5
Marks					

Question 1. [2 points] Find the value of a so that the following function is continuous

$$f(x) = \begin{cases} \frac{x^2+x-2}{x-1} & x \neq 1 \\ a & x = 1. \end{cases}$$

Answer: $a =$ $\boxed{3}$

Justify your answer using limit laws.

① For continuity we need

$$\lim_{x \to 1^-} f(x) = f(1) = \lim_{x \to 1^+} f(x)$$

② When $x \neq 1$ then $f(x) = \dfrac{x^2+x-2}{x-1} = \dfrac{(x-1)(x+2)}{x-1} = x+2$

therefore $\lim_{x \to 1} f(x) = 3$

③ Therefore we need $a = 3$.

Question 2. [8 points] A patient receives a daily dose $d = 3.6$mg of the drug FilGud®. In the course of 24 hours, 60% of the drug is absorbed and a fraction of $p = 0.4$ remains in the blood. The DTDS modelling the daily concentration M_t of FilGud® in the blood immediately after administering the dose is

$$M_{t+1} = pM_t + d = 0.4M_t + 3.6.$$

(a) [0.5 point] The updating function of the DTDS is $f(x) =$ $\boxed{0.4x + 3.6}$

(b) [1 point] The equilibrium point of the DTDS is $M^* =$ $\boxed{6}$

$$M^* = 0.4M^* + 3.6$$

$$M^* = \frac{3.6}{1-0.4} = \frac{3.6}{0.6} = 6$$

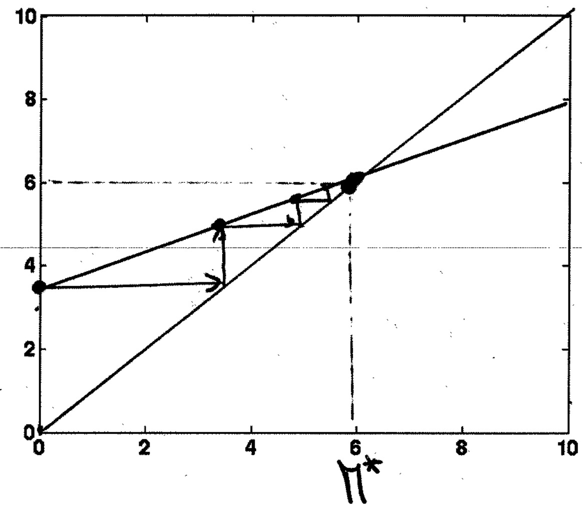
(c) [2 points] Give the general solution for the DTDS with general initial condition M_0:

$$M_t = \boxed{(0.4)^t (M_0 - 6) + 6}$$

(d) [0.5 point] Calculate M_5 if $M_0 = 0$. $M_5 =$ $\boxed{5.938}$

$$M_5 = (0.4)^5(-6) + 6$$

(e) [2 points] Graph the updating function and draw the cobweb diagram of the DTDS, starting from $M_0 = 0$ for at least 4 steps.



(f) [1 point] Is the equilibrium point stable or unstable? $\boxed{\text{stable, since } p < 1}$

(g) [1 point] Due to sudden complications, the patient now also needs to take the drug WelSun®. This drug inhibits the uptake of FilGud® so that only 50% will be absorbed and a fraction of $\tilde{p} = 0.5$ will remain in the blood. Calculate the new daily dose $\tilde{d}$ of FilGud® needed to maintain the equilibrium concentration of that drug at the same level as before.

$\tilde{d} = \boxed{3}$

New DTDS: $\tilde{M}_{t+1} = \tilde{p}\tilde{M}_t + \tilde{d}$ has equilibrium $\tilde{M}^* = \dfrac{\tilde{d}}{1-\tilde{p}}$

so $\tilde{d} = (1-\tilde{p})\tilde{M}^*$

The condition is that $\tilde{M}^* = \tilde{M}$, so $\tilde{d} = (1-\tilde{p})M^* = 0.5 \cdot 6 = 3$

Question 3. [4 points] (a) Use the definition of the derivative to calculate the derivative of the function

$$f(x) = \frac{x}{2+3x}.$$

Give as much detail as possible.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{x+h}{2+3(x+h)} - \frac{x}{2+3x}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{(x+h)(2+3x) - x(2+3(x+h))}{[2+3(x+h)]\cdot[2+3x]}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{2x+3x^2+2h+3hx - 2x-3x^2-3xh}{[2+3(x+h)]\cdot[2+3x]}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{2h}{[2+3(x+h)][2+3x]}\right)$$

$$= \lim_{h\to 0} \frac{2}{(2+3(x+h))(2+3x)} = \frac{2}{(2+3x)^2}$$

(b) Check your answer by using differentiation rules.

$$f'(x) = \boxed{\frac{(2+3x) - x\cdot 3}{(2+3x)^2} = \frac{2}{(2+3x)^2}}$$

Question 4. [6 points] Find the derivatives for the following functions. Do **not** simplify your answer.

(a) $f(x) = (4x^2 + 1)e^{-3x}$

$$f'(x) = \boxed{8x e^{-3x} - 3(4x^2+1)e^{-3x}}$$

product rule

(b) $g(s) = \frac{\sin(3s+1)}{\cos(3s)}$ quotient rule

$$g'(s) = \boxed{\frac{3\cos(3s+1)\cos(3s) + 3\sin(3s+1)\sin(3s)}{\cos^2(3s)}}$$

(c) $h(y) = \ln\left(\frac{y^{23}}{\sqrt{10y+2013}}\right) = \ln(y^{23}) - \ln\left(\sqrt{10y+2013}\right) = 23\ln y - \frac{1}{2}\ln(10y+2013)$

$$h'(y) = \boxed{\frac{23}{y} - \frac{1}{2}\,\frac{10}{10y+2013}}$$

Question 5. [5 points]
Consider the following function

$$f(x) = \sqrt{3x}\, e^{-x/6}$$

(a) The domain of definition is: $\boxed{x \geq 0}$

$$f'(x) = \frac{3}{2\sqrt{3x}} e^{-x/6} - \frac{1}{6}\sqrt{3x}\, e^{-x/6} = \sqrt{3x}\, e^{-x/6}\left[\frac{3}{6x} - \frac{1}{6}\right]$$

(b) The critical point(s) is (are): $\boxed{x=0 \ (f' \text{ not defined}),\ x=3 \ (f'=0)}$

(c) f is increasing in the interval: $\boxed{0 \leq x \leq 3}$

(d) f is decreasing in the interval: $\boxed{x \geq 3}$

(e) Use a table of values to guess the horizontal asymptote.

horizontal asymptote: find $\lim_{x \to \infty} f(x)$

so: choose large x

x	10	100	1000	10 000
$f(x)$	1.0345	$1 \cdot 10^{-6}$	$2 \cdot 10^{-71}$	0

The horizontal asymptote is $y =$ $\boxed{0}$

Student number:____________________, Total marks: __________ out of 25

Problem	1	2	3	4	5
Marks					

Question 1. [2 points] Find the value of a so that the following function is continuous

$$f(x) = \begin{cases} \frac{x^2-x-2}{x-2} & x \neq 2 \\ a & x = 2. \end{cases}$$

Answer: $a =$ $\boxed{3}$

Justify your answer using limit laws.

① $x \neq 2 \Rightarrow f(x) = \frac{x^2-x-2}{x-2} = \frac{(x-2)(x+1)}{x-2} = x+1$

② $\lim_{x \to 2} f(x) = 2+1 = 3$

③ $\lim_{x \to 2} f(x) = f(2) = a$ for continuity, so $a = 3$

Question 2. [8 points] A patient receives a daily dose $d = 3.5$mg of the drug FilGud®. In the course of 24 hours, 70% of the drug is absorbed and a fraction of $p = 0.3$ remains in the blood. The DTDS modelling the daily concentration M_t of FilGud® in the blood immediately after administering the dose is

$$M_{t+1} = pM_t + d = 0.3M_t + 3.5.$$

(a) [0.5 point] The updating function of the DTDS is $f(x) =$ $\boxed{0.3x + 3.5}$

(b) [1 point] The equilibrium point of the DTDS is $M^* =$ $\boxed{5}$

$$M^* = 0.3M^* + 3.5$$

$$M^* = \frac{3.5}{1-0.3} = \frac{3.5}{0.7} = 5$$

(c) [2 points] Give the general solution for the DTDS with general initial condition M_0:

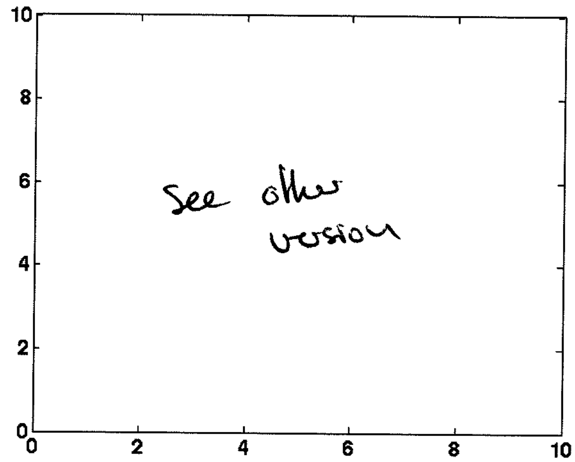
$$M_t = \boxed{(0.3)^t(M_0 - 5) + 5}$$

(d) [0.5 point] Calculate M_5 if $M_0 = 0$.

$M_5 =$ $\boxed{4.9878}$

$$(0.3)^5(-5) + 5$$

(e) [2 points] Graph the updating function and draw the cobweb diagram of the DTDS, starting from $M_0 = 0$ for at least 4 steps.



(f) [1 point] Is the equilibrium point stable or unstable? $\boxed{\text{stable}}$

(g) [1 point] Due to sudden complications, the patient now also needs to take the drug WelSun®. This drug inhibits the uptake of FilGud® so that only 50% will be absorbed and a fraction of $\tilde{p} = 0.5$ will remain in the blood. Calculate the new daily dose $\tilde{d}$ of FilGud® needed to maintain the equilibrium concentration of that drug at the same level as before.

$\tilde{d} = \boxed{2.5}$

$$\tilde{d} = (1-\tilde{p}) \cdot M^* = 0.5 \cdot 5 = 2.5$$

Question 3. [4 points] (a) Use the definition of the derivative to calculate the derivative of the function

$$f(x) = \frac{x}{4+3x}.$$

Give as much detail as possible.

See other version

(b) Check your answer by using differentiation rules.

$$f'(x) = \boxed{\frac{4+3x-3x}{(4+3x)^2} = \frac{4}{(4+3x)^2}}$$

Question 4. [6 points] Find the derivatives for the following functions. Do **not** simplify your answer.

(a) $f(x) = (3x^2 + 1)e^{-5x}$

$f'(x) = \boxed{6x e^{-5x} - 5(3x^2+1)e^{-5x}}$

(b) $g(s) = \frac{\sin(2s+5)}{\cos(2s)}$

$g'(s) = \boxed{\dfrac{2\cos(2s+5)\cos(2s) + 2\sin(2s+5)\sin(2s)}{\cos^2(2s)}}$

(c) $h(y) = \ln\left(\frac{y^{31}}{\sqrt{10y+2013}}\right)$

$h'(y) = \boxed{\dfrac{31}{y} - \dfrac{1}{2}\,\dfrac{10}{10y+2013}}$

Question 5. [5 points]
Consider the following function

$$f(x) = \sqrt{3x}\, e^{-x/8}$$

(a) The domain of definition is: $\boxed{x \geq 0}$

$$f'(x) = \frac{3}{2\sqrt{3x}} e^{-x/8} - \frac{1}{8}\sqrt{3x}\, e^{-x/8} = \sqrt{3x}\, e^{-x/8}\left[\frac{3}{6x} - \frac{1}{8}\right]$$

(b) The critical point(s) is (are): $\boxed{x=0, \quad x=4}$

(c) f is increasing in the interval: $\boxed{0 \leq x \leq 4}$

(d) f is decreasing in the interval: $\boxed{x \geq 4}$

(e) Use a table of values to guess the horizontal asymptote.

x	10	100	1000	10 000
$f(x)$	1.569	$6 \cdot 10^{-5}$	$3 \cdot 10^{-53}$	0

The horizontal asymptote is $y = \boxed{0}$

Student number:__________________, Total marks: __________ out of 25

Problem	1	2	3	4	5
Marks					

Question 1. [2 points] Find the value of a so that the following function is continuous

$$f(x) = \begin{cases} \frac{x^2+x-6}{x-2} & x \neq 2 \\ a & x = 2. \end{cases}$$

Answer: $a =$ 5

Justify your answer using limit laws.

① If $x \neq 2$ then $f(x) = \frac{x^2+x-6}{x-2} = \frac{(x-2)(x+3)}{x-2} = x+3$

② Therefore $\lim_{x \to 2} f(x) = 2+3 = 5$

③ For continuity need $\lim_{x \to 2} f(x) = f(2) = a$ So $a = 5$

Question 2. [8 points] A patient receives a daily dose $d = 4$mg of the drug FilGud®. In the course of 24 hours, 80% of the drug is absorbed and a fraction of $p = 0.2$ remains in the blood. The DTDS modelling the daily concentration M_t of FilGud® in the blood immediately after administering the dose is

$$M_{t+1} = pM_t + d = 0.2M_t + 4.$$

(a) [0.5 point] The updating function of the DTDS is $f(x) =$ $\boxed{0.2x + 4}$

(b) [1 point] The equilibrium point of the DTDS is $M^* =$ $\boxed{5}$

$$M^* = 0.2M^* + 4$$

$$M^* = \frac{4}{1-0.2} = \frac{4}{0.8} = 5$$

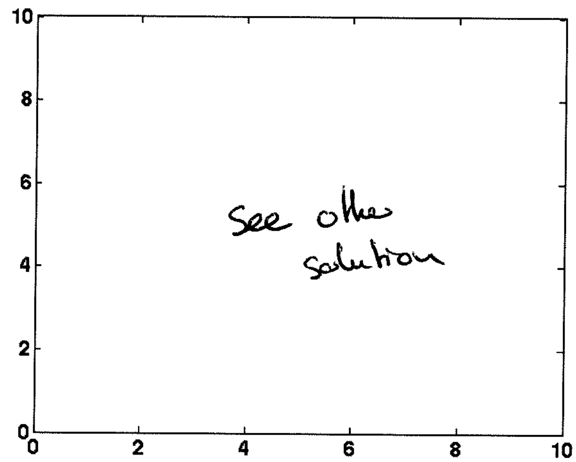
(c) [2 points] Give the general solution for the DTDS with general initial condition M_0:

$$M_t = \boxed{(0.2)^t (M_0 - 5) + 5}$$

(d) [0.5 point] Calculate M_5 if $M_0 = 0$. $M_5 =$ $\boxed{4.9984}$

$$M_5 = (0.2)^5(-5) + 5$$

(e) [2 points] Graph the updating function and draw the cobweb diagram of the DTDS, starting from $M_0 = 0$ for at least 4 steps.



(f) [1 point] Is the equilibrium point stable or unstable? $\boxed{\text{Stable}}$

(g) [1 point] Due to sudden complications, the patient now also needs to take the drug WelSun®. This drug inhibits the uptake of FilGud® so that only 50% will be absorbed and a fraction of $\tilde{p} = 0.5$ will remain in the blood. Calculate the new daily dose $\tilde{d}$ of FilGud® needed to maintain the equilibrium concentration of that drug at the same level as before.

$\tilde{d} = \boxed{2.5}$

$$\tilde{d} = (1-\tilde{p}) \cdot M^* = 0.5 \cdot 5 = 2.5$$

Question 3. [4 points] (a) Use the definition of the derivative to calculate the derivative of the function

$$f(x) = \frac{x}{3+5x}.$$

Give as much detail as possible.

See other version

(b) Check your answer by using differentiation rules.

$$f'(x) = \boxed{\frac{3+5x \quad - 5x}{(3+5x)^2} =} \quad \frac{3}{(3+5x)^2}$$

Question 4. [6 points] Find the derivatives for the following functions. Do **not** simplify your answer.

(a) $f(x) = (4x^2+1)e^{-2x}$

$f'(x) = \boxed{8xe^{-2x} - 2(4x^2+1)e^{-2x}}$

(b) $g(s) = \frac{\sin(6s+5)}{\cos(6s)}$

$g'(s) = \boxed{\dfrac{6\cos(6s+5)\cos(6s) + 6\sin(6s+5)\sin(6s)}{\cos^2(6s)}}$

(c) $h(y) = \ln\left(\frac{y^{20}}{\sqrt{9y+1969}}\right)$

$h'(y) = \boxed{\dfrac{20}{y} - \dfrac{1}{2}\,\dfrac{9}{9y+1969}}$

Question 5. [5 points]
Consider the following function

$$f(x) = \sqrt{7x}\, e^{-x/2}$$

(a) The domain of definition is: $\boxed{x \geq 0}$

$$f'(x) = \frac{7}{2\sqrt{7x}} e^{-x/2} - \frac{1}{2}\sqrt{7x}\, e^{-x/2} = \sqrt{7x}\, e^{-x/2}\left[\frac{7}{14x} - \frac{1}{2}\right]$$

(b) The critical point(s) is (are): $\boxed{x = 0, \quad x = 1}$

(c) f is increasing in the interval: $\boxed{0 \leq x \leq 1}$

(d) f is decreasing in the interval: $\boxed{x \geq 1}$

(e) Use a table of values to guess the horizontal asymptote.

x	10	100	1000	10000
$f(x)$	0.056	$5 \cdot 10^{-21}$	$6 \cdot 10^{-216}$	0

The horizontal asymptote is $y = \boxed{0}$