

MATH 1119A Practice Test 4

This test paper has two parts. Total of 40 marks. Duration: 50 Minutes.

Part I has 3 multiple choice questions.

Part II has 3 long answer questions.

This test paper cannot be taken from the examination room.

Only nonprogrammable calculators are allowed.

NAME :

STUDENT NO :

**PART I: Multiple Choice Questions. Circle the correct answer. Four marks each. No partial marks.**

1) A Markov Chain has the transition matrix  $T = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$  and the initial state vector  $X_0 = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}$ . What is the second state vector, i.e, what is  $X_2$ ?

a)  $X_2 = \begin{bmatrix} 0.52 \\ 0.48 \end{bmatrix}$     b)  $X_2 = \begin{bmatrix} 0.464 \\ 0.536 \end{bmatrix}$     c)  $X_2 = \begin{bmatrix} 0.59 \\ 0.41 \end{bmatrix}$     d)  $X_2 = \begin{bmatrix} 0.513 \\ 0.487 \end{bmatrix}$

2) The territory of a certain republic consists of two islands, Green Island and Brown Island. Every year 20% of Green Island residents migrate to Brown Island, while 10% of Brown Island residents migrate to Green Island. It is determined that the total population of the republic remains constant from year to year. What is the transition matrix?

a)  $\begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$     b)  $\begin{bmatrix} 0.8 & 0.9 \\ 0.2 & 0.1 \end{bmatrix}$     c)  $\begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$     d)  $\begin{bmatrix} 0.8 & 0.8 \\ 0.2 & 0.2 \end{bmatrix}$

3) A Markov Chain has the transition matrix  $T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$  and the initial state vector  $X_0 = \begin{bmatrix} 3/4 \\ 1/4 \end{bmatrix}$ . Which of the following statements is FALSE?

a)  $\begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$  is a steady-state vector for this Markov Chain.

b) The transition matrix  $T$  of this Markov Chain is not regular.

c)  $\begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$  is a long term trend for this Markov Chain.

d)  $X_{19} = \begin{bmatrix} 1/4 \\ 3/4 \end{bmatrix}$ .

MATH 1119A Practice Test 4

**PART II: Long answer questions. Show all your work.**

[10] 1) Use Cramer's Rule to find  $y$ , where

$$\begin{aligned}x + y - z + 3w &= 0 \\x + y + z + 2w &= 1 \\-4x + 2y - z - w &= 0 \\y + 2z - w &= 0\end{aligned}$$

[8] 2) Find the equation  $y = \beta_0 + \beta_1x$  of the least-squares line that best fits the data points  $(5, 1)$ ,  $(2, 3)$ ,  $(6, 0)$ ,  $(3, 2)$ .

[10] 3) Let the transition matrix of a Markov Chain be  $T = \begin{bmatrix} 1/4 & 2/5 \\ 3/4 & 3/5 \end{bmatrix}$ .

a) Find the steady state vector for  $T$ .

b) Does this Markov chain converge to a long term trend? If so, find it. If not, explain.