CHAPTER 5

<u>Continuous Probability Distribution</u>

The curve $f(x)$ is called continuous probability distribution of continuous random variable x if $P(a \le x \le b)$ equals the area under the curve $f(x)$ and limited to $x = a$ and $x = b$.

$P(a \le x \le b) = P(a < x \le b) = P(a \le x < b) = P(a < x < b)$.

This is true because:

- $P(x = a) = P(x = b) = 0$
- $P(x = k) = 0$

Properties of $f(x)$:

I. For any x, $f(x) \ge 0$

II. $P(a \le x \le b)$ equals the area under $f(x)$ and limited by $x = a \text{ and } x = b$

<u>Uniform Distribution on</u> $[a,\ b]$

The random variable is evenly distributed on $[a,\ b]$

$f(x) = \left\{\frac{1}{b-a},\ a \le x \le b,\ otherwise = 0\right\}$

Mean and variance of uniform $(a,\ b)$:

- $\mu_x = \frac{a+b}{2}$
- $\sigma_x{}^2 = \frac{(b-a)^2}{12}$
- $\sigma_x = \frac{(b-a)}{\sqrt{12}}$

<u>The Normal Distribution</u>

$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2},\ -\infty < x < \infty,\ x \text{ has } N(\mu, \sigma^2)$

$N(0, 1)$ is called the standard normal distribution.

The effect of S.D on the curve of normal distribution:

- The higher the S.D or variance the more the distribution is spread out

Transforming x that has $N(\mu, \sigma^2)$ to Z had $N(0, 1)$:

- $Z = \frac{x-\mu}{\sigma},\ Z\ has\ N(0,1)$
- Example:
    - x has $N(-5,2)$
      $Z = \frac{x+5}{\sqrt{2}}\ has\ N(0,1)$

$x\ has\ N(\mu,\sigma^2),\ what\ is\ P(x \le a):$

1. $P(x \le a) = P(\frac{x-\mu}{\sigma} \le \frac{a-\mu}{\sigma}) = P(Z \le \frac{a-\mu}{\sigma})$
2. Find $P(Z \le \frac{a-\mu}{\sigma})$ in the standard normal table

$P(x \ge a) = P(x \le -a)$

<u>Normal Approximation to Binomial</u> (n,p)

1. n should be large
2. $np \ge 5,\ n(1-p) \ge 5$

$P(x \le k) \approx P(Z \le \frac{k+0.5-np}{\sqrt{np(1-p)}})$

<u>Exponential Distribution</u>

The random variable x has exponential distribution with parameters λ if its probability curve is:

- $f(x) = \{\lambda e^{-\lambda x}\ if\ x \ge 0,\ 0\ otherwise$

x has E(λ)

Properties of exponential (λ):

1. $P(a \le x \le b) = e^{-\lambda a} - e^{-\lambda b}$
2. $P(x > c) = e^{-\lambda c}$
3. Mean of $x = \frac{1}{\lambda}$
   Variance of $x = \frac{1}{\lambda^2}$
   Standard deviation of $x = \frac{1}{\lambda}$

Memoryless property of the exponential distribution:

- $P(x \ge a+b | x \ge a) = P(x \ge b)$