CHAPTER 4

DISCRETE RANDOM VARIABLES

Random variable is a variable that assumes numerical values that are determined by the outcome of an experiment.:

- Discrete random variables assume countable values
- Continuous random variables assume uncountable values.

Represent random variable with x, y, z, … Probability distribution of discrete random variable is a graph, table or formula, that give the probability associated with each value of the random variable.

Properties of discrete random variables:

1. For each x, p(x) ≥ 0
2. $\sum_{All\ x} p(x) = 1$

Example:

- Let x be the number of computers sold in one day in certain shop
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x	0	1	2	3	4
p(x)	1/10	1/10	4/10	3/10	? = 1/10

Mean, expected value variance and standard deviation of a random variable x:

- $u_x = \sum_{All\ x} x\, p(x)$
- $\sigma_x^2 = \sum_{All\ x} (x - u_x)^2\, p(x) = \sum_{All\ x} x^2\, p(x) - u_x^2$
- $\sigma_x = \sqrt{\sigma_x^2}$

<u>Binomial Distribution</u>

Binomial experiment:

- Consists of n trial
- Each trial can result in either S or F
- The probability of success should remain the same

- The trial should be independent

$P(x) = \binom{n}{x} p^x (1-p)^{n-x},\ x = 0, 1, \ldots, n$

n is the number of trials and 0 < P < 1 is the probability of S. x has a binomial (n, p)

Example:

- Shoot a gun 10 times to hit a target. The probability of S is 0.3. What is the probability that the target is hit twice? x = the number of times the target is hit in 10 times shooting the gun.
  - $P(2) = \frac{10!}{2!8!}(0.3)^2(0.7)^8 = 45(0.09)(0.05764801) \approx 0.2335 = 23.35\%$
- The probability that the person will hit the target at least twice?
  - $P(x \geq 2) = 1 - P(x \leq 1) = 1 - (P(0) + P(1))$

<u>Cumulative Distribution Function</u>

$F(k) = P(x \leq k) = 1 - P(x > k)$

Mean (or expected value), variance, and standard deviance of x which has binomial (n, p):

- $\mu = np$
- $\sigma_x^2 = np(1-p)$
- $\sigma_x = \sqrt{\sigma_x^2} = \sqrt{np(1-p)}$

<u>The Poisson Distribution</u>

Considers the number of events over an interval of time or space with average μ. Probability distribution function of x which has poisson (μ):

- $P(x) = \frac{e^{-\mu}\mu^x}{x!},\ x = 0, 1,\ \ldots\ and\ \mu > 0$

Mean (or expected value), variance, and standard deviance:

- $\mu_x = \mu$
- $\sigma_x^2 = \mu$
- $\sigma_x = \sqrt{\sigma_x^2} = \sqrt{\mu}$

<u>Hypergeometric</u>

$P(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$, where N is the total events, r is one variable, N-r is the second variable, and n is the sample size without replacement.

Mean (or expected value), variance, and standard deviance:

- $\mu_x = n\left(\frac{r}{N}\right)$

- $\sigma_x{}^2 = n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$
- $\sigma_x = \sqrt{\sigma_x{}^2} = \sqrt{n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)}$