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Student # [redacted] MAT 1302A First Midterm Test – Blue Version

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QUESTION 1. **[2 points]** Which of the following statements are true? Note that more than one statement may be true. You should indicate *all* the true statements. (You will lose points for indicating that false statements are true, but you cannot receive a negative score on this question.)

(a) The span of two nonzero parallel vectors in $\mathbb{R}^3$ is a line.
(b) A homogeneous systems always has infinitely many solutions.
(c) A matrix may have more than one reduced echelon form.
(d) If the coefficient matrix of a linear system is a 4×5 matrix with 4 pivot columns, then the system is consistent.
(e) If one of the rows of an echelon form of an augmented matrix is of the form $[0\ 0\ 0\ 1\ 0]$, then the associated linear system must be consistent.

Answer: e, a

$\boxed{1}$

QUESTION 2. Calculate the following.

1 (a) **[1 point]** $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + 3 \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ 18 \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \\ 21 \end{bmatrix}$ ✓

0 (b) **[2 points]** $\begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \cdot 1 & 2 \cdot 4 & 0 \cdot 0 \\ 0 \cdot 1 & 1 \cdot 4 & -2 \cdot 0 \end{bmatrix} = \begin{bmatrix} -1 & 8 & 0 \\ 0 & 4 & 0 \end{bmatrix}$ X

$$\begin{array}{r} 32 \\ -18 \\ \hline 14 \end{array}$$

$\boxed{5}$

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QUESTION 3. **[5 points]** Find all the solutions to the following linear system.

$$\begin{array}{rcrcrcr} & & x_2 & + & 2x_3 & = & 6 \\ x_1 & + & & + & 3x_3 & = & 6 \\ 4x_1 & + & -3x_2 & + & 8x_3 & = & 14 \\ 2x_1 & - & 3x_2 & + & 2x_3 & = & 2 \end{array}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 2 & 6 \\ 1 & 0 & 3 & 6 \\ 4 & -3 & 8 & 14 \\ 2 & -3 & 2 & 2 \end{array}\right] \underset{R_1 \leftrightarrow R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & 2 & 6 \\ 4 & -3 & 8 & 14 \\ 2 & -3 & 2 & 2 \end{array}\right] \underset{-2R_1+R_4}{\overset{-4R_1+R_3}{\sim}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & 2 & 6 \\ 0 & -3 & -4 & -10 \\ 0 & -3 & -4 & -10 \end{array}\right]$$

$$\underset{3R_2+R_4}{\overset{3R_2+R_3}{\sim}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 2 & 8 \end{array}\right] \overset{-R_3+R_4}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right] \overset{1/2 R_3}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\underset{-3R_3+R_1}{\overset{-2R_3+R_2}{\sim}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{\begin{array}{l} \therefore x_1 = -6 \\ x_2 = -2 \\ x_3 = 4 \end{array}}$$

<u>Check</u>

$4(-6) - 3(-2) + 8(4) = 14$

$-24 + 6 + 32 = 14$

$-18 + 32 = 14$

$14 = 14$

*

$1 \cancel{A} -2 -1 = -2$

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QUESTION 4. [**4 pts**] Consider the vectors

$$\vec{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \quad \vec{a}_2 = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}, \quad \vec{a}_3 = \begin{bmatrix} -1 \\ 4 \\ 24 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} c \\ 0 \\ d \end{bmatrix}.$$

For which values of c and d is the vector $\vec{b}$ a linear combination of the vectors $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$?

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & c \\ 0 & 1 & 4 & 0 \\ 4 & -1 & 24 & d \end{array}\right] \overset{-4R_1+R_3}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & -1 & c \\ 0 & 1 & 4 & 0 \\ 0 & 7 & 28 & -4c+d \end{array}\right] \overset{-7R_2+R_3}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & -1 & c \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & -4c+d \end{array}\right]$$

$$\overset{2R_2+R_1}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 7 & c \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & -4c+d \end{array}\right]$$

∴ In order for $\vec{b}$ to be a linear combo of $\vec{a}_1, \vec{a}_2, \vec{a}_3$, $-4c+d=0$.

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Question 5. [6 pts]

(a) Find the general solution to the following linear system:

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$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & + & 3x_3 & = & 0 \\ 2x_1 & + & x_2 & + & 3x_3 & = & 0 \\ -x_1 & - & 5x_2 & - & 6x_3 & = & 0 \end{array}$$

Write your answer in *vector parametric form*. What is the geometric interpretation of the solution set?

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 1 & 3 & 0 \\ -1 & -5 & -6 & 0 \end{array}\right] \underset{R_1+R_3}{\overset{-2R_1+R_2}{\sim}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & -3 & -3 & 0 \end{array}\right] \underset{-1/3R_3}{\overset{-1/3R_2}{\sim}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\overset{-R_2+R_3}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \overset{-2R_2+R_1}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore x_1 = -x_3$

$x_2 = -x_3$

x_3 free

$$\therefore \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \quad x_3 \in \mathbb{R}$$

$\therefore$ The geo. interp. of the sol'n set is a line that goes through the origin

(b) Check that

$$x_1 = 2, \quad x_2 = 0, \quad x_3 = 1$$

is a solution to the linear system

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$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 3x_3 & = & 5 \\ 2x_1 & + & x_2 & + & 3x_3 & = & 7 \\ -x_1 & - & 5x_2 & - & 6x_3 & = & -8 \end{array}$$

Without performing any row reduction, find the general solution to this linear system in *vector parametric form*. (Note that the coefficient matrix here is the same as in part (a).)

Check

$(2)+0+3=5$ ✓ $\quad 2(2)+0+3=7$
$5=5 \quad\quad 7=7$ ✓

$-2-0-6(1)=-8$ ✓
$-8=-8$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \quad x_3 \in \mathbb{R}$$

∴ This is the general sol'n to this LS b/c when we know the sol'n to the corresponding homogenous sys., you just add the particular sol'n to get your gen. sol'n.

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QUESTION 6. [**5 pts**] For which values of h and k does the matrix-vector equation

$$\begin{bmatrix} 1 & 2 \\ 3 & h \end{bmatrix} \vec{x} = \begin{bmatrix} 2k \\ 6 \end{bmatrix}$$

(a) have no solutions,
(b) have a unique solution,
(c) have infinitely many solutions?

$$\cancel{\left[\begin{array}{c|c} 2k & 12 \\ 6k & 6h \end{array}\right]}$$

~~a) no sol'n when h≠0, k≠0~~

~~a) no sol'n when k=0, h≠0~~

~~b) unique sol'n when~~

$$\left[\begin{array}{cc|c} 1 & 2 & 2k \\ 3 & h & 6 \end{array}\right] \overset{-3R_1+R_2}{\sim} \left[\begin{array}{cc|c} 1 & 2 & 2k \\ 0 & h-6 & -6k+6 \end{array}\right]$$

a) no sol'ns when $h=6$, ~~$k \in R$~~ $k \neq 1$

~~b) unique when h=6, k=~~

b) unique when ~~$h=6$~~ $h \neq 6$, $k \in R$

c) Inf sol'ns when $h=6$, $k=1$