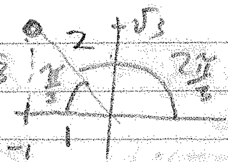
D. Amundsen

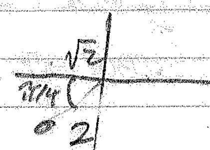
MATH 3007 TEST #1 SOLNS

1) a) $z=-1+\sqrt{3}i$



$\Rightarrow |z|=2$, $\arg(z)=\frac{2\pi}{3}$ $\Rightarrow \boxed{z=2e^{i\frac{2\pi}{3}}}$

b) $|z|=2,\ \arg(z)=\frac{5\pi}{4}$



$\Rightarrow \boxed{z=-\sqrt{2}-\sqrt{2}\,i}$

c) $z=\frac{1}{1+i}+\log(1+i)=\frac{1-i}{2}+\log(\sqrt{2})+i\frac{\pi}{4}\Rightarrow \boxed{\mathrm{Im}(z)=\frac{-1}{2}+\frac{\pi}{4}}$

d) $\left|\frac{z_1^2}{z_2}\right|=\frac{3^2}{2}\boxed{=\frac{9}{2}}$

e) $\arg\left(\frac{z_1^2}{z_2}\right)=2\cdot\frac{\pi}{5}-\frac{\pi}{3}=\frac{6-5}{15}\pi\boxed{=\frac{\pi}{15}}$

f) $z=e^3e^{4i}\Rightarrow\boxed{\arg(z)=4}$ θ

g) $3^i=e^{i\log(3)}=\cos(\log(3))+i\sin(\log(3))\Rightarrow\boxed{\mathrm{Re}(3^i)=\cos(\log(3))}$

2) $z^4-2i=0\Rightarrow z^4=2i$ let $z=re^{i\theta}$, $2i=2e^{i\frac{3\pi}{2}},2e^{i\frac{7\pi}{2}},2e^{i\frac{11\pi}{2}},2e^{i\frac{15\pi}{2}},\ldots$

$\Rightarrow r^4e^{4i\theta}=2e^{i\frac{3\pi}{2}},2e^{i\frac{7\pi}{2}},2e^{i\frac{11\pi}{2}},2e^{i\frac{15\pi}{2}}$

$\Rightarrow r^4=2\Rightarrow r=2^{\frac{1}{4}}$

$4\theta=\frac{3\pi}{2},\frac{7\pi}{2},\frac{11\pi}{2},\frac{15\pi}{2}\Rightarrow\theta=\frac{3\pi}{8},\frac{7\pi}{8},\frac{11\pi}{8},\frac{15\pi}{8}$

$\Rightarrow z=2^{\frac{1}{4}}e^{i\frac{3\pi}{8}},\ 2^{\frac{1}{4}}e^{i\frac{7\pi}{8}},\ 2^{\frac{1}{4}}e^{i\frac{11\pi}{8}},\ 2^{\frac{1}{4}}e^{i\frac{15\pi}{8}}$

3 $\cos(\bar{z}) = i \Rightarrow \dfrac{e^{i\bar{z}} + e^{-i\bar{z}}}{2} = i \Rightarrow$ let $w = e^{i\bar{z}} \Rightarrow w + \dfrac{1}{w} = 2i$

$$\Rightarrow w^2 - 2iw + 1 = 0 \Rightarrow w = \frac{2i \pm \sqrt{-4-4}}{2} = i \pm \sqrt{2}\, i = (1 \pm \sqrt{2})\, i$$

$$\Rightarrow e^{i\bar{z}} = (1 \pm \sqrt{2})\, i \Rightarrow i\bar{z} = \log((1 \pm \sqrt{2})\, i) = \begin{cases} + & \log(1+\sqrt{2}) + i\dfrac{\pi}{2} + i2n\pi \\ - & \log(\sqrt{2}-1) - i\dfrac{\pi}{2} + i2n\pi \end{cases}$$

note $1+\sqrt{2} > 0$ BUT $1-\sqrt{2} < 0$

$$\Rightarrow \bar{z} = \pm\frac{\pi}{2} + 2n\pi - i\log(|1 \pm \sqrt{2}|)$$

$$\Rightarrow z = \bar{\bar{z}} = \pm\frac{\pi}{2} + 2n\pi + i\log(|1 \pm \sqrt{2}|) \qquad n = 0, \pm 1, \pm 2, \dots$$

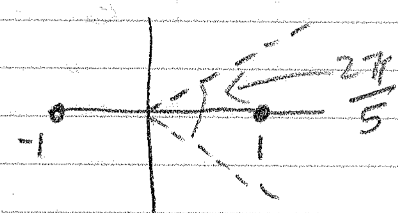
4 NO note if $f(z) = z^{\frac{1}{5}}$ then we define

$f(z) = r^{\frac{1}{5}} e^{i\frac{\theta}{5}}$ where $\theta = \arg(z)$ is taken on some 2π interval.

so for example if $\theta \in [0, 2\pi) \Rightarrow \arg(f(z)) \in [0, \frac{2\pi}{5})$

or if $\theta \in [-\pi, \pi) \Rightarrow \arg(f(z)) \in [-\frac{\pi}{5}, \frac{\pi}{5})$

so $\arg(f(z))$ spans an interval of length $\frac{2\pi}{5}$



so if $f(1) = 1 \Rightarrow \arg(f(1)) = 0$

then for any other $z \in \mathbb{C}$ $\quad |\arg(f(z))| < \frac{2\pi}{5}$

but if $f(-1) = -1 \Rightarrow \arg(f(-1)) = \pi > \frac{2\pi}{5} \Rightarrow$ NOT POSSIBLE