ECON 4021 Winter 2014

Christopher Gunn

Assignment 2

Due date: Thursday February 27, 4:00 pm

Total Marks: 100

1. (50 marks) Consider the following model economy: There is one period with a single representative consumer and a single representative firm. The consumer cares about consumption c only, and has preferences given by the utility function $U(c)$ where $U(\cdot)$ is strictly increasing, strictly quasi-concave and twice differentiable. The consumer earns labour income in the labour market at wage rate w, however, the consumer supplies 1-unit of labour inelastically - ie $N^s = 1$ for all w. The consumer also receives non-labour income π from the firm that it owns, and pays lump-sum taxes T to the government. The consumer then uses the proceeds of its total disposable income to purchase consumption from the goods market. The consumer takes prices as given and chooses consumption to maximize utility subject to its budget constraint.

   The firm produces output y with production technology $y = zF(k, N^d)$, where k is the firm's fixed stock of capital, N^d is the firm's labour, and where $F(\cdot)$ is CRS, increasing in K and N, strictly quasiconcave, and twice differentiable. The firm chooses labour to maximize its profits taking prices as given, sells output to the goods market, and demands labour from the labour market.

   The government purchases output G from the goods market, financing it with lump-sum taxes T.

   (a) (5 marks) Write down the consumer's budget constraint

   (b) (5 marks) Show the consumer's problem, and find the consumer's optimal level of consumption c^*, in terms of w and $\pi - T$.

   (c) (1 mark) Write down the government's budget constraint

   (d) (1 mark) Write down an expression for the firm's (non-equilibrium) level of profits, π

   (e) (4 marks) Show the firm's problem, and find its labour first-order condition.

   (f) (5 marks) Define a competitive equilibrium for this economy, consistent with the definition of equilibrium that we discussed in class.

   (g) (3 marks) Write down the market-clearing conditions for the labour and goods markets

(h) (2 marks) Beginning with the consumer's budget constraint, make the appropriate substitutions to find an expression for the equilibrium level of consumption C in terms of G and Y

(i) (3 marks) Substitute the equilibrium production technology in for Y in the above expression, and totally differentiate this expression.

(j) (3 marks) Using the expression above, determine the equilibrium impact on C of fiscal policy in this economy corresponding to an increase in G - ie find $\frac{\partial C}{\partial G}$.

(k) (2 marks) Now determine the equilibrium impact on Y of the same increase in G - ie find $\frac{\partial Y}{\partial G}$ (you may need to use an additional equation to get this result).

(l) (4 marks) In the baseline model in Williamson Ch.5, in response to an increase in G, he writes that "private consumption is crowded out by government purchases, but is not completely crowded out..." (p.142). Does the same description apply in this economy? If not, what accounts for the difference in the response of C and Y between this economy and Williamson's economy?

(m) (1 mark) Using the firm's FOC, find an expression for $\frac{\partial N^d}{\partial w}$, determine the sign of this derivative. What does this imply about the slope of the labour demand curve?

(n) (1 mark) Again using the firm's FOC, find an expression for $\frac{\partial N^d}{\partial z}$, determine the sign of this derivative. What does this imply about how changes in TFP, z, impact the labour demand curve?

(o) (4 marks) Now sketch a diagram of the labour market in this economy in wage-N space, showing the labour supply curve and labour demand curve. Show graphically how equilibrium in the labour market changes with a positive shock to TFP.

(p) (3 marks) If you were given the information that TFP was procyclical - ie TFP increased during booms, and decreased during recessions, from your graphical answer regarding the labour market above, does the model imply that the real wage is countercyclical, weekly procyclical, or strongly procyclical?

(q) (1 marks) King and Rebelo (2000) report that in the U.S., the correlation coefficient of the business cycle component of the real wage with the business cycle component of output is 0.12. Is your result from above consistent with this?

2. (50 marks) Consider the following model economy: There is one period with a single representative consumer and a single representative firm. The consumer has preferences given by the utility function $U(c, l)$ where c is consumption and l is leisure, and where $U(\cdot)$ is strictly increasing in both arguments, strictly quasi-concave and twice differentiable. The consumer is endowed with h units of time which can be allocated between work N^s and leisure. Unlike in the representation in Williamson, in this model economy, the consumer owns the fixed stock of capital, which we will denote as k_0. The consumer earns labour income in the labour

market at wage rate w for each unit of labour supplied to the firm. In addition, the consumer also rents its capital to the firm through the capital services market, earning rental income r for each unit of capital that it supplies. The consumer also receives non-labour income π from the firm that it owns, and pays lump-sum taxes T to the government. The consumer then uses the proceeds of its total disposable income to purchase consumption from the goods market. The consumer takes prices as given and chooses consumption and leisure to maximize utility subject to its budget constraint.

The firm produces output y with production technology $y = zF(K^d, N^d)$, where K^d is the firm's capital services that it rents from the household, and N^d is the firm's labour, and where $F(\cdot)$ is CRS, increasing in K and N, strictly quasiconcave, and twice differentiable. Unlike in Williamson, the firm's profits π are now given as $\pi = Y - wN^d - rK^d$. The firm chooses labour and capital services to maximize its profits taking prices as given, sells output to the goods market, demands labour from the labour market, and demands capital services from the capital services market.

The government purchases output G from the goods market, financing it with lump-sum taxes T.

(a) (6 marks) Write down the consumer's budget constraint

(b) (4 marks) Show the consumer's problem, and find the consumers c and l first-order conditions.

(c) (1 mark) Write down the government's budget constraint

(d) (5 marks) Show the firm's problem, and find its labour and capital services first-order conditions.

(e) (5 marks) Substitute your N^d and K^d first-order conditions into the firm's expression for profits by eliminating w and r. Knowing that the production function is CRS (is linearly homogeneous), what does this imply about the firm's profits? (hint: if some function y = f(x,y) is linearly homogeneous, then Euler's theorem says that $y = f_x x + f_y y$).

(f) (5 marks) Define a competitive equilibrium for this economy, consistent with the definition of equilibrium that we discussed in class.

(g) (5 marks) Write down the market-clearing conditions for the labour, goods and capital services markets

(h) (5 marks) Beginning with the consumer's budget constraint, substitute in the government budget constraint and the expression for firm profits, and find an expression for the national income identity in this economy.

(i) (4 marks) Using your expression above, find the PPF for this model economy.

(j) (5 marks) Is the competitive equilibrium in this economy socially optimal?

(k) (3 marks) Since the production function is CRS, in equilibrium, is any fraction of total income Y left over after payments are made to labour and capital?

(l) (2 marks) In the model in Williamson where the firm owns its capital, the production function was also CRS, and profits were given by $Y - wn$. Does this different definition of the firm's profits mean the distribution of factor income is different in the Williamson model economy compared to the model economy in this problem?