MAT 2377
Midterm

23 February, 2010 **Professor: R. Kulik**
Time: 80 minutes

Student Number: _____

Family Name: _____

First Name: _____

This is an open book examination.
Only non-programmable and non-graphic calculators are permitted.
Record your answer to each question in the table below.
Number of pages: **3** (including this one).
NOTE: At the end of the examination, hand in only this page. You may keep the questionnaire.

Question	Answer
1	
2	
3	
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10	

Q1. A company's warranty document states that the probability that a new swimming pool requires some repairs within the first year is 20%. What is the probability, that the sixth sold pool is the first one which requires some repairs within the first year?

(a) 0.6068 (b) 0.3932 (c) 0.9345

(d) 0.0655 (e) none of the preceding

Q2. Let X denote a number of failures of a particular machine within a month. Its probability mass function is given by

x	0	1	2	3	4	5
$P(X=x)$	0.17	0.23	0.19	0.13	0.08	0.2

Compute the probability that there are more than 3 failures within a month.

(a) 0.28 (b) 0.72 (c) 0.20

(d) 0.80 (e) none of the preceding

Q3. Refer to Question 2. What is the expected number of failures within a month?

(a) 2.50 (b) 3.00 (c) 2.32

(d) 11.94 (e) none of the preceding

Q4. Two companies A and B consider making an offer for road construction. The company A makes the submission. The probability that B submits the proposal is 1/3. If B does not submit the proposal, the probability that A gets the job is 3/5. If B submits the proposal, the probability that A gets the job is 1/3. What is the probability that A will get the job?

(a) 0.6667 (b) 0.5111 (c) 0.75

(d) 0.3333 (e) none of the preceding

Q5. In a box of 50 fuses there are 8 defective ones. We choose 5 fuses randomly (without replacement). What is the probability that all 5 fuses are not defective?

(a) 0.4015 (b) 0.84 (c) 0.3725

(d) 0.4275 (e) none of the preceding

Q6. Consider a random variable X with the following probability density function:

$$f(x) = \begin{cases} 0 & \text{if } x \leq -1 \\ \frac{3}{4}(1-x^2) & \text{if } -1 < x < 1 \\ 0 & \text{if } x \geq 1 \end{cases}$$

The value of $P(X \leq 0.5)$ is

(a) 11/32 (b) 27/32 (c) 16/32

(d) 1 (e) none of the preceding

Solution to Q6:

To compute:

$$P(X \le 0.5) = \int_{-1}^{0.5} \frac{3}{4}(1-x^2)\,dx = \frac{3}{4}x|_{-1}^{0.5} - \frac{1}{4}x^3|_{-1}^{0.5} = 27/32.$$

Q7. A receptionist receives on average 2 phone calls per minute. If the number of calls follows a Poisson process, what is the probability that the waiting time for call will be greater than 1 minute?

(a) $e^{-1/15}$ (b) $e^{-1/30}$ (c) e^{-2} (d) e^{-1} (e) none of the preceding

Solution to Q7:

We have Poisson process with $\lambda = 2$. Now, waiting time in Poisson process is exponential. Let X be an exponential random variable with the parameter $\lambda = 2$. To compute: $P(X > 1) = \exp(-2 \times 1)$.

Q8. A company manufactures hockey pucks. It is known that their weight is normally distributed with mean 1 and the standard deviation 0.05. The pucks used by NHL must weight between 0.9 and 1.1. What is the probability that a randomly chosen puck can be used by NHL?

(a) 1 (b) 0.9545 (c) 0.4560 (d) 0.9772 (e) none of the preceding

Solution to Q8:

$$P(0.9 < X < 1.1) = P\left(\frac{0.9-1.0}{0.05} < Z < \frac{1.1-1.0}{0.05}\right) = \Phi(2) - \Phi(-2) = 0.977250 - 0.022750 = 0.9545$$

Q9. If $X \sim N(10, 1)$, the value of k such that $P(X \le k) = 0.701944$ is closest to

(a) 0.59 (b) 0.30 (c) 0.53

(d) 10.53 (e) none of the preceding

Solution to Q9:

$$P(X \le k) = P((X-10)/1 \le (k-10)/1) = P(Z \le k-10) = 0.701944.$$

Thus, $k - 10 = 0.53$ and $k = 10.53$.

Q10. Consider the following data:

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12 14 6 10 1 20 4 8
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The median and the first quartile are, respectively,

(a) 9 and 5 (b) 5.5 and 6 (c) 10 and 5 (d) 5 and 10 (e) none of the preceding

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This is the last question

Solutions to multiple choice questions:

Q1 $\longrightarrow$ d
Q2 $\longrightarrow$ a
Q3 $\longrightarrow$ c
Q4 $\longrightarrow$ b
Q5 $\longrightarrow$ a
Q6 $\longrightarrow$ b
Q7 $\longrightarrow$ c
Q8 $\longrightarrow$ b
Q9 $\longrightarrow$ d
Q10 $\longrightarrow$ a