University of Ottawa

Department of Mathematics and Statistics

MAT 2377 3X: Probability and Statistics for Engineers

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Midterm Exam -Blue- Version 1- Solutions

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Surname ______________________ First Name ____________________

Student # ____________________

Instructions:

- You have 1 hour and 45 minutes to complete this exam.
- Record your answers in the table below.
- Only TI 30 calculators or equivalent models are permitted.
- Do not detach the staple.
- Write your student number at the top of each page in the space provided.
- There are 12 multiple choice questions.

GOOD LUCK!

Question	Answer
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

Question 1. How many different 8 letter codes can be constructed using 2 a's, 4 b's and 2 c's?

A) 420

B) 40320

C) 16

D) 70

E) 840

Solution: $\frac{8!}{2!\ 4!\ 2!} = 420$.

Question 2. In how many ways can 4 of 10 printers be chosen for a special company if
a) order matters?
b) order does not matter?

A) a) 420, b) 5040

B) a) 210, b) 5040

C) a) 5040, b) 210

D) a) 5040, b) 200

E) a) 200, b) 10

Solution: a) Order matters, so it is a permutation:

$$P_4^{10} = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 5040.$$

b) This one is combination,

$$C_4^{10} = \frac{10!}{4!(10-4)!} = \frac{10!}{4!6!} = 210.$$

Question 3. A company that produces video cameras manufactures a basic model and a deluxe model. Over the past year, 40% of the cameras sold have been of the basic model. Of those buying the basic model, 30% purchase an extended warranty, whereas 50% of all deluxe purchasers do so. If you learn that a randomly selected purchaser has an extended warranty, how likely is it that he or she has a basic model?

A) 71.43%

B) 40%

C) 50%

D) 30.25%

E) 28.57%

Solution: Define the following events:
B: Buying basic model
B': Buying deluxe model
W: Purchasing extended warranty

$$P(B) = .4,\ P(B') = .6,\ P(W|B) = .3,\ P(W|B') = .5$$

$$P(B|W) = \frac{P(W|B)P(B)}{P(W|B)P(B) + P(W|B')P(B')} = \frac{.3 \times .4}{.3 \times .4 + .5 \times .6} = .2857$$

The probability is 28.57%.

Question 4. A communication system consists of four components where the components work independently. The system transfers signals properly if 2 or more components work properly. If the probability that an individual component works is 0.75, what is the probability that the system transfers signals properly?

A) 0.0352

B) 0.3164

C) 0.9492

D) 0.8500

E) None of the preceding.

Solution: Let X be the number of working components $\implies X \sim B(4, 0.75)$

$$\begin{aligned} P(\text{system works}) =& P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 0) - P(X = 1) \\ =& 1 - C_0^4 (0.75)^0 (0.25)^4 - C_1^4 (0.75)^1 (0.25)^3 = 0.9492 \end{aligned}$$

Question 5. The number of people arriving for treatment at an emergency room can be modeled by a Poisson process with a rate of five per hour.
a) What is the probability that exactly four people arrive during a particular hour?
b) How many people do you expect to arrive during a 45-minute period?

A) a) 0.175, b) 3.75

B) a) 0.175, b) 7.50

C) a) 0.175, b) 5

D) a) 0.198, b) 3.75

E) a) 0.550, b) 4

Solution: a) $P(X_1 = 4) = e^{-5}\frac{5^4}{4!} = 0.175$
b) $X_{0.75}$ is a Poisson r.v. with parameter $0.75 \times 5 = 3.75$, therefore $E(X_{0.75}) = 3.75$

Question 6. In the previous question determine the length of an interval of time such that the probability that no patients arrive during this interval is 0.6.

A) 7.13 minutes

B) 6.25 minutes

C) 6.13 minutes

D) 5 minutes

E) None of the preceding.

Solution: Let the interval end at time t (in hours). X_t has Poisson distribution with parameter $5t$:

$$P(X_t = 0) = 0.6 \Longrightarrow e^{-5t}\frac{(5t)^0}{0!} = 0.6 \Longrightarrow t = 0.1022 \text{ hour}$$

Question 7. The fill volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 and standard deviation of 0.1 fluid ounce. What is the probability a fill volume is less than 12.2 fluid ounces?

A) 0.001350

B) 0.022750

C) 0.982510

D) 0.011250

E) 0.098525

Solution: X: fill volume $\Longrightarrow X \sim N(12.4, (0.1)^2)$

$$P(X \leq 12.2) = P(Z \leq \frac{12.2 - 12.4}{0.1}) = P(Z \leq -2) = \Phi(-2) = 0.02275$$

Question 8. Suppose that $X \sim N(3, (0.1)^2)$. Find t such that $P(X > t) = 0.017864$.

A) -2.21

B) -3.21

C) 2.10

D) 2.21

E) 3.21

Solution: $P(\frac{X-3}{0.1} > \frac{t-3}{0.1}) = P(Z < \frac{3-t}{0.1}) = \Phi(\frac{3-t}{0.1}) = 0.017864.$
Therefore,

$$\frac{3-t}{0.1} = -2.1 \Longrightarrow t = (2.1)(0.1) + 3 = 3.21$$

Question 9. In the process of manufacturing aluminum cans, the probability that a can has a flaw on its side is 0.02, the probability that a can has a flaw on the top is 0.03, and the probability that a can has a flaw on both the side and the top is 0.01. What is the probability that a randomly selected can has no flaw?

A) 0.94

B) 0.04

C) 0.93

D) 0.96

E) None of the preceding.

Solution: Define the following events:
FT: flaw on top
FS: flaw on side.
We have $P(FT) = 0.03, P(FS) = 0.02$ and $P(FT \cap FS) = 0.01$.

$$\begin{aligned} P(\text{no flaw at all}) =& P(FT' \cap FS') = P((FT \cup FS)') \\ =& 1 - P(FT \cup FS) = 1 - P(FT) - P(FS) + P(FT \cap FS) \\ =& 1 - 0.03 - .02 + 0.01 = 0.96. \end{aligned}$$

Question 10. The probability mass function $f(x) = c(1/2)^x$ for $x = 1, 2, \ldots$ is given.
a) Determine the value of c.
b) Find $P(X > 1)$.

A) a) 1, b) 1/3

B) a) 1, b) 1/2

C) a) 2, b) 1/3

D) a) 2, b) 1/2

E) None of the preceding.

Solution: a)

$$\sum_{x=1}^{\infty} f(x) = 1 \Longrightarrow c \sum_{x=1}^{\infty} (\frac{1}{2})^x = 1 \Longrightarrow c \times 1 = 1 \Longrightarrow c = 1.$$

b)

$$P(X > 1) = 1 - P(X \leq 1) = 1 - P(X = 1) = 1 - (1/2) = 1/2.$$

Question 11. Let Y be a random variable with the following density function:

$$f(x) = \begin{cases} \frac{1}{3} \text{ if } 0 \leq x \leq 1 \\ \frac{2}{3} \text{ if } 4 \leq x \leq 5. \end{cases}$$

Which one is correct?

A) $\mu_Y = 1$

B) $\mu_Y = 19/6$

C) $\mu_Y = 11/6$

D) $\mu_Y = 2.5$

E) $\mu_Y = 5$

Solution:

$$\mu_Y = \int_0^1 \frac{x}{3} dx + \int_4^5 \frac{2x}{3} dx = 19/6.$$

Question 12. A battery's output is measured daily and follows a normal distribution with mean 1.5 V and standard deviation 0.2 V. Determine the mean number of days until the first time the voltage drops below 1 V. Assume that the daily outputs are independent.

A) 9.456 days

B) 8.5 days

C) 161.03 days

D) 0.00621 days

E) 0.105650 days

Solution: Let X be the daily voltage. $X \sim N(1.5, 0.2^2)$. The probability that the voltage drops below 1 V in each day is

$$P(X < 1) = P(\frac{X - 1.5}{0.2} < \frac{1 - 1.5}{0.2}) = P(Z < -2.5) = \Phi(-2.5) = 0.006210.$$

Y counting the number of days until the first drop is a geometric random variable with parameter 0.006210. Therefore, the mean number of days until the first drop is

$$E(Y) = \frac{1}{0.006210} = 161.03.$$