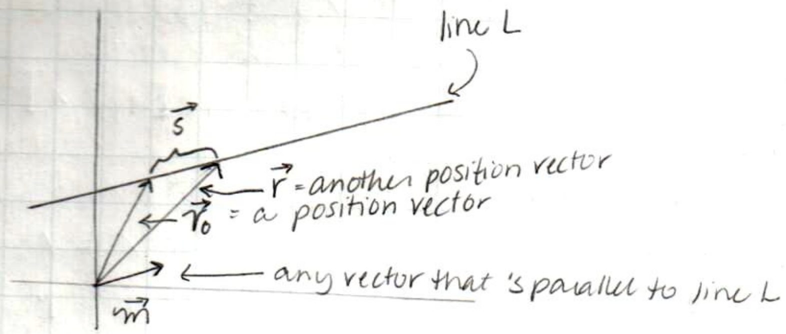

$\vec{r_0} + \vec{s} = \vec{r}$

since $\vec{s}$ is parallel to $\vec{m}$, it is a multiple of $\vec{m}$ $\therefore$ $\vec{s}$ is $= t\vec{m}$ where $t \in \mathbb{R}$.
So we can rewrite:
$\vec{r_0} + t\vec{m} = \vec{r}$

this is the vector equation of a line in 2-dimensional space.
Substitute $\vec{r} = [x, y]$ as the position vector for any unknown point on the line,
$\vec{r_0} = [x_0, y_0]$ where $\vec{r_0}$ is a position vector is a known point on the line,
$\vec{m} = [a_0, b_0]$ is a direction vector parallel to the line.
So basically we are using 3 vectors to draw the vector $\vec{s}$ on the line.

$[x_0, y_0] + t[a_0, b_0] = [x, y]$ ← the vector equation
rearranging:

$$\left.\begin{aligned} x &= x_0 + ta_0 \\ y &= y_0 + tb_0 \end{aligned}\right\} \leftarrow \text{the parametric equation}$$

$t \in \mathbb{R}$ for all of the above.

Eg: a line passes through A(1,4) B(3,1)

a) write a vector equation
① → find a vector between point A & point B to find the direction vector
$\vec{m} = \vec{OB} - \vec{OA} = [3,1] - [1,4] = [2,-3]$
② → choose either point A or B to be the position vector $\vec{r_0} = [3,1]$ point B
$[x,y] = [3,1] + t[2,-3]$

b) determine if (2,3) is on the line
① write the parametric equation
$x = 3 + 2t$
$y = 1 - 3t$
② solve for t for $x = 2$ & $y = 3$
$2 = 3 + 2t \quad 2 - 3 = 2t \quad \frac{-1}{2} = t$
$3 = 1 - 3t \quad 3 - 1 = -3t \quad \frac{2}{-3} = t$

these t are not equal so 2,3 is not on the line.

Eg: $l_1 = \begin{cases} x = 3+2t \\ y = -5+4t \end{cases}$

a) find the coordinates of two points on the line.

① choose any 2 values for t

$t=1 \quad x = 3+2 = 5 \quad (5,-1)$
$\quad\quad y = -5+4 = -1$

$t=2 \quad x = 3+2\cdot2 = 3+4 = 7 \quad (7,3)$
$\quad\quad y = -5+4\cdot2 = -5+8 = 3$

b) write a vector equation for the line

$[x,y] = [x_0, y_0] + t[a_0, b_0]$ — we know if $t=2$ then $(7,3)$ is on the line

$= [7,3] + t[a_0, b_0]$ — 2 & 4 are a_0 & b_0 in the parametric equation

$[x,y] = [7,3] + t[2,4]$

c) write the scalar equation of the line

① isolate t

$y = -5+4t \quad\quad x = 3+2t$

$\Rightarrow \frac{y+5}{4} = t \quad\quad \Rightarrow x-3 = 2t \Rightarrow \frac{(x-3)}{2} = t$

② set up an equation to show $t = t$ & solve

$\frac{y+5}{4} = \frac{x-3}{2} \Rightarrow 2(y+5) = 4(x-3) \Rightarrow 2y+10 = 4x-12$

$\Rightarrow y+5 = 2x-6 \quad \boxed{2x-y-11=0}$

d) is l_2 parallel to l_1?

$l_2 = \begin{cases} x = 1+3t \\ y = 8+12t \end{cases}$

① if the lines are parallel then l_2 will be a multiple of l_1. $[a_{l_1}, b_{l_1}]$ will $= h[a_{l_2}, b_{l_2}]$ $(\vec{m})$

where h is $\in \mathbb{R}$.

$\vec{m}$ for $l_1 = [2,4]$
$\vec{m}$ for $l_2 = [3,12]$

$[2,4] = h[3,12] \quad \begin{cases} 2 = h[3] \rightarrow \frac{2}{3} = h \\ 4 = h[12] \rightarrow \frac{4}{12} = h = \frac{1}{3} \end{cases}$

these two scalars are not equal so l_2 is not parallel to l_1.

Eg: a line has the scalar equation $4x+5y+20=0$
write the vector equation:

① for $x=0$?y?

$5y+20=0 \quad 5y=-20 \quad \frac{-20}{5}=y=-4$

② point 1 = $[0,-4]$

point 2 = for $y=0$?x?

$4x+20=0 \quad 4x=-20 \quad x=-5 \quad [-5,0]$

③ find $\vec{m}$, which is a vector between point 1 to 2

$\vec{P_2}-\vec{P_1}=[(-5-0),(0--4)]=[-5,4]$

$[x,y]=[-5,0]+t[-5,4]$