Chapter 2: Derivatives

Some rules & formulas for finding derivatives

$f(x) = C = Cx^{0}$ ← zero $f'(x) = 0$ → the derivative of a constant is always 0

$f(x) = x \quad f'(x) = 1$

$f(x) = x^2 \quad f'(x) = 2x$

$f(x) = x^3 \quad f'(x) = 3x^2$

$\boxed{f(x) = x^n \quad f'(x) = nx^{n-1}}$

$f(x) = \sqrt{x} = x^{\frac{1}{2}} \quad f'x = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}} \quad f'x = \frac{1}{3x^{\frac{2}{3}}} = \frac{1}{3\sqrt[3]{x^2}}$

It's helpful to memorize the derivatives of some simple functions — for speed's sake

Rules:

Summation rule

$(f+g)'(x) = (f'(x)) + (g'(x))$

the derivative of two functions added together is the same as the derivative of each function individually added together.

It's the same for subtraction: $(f-g)'(x) = (f'(x) - g'(x))$

Eg: find the derivative of $f(x) = (x^2+x)$

$f'x = 2x + 1$

Eg: ? $f'(x)$ x^3+1

$f'(x) = 3x^2 + 0 = 3x^2$

In general to find the derivative of any polynomial

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} \dots a_1 x^1 + a_0 x^0$

$f'(x) = a_n(n)x^{n-1} + a_{n-1}(n-1)x^{n-2} + a_{n-2}(n-2)^{n-2} + \dots a_1(1)x^1 + 0$

Product Rule

$(f \cdot g)'(x) = (f'(x) \cdot g(x)) + (g'(x) \cdot f(x))$

Eg: $f(x) = 2x^2$? $f'(x)$?

$g(x) = 2 \qquad f'(x) = (g'(x) \cdot h(x)) + (h'(x) \cdot g(x))$

$h(x) = x^2 \quad g'x = 0 \qquad f'(x) = (0 \cdot x^2) + (2x \cdot 2)$

$h'x = 2x \qquad f'x = 0 + 4x = 4x$

In general if $f(x) = cx^n$ then $f'(x) = c \cdot n x^{n-1}$

quotient rule

$$\left(\frac{f}{g}\right)'(x) = \frac{(f'(x) \cdot g(x)) - (g'(x) \cdot f(x))}{g(x)^2}$$

$$Eg: \frac{(x^2+1)}{(x^3-x)} = \quad f'(x^2+1) = 2x+0 \quad f'(x^3-x) = 3x^2-1 \quad \frac{(2x(x^3-x)) - ((3x^2-1)(x^2+1))}{(x-3)^2}$$

(prime)

$$Eg: \frac{1}{x} = \frac{(1' \cdot x) - (x' \cdot 1)}{x^2} = \frac{(0 \cdot x) - (1 \cdot 1)}{x^2} = \frac{-1}{x^2}$$

These rules are applied step by step in long functions

$$Eg \ \frac{\sqrt[3]{x} + \sqrt{x}}{\sqrt[7]{x^3}} + x$$ → it helps to express radicals as exponents

$$\frac{x^{\frac{1}{3}} + x^{\frac{1}{2}}}{x^{\frac{3}{7}}} + x$$ start with summation rule. $f'x = 1$

then use quotient rule on the rational expression

$$\frac{x^{\frac{1}{3}} + x^{\frac{1}{2}}}{x^{3/7}} + 1 \qquad \frac{(f'(x) \cdot g(x)) - (f(x) \cdot g'(x))}{g(x)^2}$$

$$f'(x^{\frac{1}{3}} + x^{\frac{1}{2}}) = \frac{1}{3}x^{\frac{1}{3}-1} + \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{3x^{\frac{2}{3}}} + \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{2\sqrt{x}}$$

$$f'(x^{\frac{3}{7}}) = \frac{3}{7}x^{\frac{3}{7}-1} = \frac{3}{7}x^{-\frac{5}{7}} = \frac{3}{7x^{\frac{5}{7}}} = \frac{3}{7\sqrt[7]{x^5}}$$

$$\frac{\left[\left(\frac{1}{3\sqrt[3]{x^2}} + \frac{1}{2\sqrt{x}}\right) \cdot x^{\frac{3}{7}}\right] - \left[(x^{\frac{1}{3}} + x^{\frac{1}{2}}) \cdot \left(\frac{3}{7\sqrt[7]{x^5}}\right)\right]}{(x^{3/7})^2}$$

Notation

$$f'(x) = \lim_{h \to 0} \frac{f(a+h) - a}{h} = \lim_{\substack{\Delta a \to 0 \\ h \to 0}} \frac{\Delta f}{\Delta a} \quad ((a+h) - a) = h \quad a \to 0$$

Sometimes instead of writing $\lim_{h \to 0} \frac{\Delta f}{\Delta a}$ we just write $\frac{df}{da}$ (or $\frac{dy}{dx}$ or $\frac{df}{dx}$ ...)

when we use this notation $\lim_{h \to 0}$ is implied.

important!

The chain rule

$f \circ g'(x)$ or $f'(g(x)) = f'(g(x)) \cdot g'(x)$

Eg: $(x^2+x)^{100}$

$f(x) = x^{100} \quad f'(x) = 100x^{99}$

$g(x) = x^2+x \quad g'(x) = 2x+1$

$\therefore f'(g(x)) \cdot g'(x) = 100(x^2+x)^{99} \cdot 2x+1$

Eg: $\sqrt{x^2+x} = (x^2+x)^{\frac{1}{2}}$

$f(x) = x^{\frac{1}{2}} \quad f'x = \frac{1}{2\sqrt{x}}$

$g(x) = x^2+x$

$g'(x) = 2x+1$

$$f \circ g'(x) \quad \frac{1}{2\sqrt{x^2+x}} \cdot \frac{2x+1}{1} = \frac{2x+1}{2\sqrt{x^2+x}}$$