CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number		Section(s)
Mathematics	208/2		All except EC
Examination	**Date**	**Time**	**Pages**
Final	December 2011	3 Hours	3

Instructors	Course Examiner
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FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\,\frac{(1+i)^n - 1}{i}, \quad PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- Answer all questions.
- **Only approved calculators are allowed.**

MARKS

[10] 1. Given the quadratic function $f(x) = 1.2 + 0.96x - 0.12x^2$

(A) Find x and y intercepts algebraically.

(B) Find the vertex form of f.

(C) Find the vertex and the maximum or minimum.

(D) Find the range of f.

[10] 2. Solve for x in the following equations:

(A) $\left(\frac{3}{4}\right)^x = \frac{16}{9}$

(B) $(0.5)^{-3x^2+15x-72} = (0.5)^{-x^2+35x-22}$

(C) $\log_3\left(\frac{x}{5}\right) + \log_3 7 + 2\log_3\sqrt{5} = 3\log_3\sqrt[3]{175} + 5\log_3 1$

(D) $\log_a x + \log_a(x+1) = \log_a 6$

(E) $\log_2(\sqrt{2x^2}) - 1 = \frac{3}{2}$

[10] 3. For $f(x) = -12x + 16$ and $g(x) = 3(0.8)^x$ find the following:

(A) $\sum_{k=0}^{49} f(k) = f(0) + f(1) + f(2) + \cdots + f(49)$.

(B) $\sum_{h=0}^{24} g(h) = g(0) + g(1) + g(2) + \cdots + g(24)$.

[10] 4. Kelly sells some land in Quebec. She will be paid a lump sum of \$60,000 in 7 years. Until then, the buyer pays 8% simple interest quarterly.

(A) Find the amount of each quarterly interest payment.

(B) The buyer sets up a sinking fund so that enough money will be present to pay off the \$60,000. The buyer wants to make semiannual payments into the sinking fund; the account pays 6% compounded semiannually. Find the amount of each payment into the fund.

(C) What is the amount in the sinking fund after the first two deposits.

[10] 5. A person purchased a house 10 years ago for \$160,000. The house was financed by paying 20% down and signing a 30−year mortgage at 7.75% on the unpaid balance with payments made monthly.

(A) What is the unpaid balance after 120th payment?

(B) After the 120th payment, the owner wishes to refinance the house due to a need for additional cash. If the loan company agrees to a new 30−year mortgage of 80% of the new appraised value of the house, which is \$225,000, how much cash will the owner receive after repaying the balance of the original mortgage?

[10] 6. Solve by using Gauss-Jordan Elimination:

$$2x_1 + 6x_2 + 15x_3 = -12$$

$$4x_1 + 7x_2 + 13x_3 = -10$$

$$3x_1 + 6x_2 + 12x_3 = -9$$

<u>No other method of solving these systems of equations will be accepted!</u>

[10] 7. An economy is based on three sectors, agriculture, energy, and manufacturing. Production of a dollar's worth of agriculture requires an input of \$0.20 from the agriculture sector and \$0.40 from the energy sector. Production of a dollar's worth of energy requires an input of \$0.20 from the energy sector and \$0.40 from the manufacturing sector. Production of a dollar's worth of manufacturing requires an input of \$0.10 from the agriculture sector, \$0.10 from the energy sector, and \$0.30 from the manufacturing sector.

(A) Write the technological matrix M for this economy.

(B) If a final demand of \$20 billion for agriculture, \$10 billion for energy, and \$30 billion for manufacturing is to be met, then set up the equation to be satisfied by the inputs from the respective sectors.

(C) Solve the respective inputs satisfying these demands.

[10] 8. Extremize $P(x, y) = 30x + 10y$ subject to

$$2x + 2y \geq 4,\ 6x + 4y \leq 36,\ 2x + y \leq 10,\ x \geq 0,\ y \geq 0.$$

[10] 9. A small town has two radio stations: an AM station and an FM station. A survey of 100 town residents produced the following results: In the last 30 days, 65 people have listened to the AM station, 45 have listened to the FM station, and 30 have listened to both stations. During this 30–day period,

(A) How many people in the survey have listened to the AM station but not to the FM station?

(B) How many people in the survey have listened to the FM station but not to the AM station?

(C) How many people in the survey have not listened to either station?

[10] 10. A study on body types gave the following results: 45% were short, 25% were short and overweight, and 24% were tall and not overweight. Find the probabilities that a person is the following:

(A) Overweight

(B) Short, but not overweight

(C) Tall and overweight

Concordia University - MAT208 - Solution to the December 2011 Final exam

Question 1 (A) $f(x) = -0.12x^2 + 0.96x + 1.2$ is a quadratic function. Using the quadratic formula, the x intercepts are

$$x = \frac{-0.96 \pm \sqrt{.96^2 - 4 \times (-0.12) \times 1.2}}{2 \times (-0.12)}$$

$$x \simeq -1.1 \text{ or } x \simeq 9.1$$

The y-intercept is $f(0) = 1.2$.
(B) Vertex form: $f(x) = -0.12(x-4)^2 + 3.12$.
(C) The vertex has coordinates $(4, 3.12)$. The maximum of f is 3.12 and there is no minimum.
(D) The range of f is $(-\infty, 3.12]$.
Question 2
(A) $x = -2$
(B) The equation is equivalent to

$$-3x^2 + 15x - 72 = -x^2 + 35x - 22$$

$$\Leftrightarrow$$

$$2x^2 + 20x + 50 = 0$$

$$\Leftrightarrow$$

$$x^2 + 10x + 25 = 0$$

$$(x+5)^2 = 0$$

So the solution is x =-5.
(C) Using the properties of logarithms, we see that the equation is equivalent to

$$\frac{x}{5} \times 7 \times \sqrt{5}^2 = 175$$

So the solution is $x = 175/7 = 25$.
(D)

$$\log_a(x) + \log_a(x+1) = \log_a(6)$$

$$x(x+1) = 6$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

Solution to quadratic equation: $x = -3$ or $x = 2$. We do not allow take $x = -3$ because $\log_a(-3)$ D.N.E. So the solution to the original equation is : $x = 2$.
(E)

$$\log_2(\sqrt{2x^2}) - 1 = \frac{3}{2}$$

$$\Leftrightarrow$$

$$\log_2(\sqrt{2x^2}) = \frac{5}{2}$$

$$\Leftrightarrow$$

$$\sqrt{2x^2} = 2^{\frac{5}{2}}$$

$$\Leftrightarrow$$

$$2x^2 = 2^5$$

$$\Leftrightarrow$$

$$x^2 = 2^4 = 16$$

So the solution is: $x = -4$ or $x = 4$.

Question 3

(A) Since $f(k)$ is an arithmetic sequence,

$S_{50} = 50\left(\frac{a_1 + a_{49}}{2}\right) = \left(\frac{f(0) + f(49)}{2}\right) 50$

$$\sum_{k=0}^{49} f(k) = \sum_{k=0}^{49}[-12k + 16] = \frac{f(0) + f(49)}{2} \times 50 = \frac{16 - 572}{2} \times 50 = -13900.$$

(B) Since $g(h)$ is a geometic sequence,

$S_n = a_1 \frac{r^n - 1}{r - 1}$, $n = 25$

$a_1 = g(0) = 3$

$$\sum_{h=0}^{24} g(h) = \sum_{h=0}^{24} 3(0.8)^h = 3 \times \frac{1 - (0.8)^{25}}{1 - (0.8)} \simeq 14.94$$

Question 4

(A) In the problem 8% is understood to be the **annual** interest rate. The quarterly interest to be paid is then

$$I = \$60,000 \times 0.08 \times \frac{1}{4} = \$1,200.$$

(B) We use the formula

$$FV = PMT \times \frac{(1+i)^n - 1}{i},$$

with $FV = \$60,000$, $i = 0.06/2 = 0.03$ and $n = 7 \times 2 = 14$.

$$\frac{(1+i)^n - 1}{i} \simeq 17.0863$$

so

$$PMT \simeq \$60,000/17.0863 \simeq \$3,511.58$$

(C) We use the formula

$$FV_2 = PMT \times \frac{(1+i)^n - 1}{i},$$

with $PMT = \$3511.58$ $i = 0.03$ and $n = 2$ and get

$$FV_2 \simeq 7128.51.$$

Question 5 The solution below is extracted from the solution manual of the textbook.

Amortized amount = \$160,000 - (160,000)(0.20) = \$128,000.
Thus, PV = \$128,000, i = 0.00646, n = 360.

$$PMT = 128{,}000\ \frac{0.00646}{1-(1.00646)^{-360}} = \$917.18$$

Balance after 10 years:
PMT = \$917.18, i = 0.00646, n = 20(12) = 240,

$$\text{So the balance after 10 years} = 917.18\ \frac{1-(1.00646)^{-240}}{0.00646} = \$111{,}705.19$$

New loan amount is (225,000)(0.80) = \$180,000, so the cash the owner will receive will be 180,000 – 111,705.19 = \$68,294.81.

Question 6 The augmented matrix corresponding to the system is

$$\left(\begin{array}{ccc|c} 2 & 6 & 15 & -12 \\ 4 & 7 & 13 & -10 \\ 3 & 6 & 12 & -9 \end{array}\right),$$

and Gaussian elimination gives the Row Reduced Echelon Form

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right).$$

The solution to the system is therefore $x_1 = -3, x_2 = 4, x_3 = -2$.

Question 7

(A)

$$M = \begin{pmatrix} 0.2 & 0 & 0.1 \\ 0.4 & 0.2 & 0.1 \\ 0 & 0.4 & 0.3 \end{pmatrix}$$

(B) The equation is

$$(I - M)X = D,$$

where $D = \begin{pmatrix} 20 \\ 10 \\ 30 \end{pmatrix}$ is the demand vector (measured in millions of \$), $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ is the unknown vector with the inputs from the respective sectors (measured in millions of \$), and I is the 3×3 identity matrix.

(C) The augmented matrix corresponding to the system in (B) is

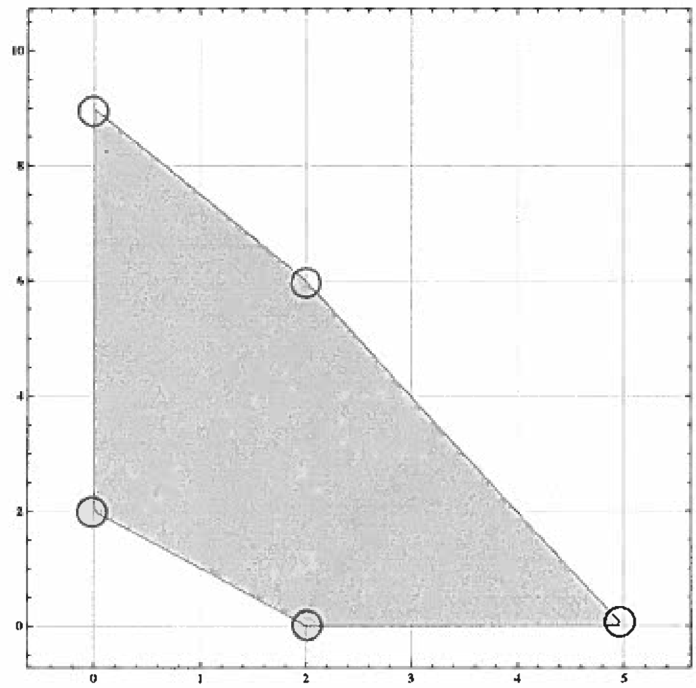
$$\left(\begin{array}{ccc|c} 0.8 & 0 & -0.1 & 20 \\ -0.4 & 0.8 & -0.1 & 10 \\ 0 & -0.4 & 0.7 & 30 \end{array}\right)$$

and Gaussian elimination gives the Row Reduced Echelon Form

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 33 \\ 0 & 1 & 0 & 37 \\ 0 & 0 & 1 & 64 \end{array}\right).$$

Therefore the outputs are $x_1 = 33M\$$ from the agriculture sector, $x_2 = 37M\$$ from the energy sector and $x_3 = 64M\$$ from the manufacturing sector.

Question 8 The feasible region in the (x, y)-plane defined by the inequality constraints is given below.



We compute the value of the objective function $P(x, y) = 30x + 10y$ at the corner points.

Corner (x, y)	$P(x, y)$
(0,2)	20
(0,9)	90
(2,6)	120
(5,0)	150
(2,0)	60

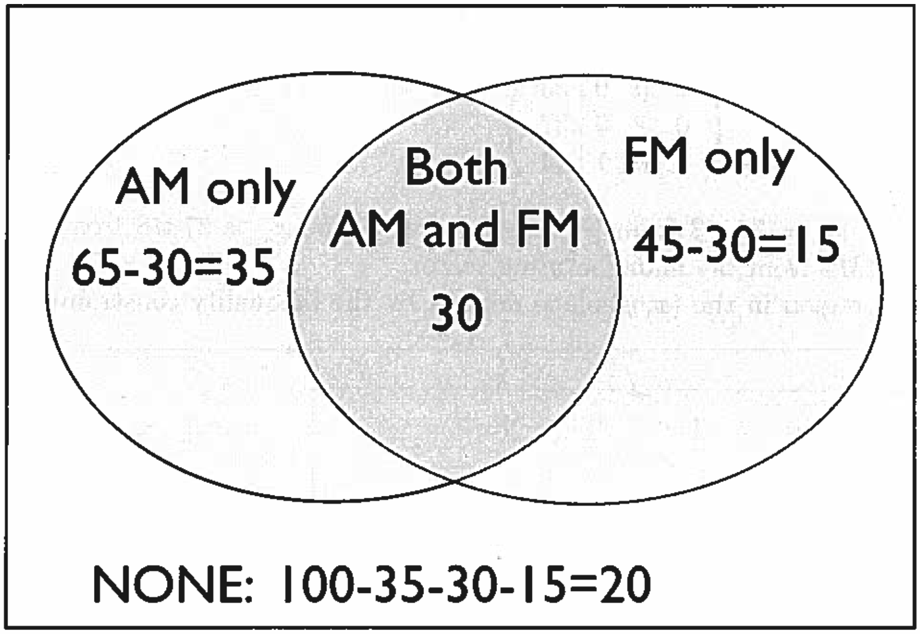
The minimum of $P(x, y)$ is equal to 20 and is attained at the point $(0, 2)$. The maximum of $P(x, y)$ is equal to 150 and is attained at the point $(5, 0)$.

Question 9

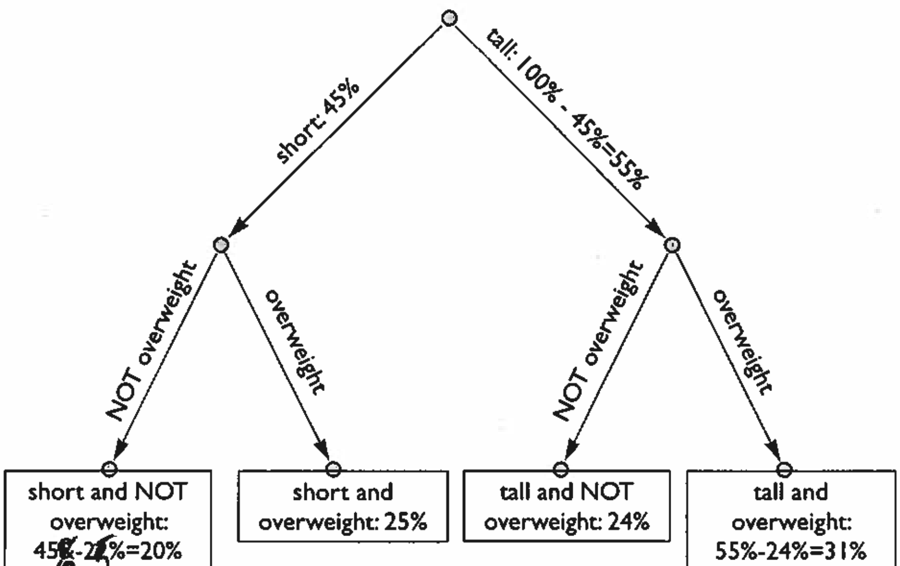
(A) 35 people

(B) 15 prople

(C) 20 people



Question 10



(A) probability of being overweight: $25\% + 31\% = 56\%$
(B) probability of being short but not overweight: 20%
(C) probability of being tall and overweight: 31%
Note: the solution can be also found using a Venn diagram like in question 9.