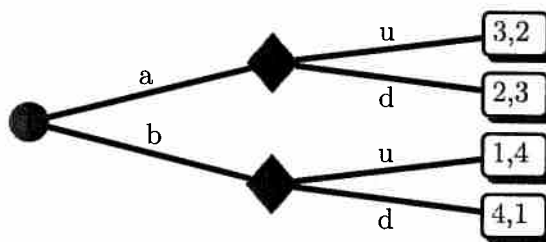


## Part 1: Multiple Choice Questions

Complete ALL of the following multiple choice questions. Indicate your answer in the box on the first page of this booklet. Each question is worth one point.

1. In a game tree, what do the edges represent?
  - A. The players
  - B. The information available to the players
  - C. The moves available to the players
  - D. The payoffs
2. Consider the following game tree:



- Which of the following is correct?
- A. Player 1 should choose b, because that is the only way (s)he can get a payoff of 4.
  - B. Player 1 should choose b, because that gives the highest chance of a payoff of 4.
  - C. Player 1 should choose b, because (b,d) is the only rollback equilibrium in the game.
  - D. None of the above.
3. What was John von Neumann's role in the development of game theory?
    - A. His 1928 article on parlor games laid out the mathematical foundations of game theory.
    - B. He proved that all games have Nash Equilibria.
    - C. He added a new dimension to the theory of games by introducing strategic moves.
    - D. His 1944 book with Oskar Morgenstern *Theory of Games and Economic Behavior* was the first text to consider the competitive nature of the free market.
  4. Thomas Schelling performed an experiment in which he had pairs of strangers try to meet each other somewhere in New York City. What was the purpose of this experiment and what can be concluded from the results?
    - A. The purpose was to study whether people use rational strategies in complex search tasks. The conclusion was that most people do not.
    - B. The purpose was to find out whether extrasensory perception plays a role in complex search tasks. The conclusion was that it didn't.
    - C. The purpose was to find out whether people can do better than guessing randomly by using focal points. The conclusion was that people rely heavily on such focal points.
    - D. The purpose was to find out whether navigation problems can be approached using the tools of game theory. The conclusion was that they can.

5. Which of the following statements about dominant strategies is correct? (Assume that all you care about is maximizing your payoffs.)

- A. You should always play a dominant strategy, even if your opponent is irrational.
- B. You should only play a dominant strategy when your opponent is rational.
- C. You should always play a dominant strategy when your opponent does the same thing, but if (s)he doesn't you must change your strategy in order to take maximal advantage.
- D. You should only play a dominant strategy if you are the only player who has one.

6. What does the "Winner's Curse" in auctions refer to?

- A. In many auctions there are hidden costs and fees for the winner.
- B. The winner of an auction frequently does poorly in subsequent auctions.
- C. Many auctions are set up in a way as to encourage excessively high bids.
- D. Sometimes the fact that you win the auction means that you overvalued the item.

7. What game is this?

- A. Chicken
- B. Battle of the Sexes
- C. Prisoner's Dilemma
- D. Coordination Game

		Player 2	
		a	b
Player 1	a	12, 12	0, 20
	b	20, 0	3, 3

8. Which of the following behaviours is not rational, in the game-theoretic sense?

- A. Playing a game of pure chance.
- B. Accepting a bet in which the chances of losing are higher than the chances of winning.
- C. Playing a game against an opponent whose behaviour is erratic and unpredictable.
- D. Playing a dominated strategy in a simultaneous game.

9. In this 2x2 game, which of the following is true?

- A. Neither player has a dominant strategy.
- B. Player 1 has a dominant strategy, player 2 doesn't.
- C. Player 2 has a dominant strategy, player 1 doesn't.
- D. Both players have a dominant strategy.

		Player 2	
		a	b
Player 1	a	0, 4	-3, 5
	b	2, 2	1, 3

10. Consider the following variation on the 21 flags game: each turn, players take 1,2,3, or 4 flags. The player to take the last flag loses. If there are 16 flags left, how many should you take in order to win?

- A. 1
- B. 2
- C. 3
- D. 4
- E. Doesn't matter, this is a lost position.

11. Which of the following statements about Nash Equilibria in a 2-player game is correct?
- A. In a Nash Equilibrium, both players play a dominant strategy.
  - B. In a Nash Equilibrium, no player can improve his/her payoff by unilaterally changing strategies.
  - C. In a Nash Equilibrium, both players receive the highest payoff in the game.
  - D. If there is more than one Nash Equilibrium in a game, then all such equilibria must result in the same payoffs for both players.
12. What is the expected value of the following bet? We flip a fair coin twice. If it comes heads at least one time, I pay you \$4. Otherwise you pay me \$8.
- A. \$4
  - B. \$1
  - C. -\$1
  - D. -\$4
  - E. None of the above.
- $$P(\geq 1 \text{ Heads}) = \frac{3}{4}$$

$$EV = \frac{3}{4} \cdot \$4 + \frac{1}{4} \cdot (-\$8) = -\$1$$
13. Roulette (European). We bet \$100 on red and \$200 on even (at the same time). Which is the most accurate statement about your chances of winning at least one of the bets and the expected value of the bet?
- A. The chances of winning are below 50%, and the EV is negative.
  - B. The chances of winning are above 50%, and the EV is negative.
  - C. The chances of winning are below 50%, and the EV is positive.
  - D. The chances of winning are above 50%, and the EV is positive.
14. Suppose we roll two fair six-sided dice. What is the probability that the total is 4 or 5?
- A.  $\frac{2}{36}$   $(1,3), (1,4), (2,2), (2,3), (3,1), (3,2), (4,1)$
  - B.  $\frac{4}{36}$
  - C.  $\frac{6}{36}$
  - D.  $\frac{7}{36}$
  - E. None of the above.
15. The conclusion drawn from the St Petersburg Paradox is that we need to consider expected utility instead of expected monetary payoffs. Why?
- A. Because the St Petersburg bet demonstrates that most people are not willing to take any risks with their money.
  - B. Because the the expected monetary payoff of the bet is not proportional to the subjective value people attach to the bet.
  - C. Because the St Petersburg bet demonstrates that some people do not understand how to calculate expected payoffs.
  - D. Because the St Petersburg bet demonstrates that some bets are inherently unfair and cannot be made appealing regardless of the payoffs.

Part 2: Long Answer Questions: choose 2 out of 3 questions.

Write your answers in the space below the questions. Each question is worth 5 points. You get points for clear, structured and relevant motivation of your answers.

**Question 1.** Consider the following game matrix:

(a) Is (b,q) a Nash Equilibrium? Why (not)?

(b) Find all Nash Equilibria.

		Player 2		
		p	q	r
Player 1	a	-1 2	-2 3	*0 2
	b	1 2	*3 2	2 1
	c	1* 3	-1 0	0 1

(a) No, because player 1 would regret not choosing a

(b) Use best responses.  
(c,p) and (a,r)

**Question 2.** Explain in no more than 200 words what the difference between normative and behavioural game theory is. Include an example in your explanation.

See lecture notes

**Question 3.** Hold'em. Player 1 has  $4♥6♥$ , Player 2 has  $A♣A♥$ , and the board on the turn is  $3♥5♥Q♣A♦$ .

- (a) Determine the probability that the river card gives Player 1 a winning hand.
- (b) Suppose that the pot on the turn is \$500, and that Player 2 bets his last \$400. Should Player 1 call? (Assume Player 1 also has \$400 left, and that there are no considerations except for making the mathematically correct play.)

Winning cards:

$$(a) \quad \left. \begin{array}{l} 2h \ 7h \ 8h \ 9h \ Th \ Jh \ Kh \\ 2s \ 2d \ 2c \\ 7s \ 7d \ 7c \end{array} \right\} 13 \text{ outs}$$

$$52 - 8 = 44 \text{ unseen cards } \Rightarrow$$

$$P(\text{win}) = \frac{13}{44}$$

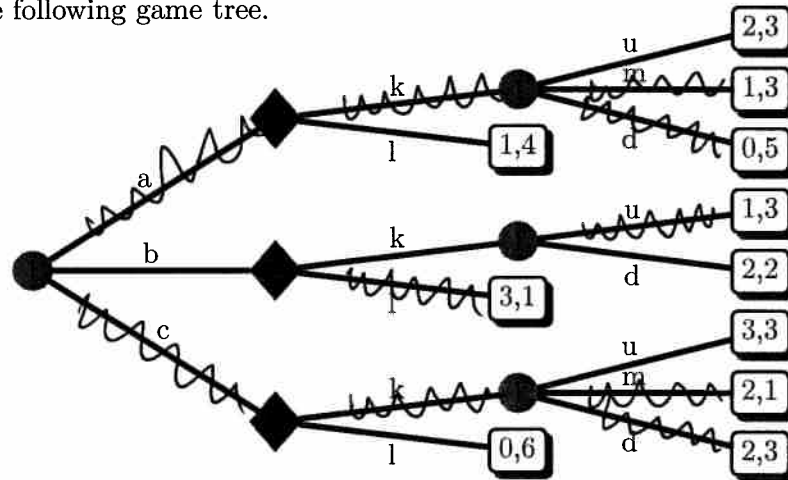
$$(1) \quad EV(\text{call}) = \frac{13}{44} \cdot \$900 + \frac{31}{44} \cdot (-\$400) < 0$$

So folding is better than calling.

### Part 3: Multiple Choice Questions

Choose 10 out of 20 multiple choice questions. Write your answers in the box on the front page of this booklet. Do not answer more than 10 questions; only the first 20 will be graded. Each question is worth 1 point.

1. Consider the following game tree.



What is the rollback equilibrium in this game?

- A. (a,k,u).
- B. (b,k,d).
- C. (c,k,u).
- D. (a,l).
- E. (b,l).
- F. This game does not have a rollback equilibrium.

2. In the game of the previous question, how many strategies does Player 1 have?

- A. 3
  - B. 6
  - C. 11
  - D. 54
  - E. None of the above
- $3 \times 3 \times 2 \times 3$

3. What is the difference between a type 1 and a type 2 Gambler's Fallacy?

- A. The type 1 fallacy denies that you can predict future outcomes based on past patterns, while the type 2 fallacy asserts you can use such patterns to predict the future.
- B. The type 2 fallacy centers around the idea that streaks of particular outcomes are likely to continue in the future, while according to the type 1 fallacy such streaks are likely to be offset with opposite outcomes.
- C. The type 1 fallacy asserts that certain random experiments are biased, while the type 2 fallacy asserts that even though these experiments are not biased, we can still predict future outcomes based on patterns.
- D. The type 1 fallacy denies the validity of the law of large numbers, while the type 2 fallacy asserts that the law is correct, but that it doesn't apply to random experiments such as flipping coins or throwing dice.

4. In class, we discussed the Effort Game, where each player had to choose an effort level, but where the payout depended on the smaller of the two effort levels chosen. What 2x2 game is the Effort Game most similar to?
- A. Prisoner's Dilemma
  - B. Chicken
  - C. Coordination Game
  - D. Battle of the Sexes
5. Roulette (European). Suppose I bet \$50 on even and \$50 on number 23. What is the expectation of this bet?
- A. -\$ 100
  - B. -\$  $\frac{50}{37}$
  - C. -\$  $\frac{17}{37}$
  - D. -\$  $\frac{100}{37}$
  - E. None of the above.
6. Texas Hold'em. You have a weak hand which can only beat a bluff, and on the river your opponent bets \$800 into the \$1000 pot. (You have just enough left to call.) You know that this either signifies a very strong hand or a complete bluff, and your best estimate is that this is a bluff about one in three times. What is the best reasoning?
- A. Call, because the size of the pot is larger than the size of the bet.
  - B. Call, because the expectation of the call is higher than the expectation of folding.
  - C. Fold, because the chance of winning the hand is too small.
  - D. Fold, because the expectation of the call is negative.
7. Which of the following statements about the chicken game is correct?
- A. Because neither player has a dominant strategy there are no Nash Equilibria.
  - B. No player has a dominant strategy, but there is exactly one Nash Equilibrium.
  - C. Both players have a dominant strategy, and there is exactly one Nash Equilibrium.
  - D. No player has a dominant strategy, and there are two Nash Equilibria.
8. The "Certainty Effect" refers to the fact that many subjects have a preference for choices which eliminate risk completely, as opposed to merely reducing it, even though the expectation of both choices is the same. How is this relevant for Expected Utility Theory?
- A. According to Expected Utility Theory, rational individuals are indifferent between choices which offer the same EV, and risk doesn't matter. Hence the certainty effect shows that many subjects don't follow the theory.
  - B. According to Expected Utility Theory, rational individuals seek to minimize risk while maximizing payoffs, so the certainty effect corroborates the theory.
  - C. According to Expected Utility Theory, rational individuals try to maximize their utility. If avoiding risk is a certain individual preference, then that is not necessarily irrational, and hence the certainty effect does not constitute evidence for or against the theory.

9. What are the paradoxes in rationality meant to demonstrate?
- A. That Expected Utility Theory is too complicated for most people to understand and apply.
  - B. That most people do not believe mathematics can be used to make correct decisions, even though Expected Utility Theory can be mathematically proven to be correct.
  - C. That most people can make rational decisions without using Expected Utility Theory.
  - D. That most people deviate from the principles of Expected Utility Theory because they make mistakes or partly base their decisions on other factors such as risk.
10. What is prospect theory?
- A. A version of game theory which takes possible future actions by players into account.
  - B. A psychological theory about future actions of individuals based on their past behavior.
  - C. A theory of preferences and decision making which focuses on the fact that humans are sensitive to perceived gains and losses in wealth.
  - D. A theory of rationality based on the idea that humans anticipate potential regret when making decisions.
11. There are 1326 different hold'em starting hands. A *connector* is a hand consisting of two cards of subsequent rank. For example,  $6\heartsuit 7\heartsuit$  is a connector, as is  $Q\clubsuit K\clubsuit$ . How many of the 1326 starting hands are connectors? (Don't count A2 hands as connectors.)
- A. 48
  - B. 52
  - C. 192
  - D. 208
  - E. None of the above.
- 16 combos for each connector type  
12 types
12. Which of the following statements about the history of probability theory is most accurate?
- A. Probability theory has its origins in Ancient Greece, where mathematicians and philosophers were eager to develop a theory of randomness and probability.
  - B. Galileo was the first scientist who tried to develop a mathematical approach to probability.
  - C. The Roman statesman and philosopher Cicero applied mathematics to better understand uncertainty and randomness.
  - D. Cardano's Law was the first systematic approach to reasoning about events with uncertain outcomes.
13. What was the main reason for the creation of the RAND corporation?
- A. To provide an attractive climate for scientists who would otherwise likely move to Europe.
  - B. To encourage scientific research which would be beneficial to policy makers and military leaders.
  - C. To educate the general public about strategic issues.
  - D. To support scientists who refused to support war-related research.

14. In class, we played the “Public Investment Game”, where you had to decide how much to invest (0-10 chips) and where the payouts depended only on the overall investment. Which of the following best describes the nature of that game?

- A. This is a multi-person prisoner’s dilemma
- B. This is a multi-person coordination game
- C. This is a multi-person battle of the sexes game
- D. This is a multi-person chicken game

15. What conclusion can be correctly drawn from the Coordination game?

- A. That games may have more than one Nash Equilibrium, but that it can be difficult to decide which one to aim for.
- B. That in some games rationality doesn’t help you in finding the equilibrium strategy.
- C. That some games don’t have Nash Equilibria.
- D. That dominant strategies don’t always help in achieving the best outcome.

16. Mr. F is an extremely tight hold’em player. She only plays JJ, QQ, KK, AA, AQ and AK (suited or unsuited). What percentage of hands does this correspond to? (Answers are rounded off; choose the number which is closest to the actual percentage.)

- A. 1%
- B. 4%
- C. 8%
- D. 12%
- E. 16%

$$6 + 6 + 6 + 6 + 16 + 16 = 54 \text{ combos}$$
$$\frac{54}{1326} \approx 4\%$$

17. Research has shown that most human subjects are not adept at identifying random sequences. For example, most people believe that the sequence HTHTTHTHHT is more likely to be random than TTTHTTHHHT, which it is not. How can this be explained?

- (A) There are no good mathematical tests for randomness, and hence humans do not have any appropriate tools for learning how to recognize it.
- (B) We overestimate how many alternations can be expected in a random sequence.
- (C) We tend to base our judgment on irrelevant criteria, such as the number of runs in the sequence.
- (D) We can only distinguish random sequences from non-random ones if we have sufficient data; we are bad at drawing conclusions from short sequences.

18. What is the main lesson from the “Truel” as studied in class?

- A. That in multi-person games it is sometimes better for the weakest player to have the strong competitors eliminate each other.
- B. That in games with uncertainty you can never hope to find a strategy which is always successful.
- C. That some games are inherently unfair and are stacked in favour of the strongest player(s).
- D. That in multi-player games, the person to move first has a big advantage.

19. Which of the following examples is a good argument that just looking at money gained or lost is not enough for determining someone's payoffs in a game?
- A. In last night's poker game, Mr. F made several bad decisions and lost \$100.
  - B. In last night's poker game, Mr. F was outclassed by his opponents but got lucky several times and ended up winning \$100.
  - C. The first time Mr. F went to the casino he was thrilled to win \$100, but on subsequent visits he noticed that he needed to win larger and larger amounts in order to get the same thrill.
  - D. The first time Mr. F went to the casino he played roulette, not knowing that the odds in this game are unfavourable.
20. In a scene of the movie "A Beautiful Mind", John Nash and his friends are in a bar and are eyeing a group of girls. The blonde is the most desirable, but Nash reasons that if they all try to woo her, none of them will succeed; moreover, they will then also fail with her friends, because none of them likes to be second choice. Therefore, he concludes, the best solution is when all of them ignore the blonde and each approaches one of her friends. Is this strategy a Nash Equilibrium?
- A. Yes, because all players (guys) end up with a satisfactory outcome (assuming that they each manage to seduce one of the non-blondes).
  - B. Yes, because the alternatives don't give a better outcome; thus even though this scenario isn't optimal, it is the best one can hope for given the circumstances.
  - C. No, because nobody achieves the best possible outcome (being with the blonde)
  - D. No, because each guy would have an incentive to cheat and go for the blonde while the others go for her friends.

**BONUS QUESTION [1 point]** Which actor plays the role of The Good in "The Good, The Bad and The Ugly?"