Last Name ______________ First Name ______________ Student Number ______________

**Note: Total mark: 40. Closed book. Non-programmable calculators are allowed.**

**Question** 1. [8 Points] Multiple Choice Questions( **Clearly enter your answers in the table provided below. Only the answers written in this table will be graded**).

(1) (2 points) Which of the following are subspaces?

$$U = \{(s^2,\ s+t,\ 2s,\ t) \mid s,t \in \mathbb{R}\}$$
$$V = \{(s,\ t,\ 0,\ r) \mid s,t,r \in \mathbb{R}\}$$
$$W = \{(r, 5,\ s,\ t) \mid s,t,r \in \mathbb{R}\}$$
$$P = \{(t,\ 0,\ s) \mid s,t \in \mathbb{R}, t > 0\}$$

A). U **B). V** C). W D). P

(2) (2 points) Which of the following is NOT a linear transformation?

A). $T(x_1, x_2, x_3) = (x_1 + x_2,\ x_2,\ x_2 - x_3)$

B). $T(x_1, x_2, x_3) = (x_1 + x_2,\ x_2 + x_3,\ x_2 - x_3)$

**C).** $T(x_1, x_2, x_3) = (x_1 + x_2 + 2,\ x_1,\ x_3)$

D). $T(x_1, x_2) = (x_1 + x_2,\ 0)$

(3) (2 points) Let

$$A = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 8 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 10 \\ 0 \\ 9 \end{bmatrix} \right\} \qquad B = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$C = \left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix} \right\} \qquad D = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -6 \end{bmatrix} \right\}$$

Which sets are bases for $\mathbb{R}^3$

A). $A, D$ only B). $B, C$ only **C).** $B, D$ **only** D). $A, C$ only

(4) (3 points) Which of the following sets of vectors spans $\mathbb{R}^3$?

$$A = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \qquad B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

$$C = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\} \qquad D = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} \right\}$$

A). $A, B, C$ only

B). $A, C$ only

**C).** $B, C$ **only**

D). $B, D$ only

**Solution:** (1) B (2)C (3) C (4) C

 Continued on next page. . .

Last Name ______________ First Name ______________ Student Number ______________

**Note: Total mark: 40. Closed book. Non-programmable calculators are allowed.**

**Answer Table for the Multiple-Choice Questions:**

| Question | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Answer | | | | |

**Question** 2. [12 Points] Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a transformation defining by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -x_1 + x_2 - 2x_3 \\ 2x_1 - x_2 - x_3 \\ -x_1 + 3x_3 \\ x_1 - x_2 + 2x_3 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

1) [ 2 points] Find $n$ and $m$.

**Solution:**

$$n = 3, \quad m = 4$$

2) [ 2 points] Find $T(\mathbf{v})$, the image of $\mathbf{v}$ under the transformation $T$.

**Solution:**

$$T(\mathbf{v}) = \begin{bmatrix} -3 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

3) [ 4 points]Is this a linear transformation? If it is, find the matrix that represents the linear transformation T.

**Solution:** Yes, this is a linear transformation.[1 mark] The matrix is[3 marks]

$$\begin{pmatrix} -1 & 1 & -2 \\ 2 & -1 & -1 \\ -1 & 0 & 3 \\ 1 & -1 & 2 \end{pmatrix}$$

4) [ 4 points]Determine whether or not $\mathbf{u} = (3, 5, 1)$ is in the kernel of $T$.

**Solution:** Since $T(\mathbf{u}) = \mathbf{0}$, $\mathbf{u}$ is in the kernel of $T$.

 Continued on next page. . .

**Question** 3. [20 Points] Let

$$A = \begin{bmatrix} 1 & 2 & 2 & -1 & 1 \\ 3 & 6 & 5 & -2 & 4 \\ 1 & 2 & -1 & 2 & 5 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} -1 \\ -1 \\ -3 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

1) (12 points) Find a basis and the dimension for the column space of $A$ and solution space of $A\mathbf{x} = \mathbf{0}$.
2) (4 points) Determine whether $\mathbf{b}$ is in the solution space of $A\mathbf{x} = \mathbf{0}$.
3) (4 points) Determine whether $\mathbf{u}$ is in the column space of $A$.

**Solution:**

1) The reduced echelon form of A is[5 marks]

$$\begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

So a basis for the column space is (column 1 3, 5 of matrix A)[3 marks]

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} \right\}$$

and the dimension of Col(A) is 3.[1 mark]

And, a basis for solution space of $A\mathbf{x} = \mathbf{0}$ is

$$\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

[2 marks] and the dimension of solution space of $A\mathbf{x} = \mathbf{0}$ is 2[1 mark].

2) Since $A\mathbf{b} = \mathbf{0}$ $\mathbf{b}$ is in the solution space of $A\mathbf{x} = \mathbf{0}$.[4 marks]
3) Yes, since $A\mathbf{x} = \mathbf{u}$ is consistent or the dimension of Col(A) is 3.[4 marks]