Last Name ______________ First Name ______________ Student Number ______________

Note: Total mark: 40. Closed book. Non-programmable calculators are allowed.

Question 1. [8 Points] Multiple Choice Questions(**Clearly enter your answers in the table provided below. Only the answers written in this table will be graded**).

(1) (2 points) Let A be a 5×5 matrix with determinant 2. Let B be the matrix formed by subtracting three copies of the fourth row of A from the first row. What is $\det(B)$?

A). 3

B). 2

C). -3

D). Insufficient information to solve the question

(2) (2 points) Which of the following statements is NOT equivalent to the other three for a square matrix A?

A). A is not invertible.

B). The reduced row-echelon form of A is the identity matrix, I.

C). The homogeneous matrix equation $AX = \mathbf{0}$ has only the trivial solution.

D). $\det(A^T) \neq 0$.

(3) (2 points) Let

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 3 & 5 & 5 & 20 & 7 \end{bmatrix}.$$

What is $\det(A)$?

A). 3 **B). -3** C). 0 D). 1

(4) (2 points) Which of the following sets of vectors are linearly dependent?

$$A = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 400 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -9 \end{bmatrix} \right\} \quad B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 10 \\ 0 \\ 9 \end{bmatrix} \right\}$$

$$C = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -100 \\ -100 \\ -100 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} \right\} \quad D = \left\{ \begin{bmatrix} 100 \\ 211 \\ 2 \end{bmatrix}, \begin{bmatrix} 102 \\ -200 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

A). A, C, D only

B). B, C, D **only**

C). A, B, D only

D). A, B, C only

Solution: (1) B (2)A (3) B (4) B

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Last Name ______________ First Name ______________ Student Number ______________

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Answer Table for the Multiple-Choice Questions:

Question	(1)	(2)	(3)	(4)
Answer				

Question 2. [10 Points] If A, B and C are three 3×3 matrices and $|A| = 2, |B| = 16, |C| = 0$.

(1) (4 points) Find $|2A^2B^{-1}|$.

(2) (2 points) Find the row rank of A.

(3) (2 points) Find $|ABC|$.

(4) (2 points) Is matrix BC invertible?

Solution:

1)

$$|2A^2B^{-1}| = 2^3|A|^2\frac{1}{|B|} = 2$$

2) The row rank of A is 3.

3) $|ABC| = 0$.

4) No, since $|BC| = 0$.

Question 3. [10 Points]Find the determinants of the following matrix using any method?

$$A = \begin{pmatrix} -2 & 3 & 0 & 4 \\ 0 & -2 & 0 & -5 \\ 1 & 2 & 1 & 2 \\ 3 & 3 & 3 & 1 \end{pmatrix}$$

Solution:

$$\det(A) = 10$$

(each right row or column operation worth 1 mark, maximum 6 marks)

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Question 4. [12 Points] Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 3 \\ -10 \\ 9 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -2 \\ 8 \\ -12 \end{bmatrix}, \ \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

(a) [6 points] Determine if the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent or dependent.

(b) [6 points] Determine if it is possible to express the vector $\mathbf{u}$ as a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$. If it is possible, write $\mathbf{u}$ explicitly as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

Solution: Let

$$A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & -10 & 8 \\ 2 & 9 & -12 \end{bmatrix}.$$

where $\mathbf{v}_i (i = 1, 2, 3)$ is the ith column of matrix A. (1) Solution 1: The reduced echelon form of matrix A is a identity matrix, so every column has a leading 1, therefore, the vectors are linearly independent.

Solution 2: $|A| = -22 \neq 0$, so the column vectors are linearly independent.

Solution 3: $A\,\mathbf{x} = 0$ has only trivial solution, so the column vectors are linearly independent. (each right row operation worth 1 mark, maximum 4 marks) (2) By (1), $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent, so the linear system $A\mathbf{x} = \mathbf{u}$ is consistent for any $\mathbf{u} \in \mathbb{R}^3$. So, it is possible to express the vector $\mathbf{u}$ as a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$. And

$$\mathbf{u} = 24\,\mathbf{v}_1 - 10\,\mathbf{v}_2 - \frac{7}{2}\,\mathbf{v}_3\,.$$

(each right row operation worth 1 mark, maximum 4 marks)