Last Name _____ First Name _____ Student Number _____

Note: Total mark: 40. Closed book. Non-programmable calculators are allowed.

Question 1. [6 Points] Multiple Choice Questions(**Clearly enter your answers in the table provided below. Only the answers written in this table will be graded**).

(1) (2 points) A homogeneous system of 3 linear equations in 4 variables has

A). only the trivial solution .

B). infinitely many solutions with at least 1 parameter.

C). infinitely many solutions with 2 parameters

D). infinitely many solutions with 3 parameters.

(2) (2 points) Let

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

and

$$Q_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 1/4 & 0 \\ 0 & 0 & 1/5 & 0 & 0 \\ 0 & 1/6 & 0 & 0 & 0 \\ 1/7 & 0 & 0 & 0 & 0 \end{bmatrix} \quad Q_2 = \begin{bmatrix} 1/7 & 0 & 0 & 0 & 0 \\ 0 & 1/6 & 0 & 0 & 0 \\ 0 & 0 & 1/5 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 1/3 \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1/7 \\ 0 & 0 & 0 & 1/6 & 0 \\ 0 & 0 & 1/5 & 0 & 0 \\ 0 & 1/4 & 0 & 0 & 0 \\ 1/3 & 0 & 0 & 0 & 0 \end{bmatrix} \quad Q_4 = \begin{bmatrix} 1/3 & 0 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 & 0 \\ 0 & 0 & 1/5 & 0 & 0 \\ 0 & 0 & 0 & 1/6 & 0 \\ 0 & 0 & 0 & 0 & 1/7 \end{bmatrix}$$

What is P^{-1}?

A). Q_1 B). Q_2 **C).** Q_3 D). Q_4

(3) (2 points) Consider two matrices A, B, C. If A is a 3×2 matrix, C is a 4×3 matrix and ABC can be computed, what must be the dimensions of B

A). 3×3 B). 3×2 C). 2×3 **D).** 2×4

Solution: (1) B (2)C (3) D

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Answer Table for the Multiple-Choice Questions:

Question	(1)	(2)	(3)
Answer			

Question 2. [13 Points] Let

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 2 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

(1) (2 points) Find A^{-1}.

(2) (3 points) Find $B - C$.

(3) (2 points) Find A^2.

(4) (3 points) Find AB

(5) (3 points) Find B^T.

Solution:

$$A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$B - C = \begin{pmatrix} -1 & -1 & -1 \\ 2 & 2 & 2 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 0 & 2 \\ 0 & 2 \\ 0 & 2 \end{pmatrix}$$

Question 3. [9 Points]Let

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 5 & -1 \\ -1 & -4 & 0 \end{pmatrix}$$

1) [8 points] Use row operations to find the inverse of A or to determine that none exists.

Solution:

$$A^{-1} = \begin{pmatrix} -4 & 4 & 3 \\ 1 & -1 & -1 \\ -3 & 2 & 1 \end{pmatrix}$$

(each right row operation worth 1 mark, maximum 6 marks, each wrong number in the inverse matrix worth -0.5 mark, maximum -3 marks.)

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2) [1 point] Find the solution vector X for following system(A is the coefficient matrix):

$$\begin{aligned} x_1 \quad +2x_2 \quad -x_3 \quad &= \quad 0 \\ 2x_1 \quad +5x_2 \quad -x_3 \quad &= \quad 0 \\ -x_1 \quad -4x_2 \quad\quad &= \quad 1 \end{aligned}$$

Solution:

$$X = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

Question 4. [12 Points] Given the following linear system:

$$\begin{aligned} x_1 - \quad 2x_2 \quad\quad + \quad x_4 &= \quad 2 \\ -2x_1 + \quad 4x_2 + \quad x_3 + \quad x_4 &= \quad -5 \\ 3x_1 - \quad 6x_2 - \quad x_3 \quad\quad &= \quad 7 \\ -3x_1 + \quad 6x_2 + \quad 2x_3 + \quad 3x_4 &= \quad -8 \end{aligned}$$

Find a parametric form and a vector form of the solution to the system.

Solution: Augmented matrix:[2 marks]

$$\left(\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 2 \\ -2 & 4 & 1 & 1 & -5 \\ 3 & -6 & -1 & 0 & 7 \\ -3 & 6 & 2 & 3 & -8 \end{array}\right).$$

the row reduced echelon form is [5 marks]

$$\left(\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

(0.5 mark for each correct row operation maximum 3 marks; 2 marks for correct RREF). The parametric form of the solution is

$$\begin{aligned} x_1 &= 2 + 2t - s, \\ x_2 &= t, \\ x_3 &= -1 - 3s, \\ x_4 &= s, \quad t, s \in \mathbb{R} \end{aligned}$$

(0.5 mark for each correct answer based on the RREF obtained in (b)).

The vector form of the solution is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} t + \begin{pmatrix} -1 \\ 0 \\ -3 \\ 1 \end{pmatrix} s, \quad t, s \in \mathbb{R}$$

(1 mark for each correct vector.)