

Last Name _____ First Name _____ Student Number _____

Question 1. [4 Points] Multiple Choice Questions (Please **CIRCLE** your answer)

(1) (2 points) Which of the following matrices are in reduced row echelon form (RREF)?

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

A). A, B, C only

B). A, C only

C). B, C only

D). B, D only

(2) (2 points) Which pair of planes are parallel and distinct?

A). $2x - y + 9z = 7$ and $x + 2y - 3z = 1$.

B). $x - y + 9z = 7$ and $2x + 2y - 18z = 13$.

C). $2x + 5y - 5z = 3$ and $-6x - 30y + 30z = -18$.

D). $2x + 5y - 5z = 3$ and $-6x - 30y + 30z = -19$.

Solution: (1)C (2) D

Question 2. [6 Points] Given two vectors $\mathbf{u} = (7, 4, -3, 6)$, $\mathbf{v} = (-2, -4, 6, 5)$,

(1) (2 points) Find $3\mathbf{u} - 2\mathbf{v}$.

(2) (2 points) Find the magnitude of $4\mathbf{v}$.

(3) (2 points) Find $\mathbf{u} \cdot \mathbf{v}$.

Solution:

(a) $3\mathbf{u} - 2\mathbf{v} = (25, 20, -21, 8)$

(b) $\|4\mathbf{v}\| = 4\|\mathbf{v}\| = 4\sqrt{2^2 + 4^2 + 6^2 + 5^2} = 36$

(c) $\mathbf{u} \cdot \mathbf{v} = -14 - 16 - 18 + 30 = -18$

Question 3. [4 Points] The augmented matrix of a linear system is given by

$$\left(\begin{array}{ccc|c} 1 & 3 & -9 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & -3 & -6 & k \end{array} \right)$$

Find the value(s) of k such that the linear system is CONSISTENT?

Solution: $k = -3$

Question 4. [6 Points] Find an equation in standard form for the plane that passes through the points $A(3, 4, 3)$, $B(1, -1, 1)$ and $C(1, 2, 3)$.

Solution: Since $\overrightarrow{AB} = (-2, -5, -2)$, $\overrightarrow{AC} = (-2, -2, 0)$, so

$$\overrightarrow{AB} \times \overrightarrow{AC} = \left(\begin{array}{c|c|c} \left| \begin{array}{cc} -5 & -2 \\ -2 & 0 \end{array} \right| & \left| \begin{array}{cc} -2 & -2 \\ 0 & -2 \end{array} \right| & \left| \begin{array}{cc} -2 & -5 \\ -2 & -2 \end{array} \right| \end{array} \right) = (-4, 4, -6) [4 \text{ marks}]$$

so the Cartesian equation is

$$(-4, 4, -6) \bullet (x - 3, y - 4, z - 3) = 0 [1 \text{ mark}]$$

that is

$$-4x + 4y - 6z = -14$$

or

$$2x - 2y + 3z = 7 [1 \text{ mark}]$$

Question 5. [20 Points] Given the following linear system:

$$x_1 - 3x_2 + 2x_3 = 4$$

$$5x_1 - 13x_2 + 8x_3 = 14$$

$$3x_1 - 9x_2 + 8x_3 = 14$$

- [3 points] Write the augmented matrix of the linear system.
- [7 points] Convert the augmented matrix into row reduced echelon form (RREF).
- [2 points] Find the row rank of the coefficient matrix and the row rank of the augmented matrix.
- [2 points] Is the given system consistent? Why?
- [6 points] Find the solution to the system.

Solution:

(a) Augmented matrix [3 points]:

$$\left(\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 5 & -13 & 8 & 14 \\ 3 & -9 & 8 & 14 \end{array} \right).$$

(b) the row reduced echelon form is

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array} \right).$$

(-1 mark for each incorrect row operation, maximum $5 \times (-1)$; -2 marks for incorrect RREF).

- both the row rank of the coefficient matrix and the row rank of the augmented matrix are 3.
- The given system is consistent since the row rank of the coefficient matrix is equal to the row rank of the augmented matrix (2=2).

(e)

$$x_1 = -4, x_2 = -2, x_3 = 1$$

(1.5 mark for each correct answer based on the RREF obtained in (b).)